Harmonic Broadcasting Is Optimal

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Abstract

Harmonic broadcasting was introduced by Juhn and Tseng as a way to reduce the bandwidth requirements required for video-on-demand broadcasting. In this paper, we note that harmonic broadcasting is actually a special case of the priority encoded transmission scheme introduced by Albanese et al. and prove—using an information theoretic argument—that it is impossible to achieve the design goals of harmonic broadcasting using a shorter encoding.

1 Introduction

One way to broadcast an m-minute movie in such a way that a viewer can start viewing the movie every m/k minutes is to simply allocate k channels and broadcast identical copies of the movie on each channel in such a way that there is one copy of the movie starting every m/k minutes. This requires a total of k channels. To reduce the bandwidth requirement, Juhn and Tseng [4] introduced the notion of *harmonic broadcasting*, a scheme where early parts of the movie are broadcasted more frequently than later parts. The harmonic broadcasting scheme can be viewed as follows: The movie is first divided into k equally sized segments $\langle M_1, M_2, \dots, M_k\rangle$. Each segment but the first is then divided into equally sized subsegments; the ith segment is divided into the i subsegments $\langle M_{i,0}, M_{i,1}, \dots, M_{i,i-1}\rangle$. An encoding $\langle E_1, E_2, \dots, E_k\rangle$ consisting of k equally sized blocks is then transmitted. The ith block in the encoding is constructed by concatenating

$$E_i = M_1 M_{2,i \bmod 2} M_{3,i \bmod 3} \cdots M_{k,i \bmod k}.$$

By the encoding procedure, the total size of the encoded move is $k\sum_{i=1}^{k} m/ki = mH_k$. The crucial property of the above construction is that the client can reconstruct M_i from any i consecutive blocks from the encoding.

Juhn and Tseng [4] claimed that harmonic broadcasting gives a maximum waiting time of m/k at the client end, but this is not the case. It was observed by Pâris, Carter and Long [5] that the client actually may not get all data it needs to reconstruct M_i on time and that the maximum waiting time is in fact $(k-1)m/k^2$ at the client end before the movie can start playing. In their papers [5, 6], Pâris, Carter and Long propose three protocols—cautious harmonic broadcasting, quasi-harmonic broadcasting and poly-harmonic broadcasting—that all guarantee a maximum waiting time of m/k at the cost of a slightly larger bandwidth requirement. They also identify the question regarding optimality of harmonic broadcasting, i.e., the question whether it is possible or not to achieve the design goals of harmonic broadcasting using even less bandwidth: *Suppose that we want to broadcast an m-minute movie in such a way that the maximum waiting time is m/k minutes. What is the minimum number of channels we need?*

Before resolving the above question, we note that harmonic broadcasting is actually a special case of *priority encoded transmission*, a scheme proposed by Albanese et al. [1]—a direct application of priority encoded transmission in fact gives results comparable to those obtained by harmonic broadcasting, we elaborate some more on this in the full version [2] of this extended abstract.

2 The Lower Bound

In their paper, Albanese et al. [1] also provide a lower bound, showing that their encoding is optimal. Since harmonic broadcasting is a special case of priority encoded transmission, this lower bound is not directly applicable to the less general harmonic broadcasting. The second contribution of this paper is a lower bound showing that in order to get a maximum waiting time of m/k, at least $\ln(k+1)+O(k/m)$ channels are necessary. Since $H_k = \ln(k+1)+\gamma+O(1/k)$ where $\gamma \approx 0.557$ is the Euler constant this proves that harmonic broadcasting is indeed optimal.

THEOREM 2.1. *If a message* $\langle M_1, \dots, M_m\rangle \in \tau_1 \times \cdots \times$

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τ_m is encoded as $\langle E_1, \ldots, E_n \rangle \in \sigma_1 \times \cdots \times \sigma_n$ in such a way that the value of M_i can be recovered from any $\rho_i n$ consecutive packets from the encoding, where $0 < \rho_1 \leq \cdots \leq \rho_m = 1$, then

$$\sum_{i=1}^{m} \frac{\log_2 |\tau_i|}{\rho_i} \leq \sum_{i=1}^{n} \log_2 |\sigma_i|.$$

We prove Theorem 2.1 in the full version [2] of this extended abstract; we only state an application of it here:

Corollary 2.1. *Suppose that we want to transmit a movie of size m in such a way that the maximum waiting time until the client can start viewing the movie is m/k. Then we need a bandwidth which is at least $\ln(k+1) + O(k/m)$ times the bandwidth needed to transmit one copy of the movie.*

Proof. If we let $\tau_i = \{0, 1\}$ and $\sigma_i = \{0, 1\}$ in Theorem 2.1, we get the bound $\sum_{i=1}^{m} 1/\rho_i n \leq 1$. Now suppose that we use α channels, i.e., a total bandwidth of α times the bandwidth required to transmit one copy of the movie. Since we assumed a maximum waiting time of m/k, $\rho_1 n \leq m\alpha/k$ since we must be able to decode the first bit in the message after time m/k. Moreover, $\rho_i n \leq m\alpha/k + (i-1)\alpha$, since we must be able to decode the ith bit in the message after time $m/k + (i-1)$. Therefore,

$$1 \geq \sum_{i=1}^{m} \frac{1}{\rho_i n} \geq \sum_{i=1}^{m} \frac{1}{m\alpha/k + (i-1)\alpha}.$$

Rearranging the above expression, we obtain

$$\alpha \geq \sum_{i=1}^{m} \frac{1}{i + m/k - 1} \geq \ln(k+1) + O(k/m).$$

The proof of Theorem 2.1 is provided in the full version [2] of this extended abstract. It uses an information theoretic argument along the lines of Albanese et al. [1], the main difference being that we consider only consecutive blocks of the encoding instead of arbitrary sets of blocks. This requires us to derive an information theoretic inequality that, to our knowledge, has not been given explicitly in the literature before and might be of independent interest. It is a modification of an inequality due to Han [3]; we now conclude by stating our modified inequality:

Lemma 2.1. *Let $X = \langle X_1, X_2, \ldots, X_k \rangle$. For any set $Q \subseteq \{1, 2, \ldots, k\}$, let $X_Q = \langle X_i \rangle_{i \in Q}$. Let $B_{q,k}$ be the set of all blocks of q consecutive integers from $\{1, 2, \ldots, k\}$, with wrapping allowed. For any $q \in \{1, 2, \ldots, k\}$, define*

$$H_q(X) = \frac{k/q}{|B_{q,k}|} \sum_{Q \in B_{q,k}} H(X_Q)$$

where $H(X_Q)$ is the binary entropy of X_Q. Then $H_1(X) \geq H_2(X) \geq \cdots \geq H_k(X) = H(X)$.

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