Motion Planning

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Piano Mover's Problem

- Given:
  - A set of obstacles
  - The initial position of a robot
  - The final position of a robot
- Goal: find a path that
  - Moves the robot from the initial to final position
  - Avoids the obstacles (at all times)

Basic notions

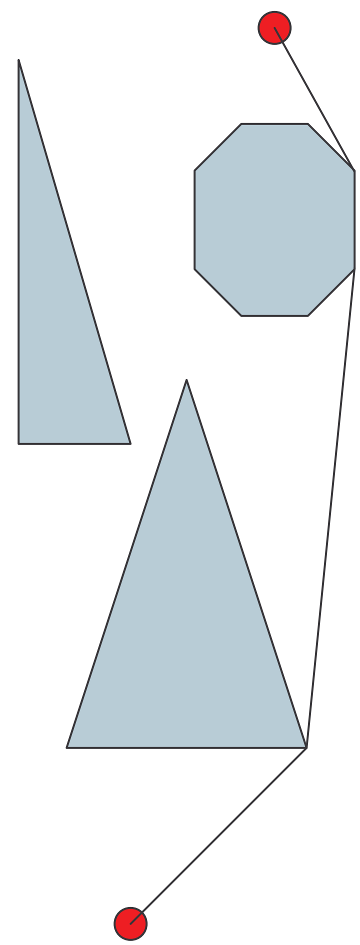
- Work space – the space with obstacles
- Configuration space:
  - The robot (position) is a point
  - Forbidden space = positions in which robot collides with an obstacle
  - Free space: the rest
- Collision-free path in the work space = path in the configuration space

Demo

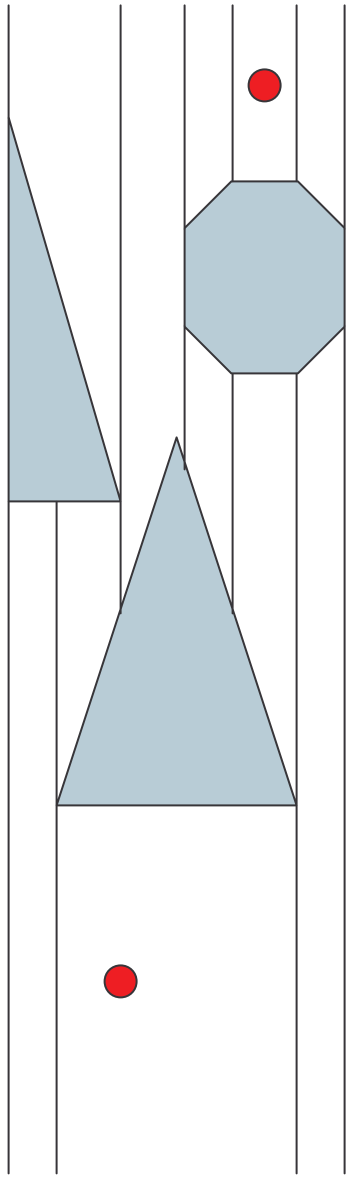
- http://www.diku.dk/hjemmesider/studerende/palu/start.html

Point case

- Assume that the robot is a point
- Then the work space=configuration space
- Free space = the bounding box – the obstacles



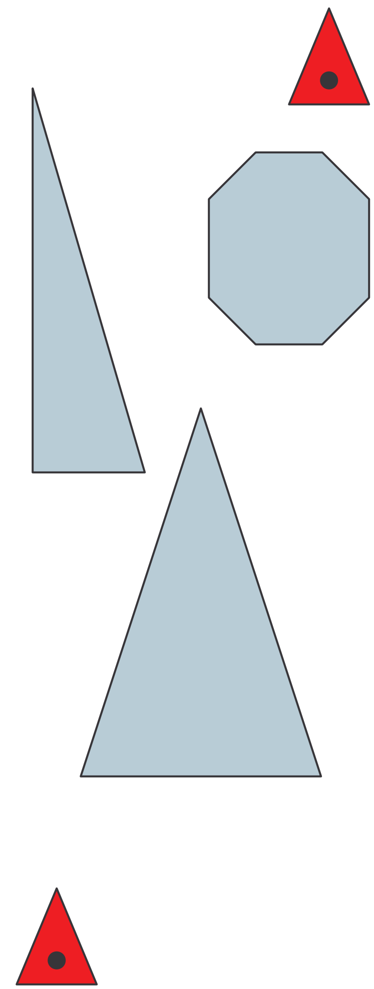
Finding a path



- Compute the trapezoidal map to represent the free space
- Place a node at the center of each trapezoid and edge
- Put the “visibility” edges
- Path finding=BFS in the graph

Convex robots

- C-obstacle = the set of robot positions which overlap an obstacle
- Free space: the bounding box minus all C-obstacles
- How to calculate C-obstacles ?



Minkowski Sum

- Minkowski Sum of two sets P and Q is defined as $P \oplus Q = \{p+q : p \in P, q \in Q\}$
- How to compute C-obstacles using Minkowski Sums ?

C-obstacles

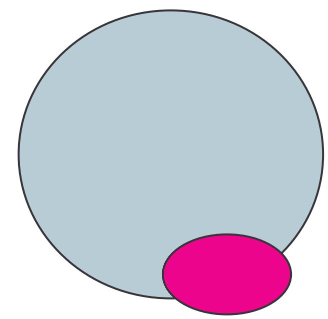
- The C-obstacle of P w.r.t. robot R is equal to $P \oplus (-R)$
- Proof:
  - Assume robot R collides with P at position c
  - I.e., consider $q \in (R+c) \cap P$
  - We have $q-c \in R \rightarrow c-q \in -R \rightarrow c \in q+(-R)$
  - Since $q \in P$, we have $c \in P \oplus (-R)$
- Reverse direction is similar

Complexity of $P \oplus Q$

- Assume P,Q convex, with n (resp. m) edges
- Theorem: $P \oplus Q$ has n+m edges
- Proof: sliding argument
- Algorithm follows similar argument

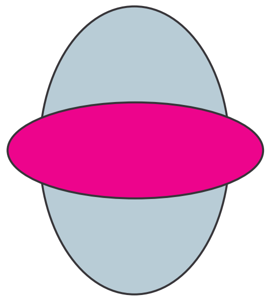
More complex obstacles

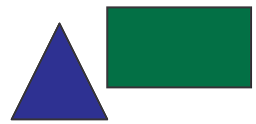
- Pseudo-disc pairs: O_1 and O_2 are in pd position, if O_1-O_2 and O_2-O_1 are connected
- At most two proper intersections of boundaries



Minkowski sums are pseudo-discs

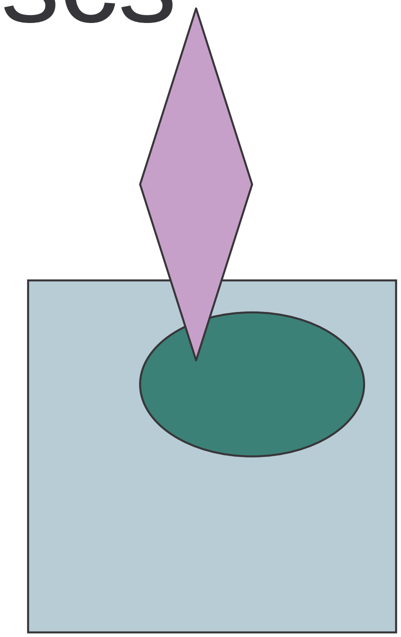
- Consider convex P,Q,R, such that P and Q are disjoint. Then $C_1=P\oplus R$ and $C_2=Q\oplus R$ are in pd position.
- Proof:
  - Consider C_1-C_2, assume it has 2 connected components
  - There are two different directions in which C_1 is more extreme than C_2
  - By properties of $\oplus$, direction d is more extreme for C_1 than C_2 iff it is more extreme for P than Q
  - Configuration impossible for convex P,Q





Union of pseudo-discs

- Let $P_1, \ldots, P_k$ be polygons in pd position. Then their union has complexity $|P_1| + \ldots + |P_k|$
- Proof:
  - Suffices to bound the number of vertices
  - Each vertex either original or induced by intersection
  - Charge each intersection vertex to the next original vertex in the interior
  - Each vertex charged at most twice



Convex $R \oplus$ Non-convex P

- Triangulate P into $T_1, \ldots, T_n$
- Compute $R \oplus T_1, \ldots, P \oplus T_n$
- Compute their union
- Complexity: $|R|\ n$
- Similar algorithmic complexity