Learning Learning Curves

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Happy Birthday, Rob!

Overview

1. Goal: to give you a nice "open problem"
2. Learning Curves (aka "experience curves")
3. Learning one learning curve
4. Learning multiple learning curves (multi-armed bandit formulation)
5. Open problem

Learning Curves

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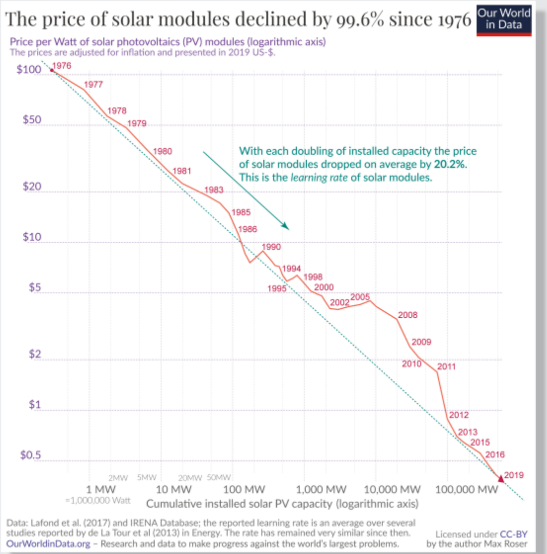
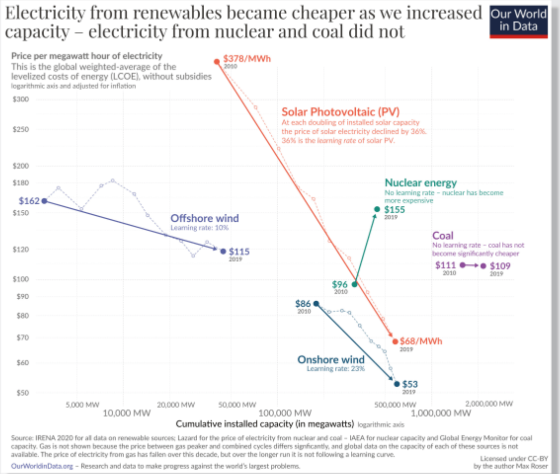
  Let $C(X)$ = cost of producing X-th unit
- $C(2X) = C(X) \cdot (1 - \lambda)$
  where λ = learning rate (e.g. $\lambda = 0.20$)

PV Solar Learning Curve



Energy Learning Curves



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- This is standard *simple linear regression* problem. (Use least-squares; details omitted.)

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- For example: $p(t) = 1/\sqrt{t}$.

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  - Obtain actual cost c_k.
  - Infer new parameters α_k, β_k, and σ_k^2 using least squares on sample of size n_k, where n_k is number of times technology k has been chosen.

Choosing best technology k

- How to choose best technology k to use at time t?
- Estimate $\hat{c}_k(T)$ = estimated log cost of energy at time T using only technology k from now (t) on:

$$\hat{c}_k(T) = \alpha_k + \beta_k(x_k + (T - t)\delta_k)$$

- Do this whenever technology k is used.

Learning the best technology

LEARN(T, K):

for $k = 1, 2, \ldots, K$: use technology k *twice.*

for $t = 2K + 1$ **to** T:

  with probability $p(t)$: # Explore

    Use technology k, where $k =$ a least-used technology.

  else: # Exploit

    for $k = 1, 2, \ldots, K$, estimate $\hat{c}_k(T)$ using what's been learned so far, using least-squares to get $\hat{\alpha}_k$, $\hat{\beta}_k$:

$$\hat{c}_k(T) = \hat{\alpha}_k + \hat{\beta}_k(x_k + (T - t)\delta_k)$$

    Use technology k, where k minimizes $\hat{c}_k(T)$

return $\min_k \hat{c}_k(T)$

Conjecture (Open Problem)

For all sets of K learning curves and all T, LEARN *returns a result $\hat{c}_k(T)$ such that with high probability*

$$\hat{c}_k(T) \textit{ is "not much more than" } c'_{k_*}(T)$$

where k_* is the value of k with minimum expected value $c'_{k_*}(T)$ (that is, where k_* is always used).

Thanks!

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References

1. Ghemawat P., *Building Strategy on the Experience Curve*, HBR, 1986.
2. Harvey H. and Gillis, J. *The Big Fix.* (Simon & Schuster; 2022).
3. Roser, M., *Why did renewables become so cheap so fast?*, 2020. `https://ourworldindata.org/cheap-renewables-growth`
4. Way, R. et al., *Empirically grounded technology forecasts and the energy transition*, 2021, INET Oxford Working Paper No. 2021-01.