

Learning Learning Curves

Ron Rivest

MIT

rivest@mit.edu

November 3, 2023

Happy Birthday, Rob!

Overview

- 1 Goal: to give you a nice “open problem”
- 2 Learning Curves (aka “experience curves”)
- 3 Learning one learning curve
- 4 Learning multiple learning curves (multi-armed bandit formulation)
- 5 Open problem

Learning Curves

- For most things, producing twice as many units yields a per-unit cost reduction of 10-30%. (See “Wright’s Law” in *The Big Fix: Seven Practical Steps to Save Our Planet*, Harvey & Gillis, 2022)

Learning Curves

- For most things, producing twice as many units yields a per-unit cost reduction of 10-30%. (See “Wright’s Law” in *The Big Fix: Seven Practical Steps to Save Our Planet*, Harvey & Gillis, 2022)
- Let X = number of units produced so far.
Let $C(X)$ = cost of producing X -th unit

Learning Curves

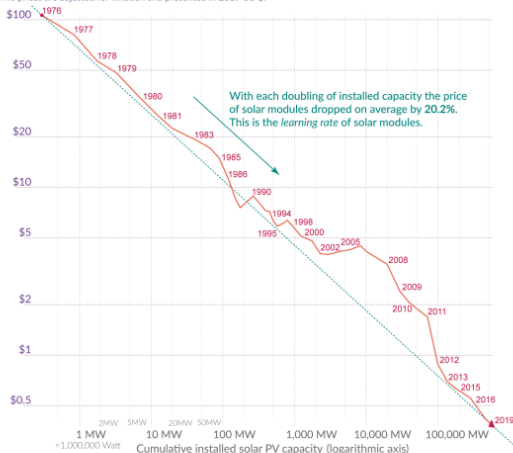
- For most things, producing twice as many units yields a per-unit cost reduction of 10-30%. (See “Wright’s Law” in *The Big Fix: Seven Practical Steps to Save Our Planet*, Harvey & Gillis, 2022)
- Let X = number of units produced so far.
Let $C(X)$ = cost of producing X -th unit
- $C(2X) = C(X) \cdot (1 - \lambda)$
where λ = learning rate (e.g. $\lambda = 0.20$)

PV Solar Learning Curve

The price of solar modules declined by 99.6% since 1976

Our World in Data

Price per Watt of solar photovoltaics (PV) modules (logarithmic axis)
The prices are adjusted for inflation and presented in 2019 US-\$.
\$100



Data: Lafond et al. (2017) and IRENA Database; the reported learning rate is an average over several studies reported by de La Tour et al (2013) in Energy. The rate has remained very similar since then.
OurWorldInData.org - Research and data to make progress against the world's largest problems.

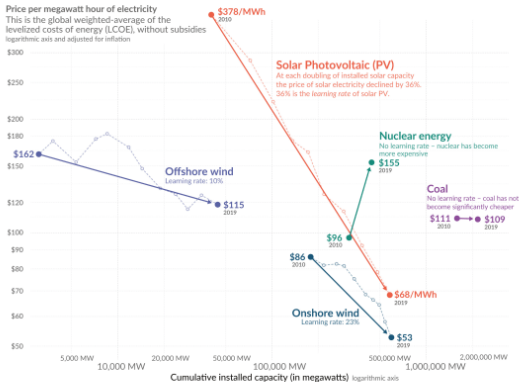
Licensed under CC-BY by the author Max Roser

Energy Learning Curves

Our World
in Data

Electricity from renewables became cheaper as we increased capacity – electricity from nuclear and coal did not

Price per megawatt hour of electricity
This is the global weighted-average of the levelized costs of energy (LCOE), without subsidies
logarithmic axis and adjusted for inflation



Source: IRENA 2020 for all data on renewable sources; Lazard for the price of electricity from nuclear and coal - IAEA for nuclear capacity and Global Energy Monitor for coal capacity. Gas is not shown because the price between gas peaker and combined cycles differs significantly, and global data on the capacity of each of these sources is not available. The price of electricity from gas has fallen over this decade, but over the longer run it is not following a learning curve.

OurWorldInData.org – Research and data to make progress against the world's largest problems.

Licensed under CC-BY
by the author Max Roser

Learning a learning curve

- Logarithms: $x = \ln(X)$; $c = \ln(C)$.

Learning a learning curve

- Logarithms: $x = \ln(X)$; $c = \ln(C)$.
- Given a number n of (x, c) pairs, where

$$c = \alpha + \beta x + \epsilon$$

infer α , β , and σ^2 , where $\epsilon \sim \mathcal{N}(0, \sigma^2)$ assumed.
Assume $n \geq 2$. Typically $\beta < 0$.

Learning a learning curve

- Logarithms: $x = \ln(X)$; $c = \ln(C)$.
- Given a number n of (x, c) pairs, where

$$c = \alpha + \beta x + \epsilon$$

infer α , β , and σ^2 , where $\epsilon \sim \mathcal{N}(0, \sigma^2)$ assumed.

Assume $n \geq 2$. Typically $\beta < 0$.

- This is standard *simple linear regression* problem. (Use least-squares; details omitted.)

Learning multiple learning curves

- Like multi-armed bandit problem: which technology to use at time t ($t = 1, 2, \dots, T$)?

Learning multiple learning curves

- Like multi-armed bandit problem: which technology to use at time t ($t = 1, 2, \dots, T$)?
- Classic “exploration / exploitation” tradeoff.

Learning multiple learning curves

- Like multi-armed bandit problem: which technology to use at time t ($t = 1, 2, \dots, T$)?
- Classic “exploration / exploitation” tradeoff.
- At time t ,
 - “**explore**” with probability $p(t)$ (use a least-used technology)
 - “**exploit**” with probability $1 - p(t)$ (use “best” technology).

Learning multiple learning curves

- Like multi-armed bandit problem: which technology to use at time t ($t = 1, 2, \dots, T$)?
- Classic “exploration / exploitation” tradeoff.
- At time t ,
 - “**explore**” with probability $p(t)$ (use a least-used technology)
 - “**exploit**” with probability $1 - p(t)$ (use “best” technology).
- Want $\sum_1^T p(t) \rightarrow \infty$ with T (learn all technologies well)
Want $\sum_1^T p(t) = o(T)$ (almost all time spent on exploitation).

Learning multiple learning curves

- Like multi-armed bandit problem: which technology to use at time t ($t = 1, 2, \dots, T$)?
- Classic “exploration / exploitation” tradeoff.
- At time t ,
 - “**explore**” with probability $p(t)$ (use a least-used technology)
 - “**exploit**” with probability $1 - p(t)$ (use “best” technology).
- Want $\sum_1^T p(t) \rightarrow \infty$ with T (learn all technologies well)
Want $\sum_1^T p(t) = o(T)$ (almost all time spent on exploitation).
- For example: $p(t) = 1/\sqrt{t}$.

Learning multiple learning curves

- At each step, choose technology k (for some k):
 - increase x_k by δ_k .
That is, scale up capacity of technology k by e^{δ_k} .
(E.g. installed solar doubles every five years, so $\delta \simeq \ln(2)/5$.)

Learning multiple learning curves

- At each step, choose technology k (for some k):
 - increase x_k by δ_k .
That is, scale up capacity of technology k by e^{δ_k} .
(E.g. installed solar doubles every five years, so $\delta \simeq \ln(2)/5$.)
 - Obtain actual cost c_k .

Learning multiple learning curves

- At each step, choose technology k (for some k):
 - increase x_k by δ_k .
That is, scale up capacity of technology k by e^{δ_k} .
(E.g. installed solar doubles every five years, so $\delta \simeq \ln(2)/5$.)
 - Obtain actual cost c_k .
 - Infer new parameters α_k , β_k , and σ_k^2 using least squares on sample of size n_k , where n_k is number of times technology k has been chosen.

Choosing best technology k

- How to choose best technology k to use at time t ?
- Estimate $\hat{c}_k(T)$ = estimated log cost of energy at time T using only technology k from now (t) on:

$$\hat{c}_k(T) = \alpha_k + \beta_k(x_k + (T - t)\delta_k)$$

- Do this whenever technology k is used.

Learning the best technology

LEARN(T, K):

for $k = 1, 2, \dots, K$: use technology k *twice*.

for $t = 2K + 1$ **to** T :

with probability $p(t)$: # Explore

Use technology k , where $k =$ a least-used technology.

else: # Exploit

for $k = 1, 2, \dots, K$, estimate $\hat{c}_k(T)$ using what's been learned so far, using least-squares to get $\hat{\alpha}_k, \hat{\beta}_k$:

$$\hat{c}_k(T) = \hat{\alpha}_k + \hat{\beta}_k(x_k + (T - t)\delta_k)$$

Use technology k , where k minimizes $\hat{c}_k(T)$

return $\min_k \hat{c}_k(T)$

Conjecture (Open Problem)

For all sets of K learning curves and all T , LEARN returns a result $\hat{c}_k(T)$ such that with high probability

$$\hat{c}_k(T) \text{ is "not much more than" } c'_{k_*}(T)$$

where k_ is the value of k with minimum expected value $c'_{k_*}(T)$ (that is, where k_* is always used).*

Thanks!
Happy Birthday, Rob!

References

- ① Ghemawat P., *Building Strategy on the Experience Curve*, HBR, 1986.
- ② Harvey H. and Gillis, J. *The Big Fix*. (Simon & Schuster; 2022).
- ③ Roser, M., *Why did renewables become so cheap so fast?*, 2020.
<https://ourworldindata.org/cheap-renewables-growth>
- ④ Way, R. et al., *Empirically grounded technology forecasts and the energy transition*, 2021, INET Oxford Working Paper No. 2021-01.