alloy: an analyzable modelling language

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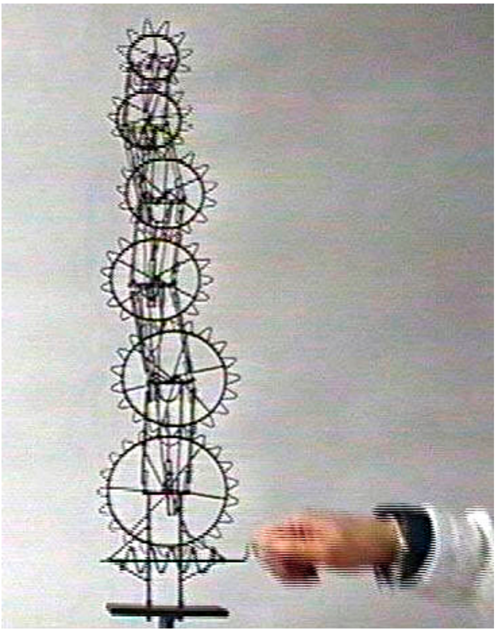
Praxis Critical Systems
April 25, 2003



Small Tower of 6 Gears, Arthur Ganson

preaching to the choir

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explicit models before code
- › higher quality
- › easier coding

preaching to the choir

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formalism helps
› forces simplicity
› no wishful thinking
› potential for tools

assurance/cost tradeoffs

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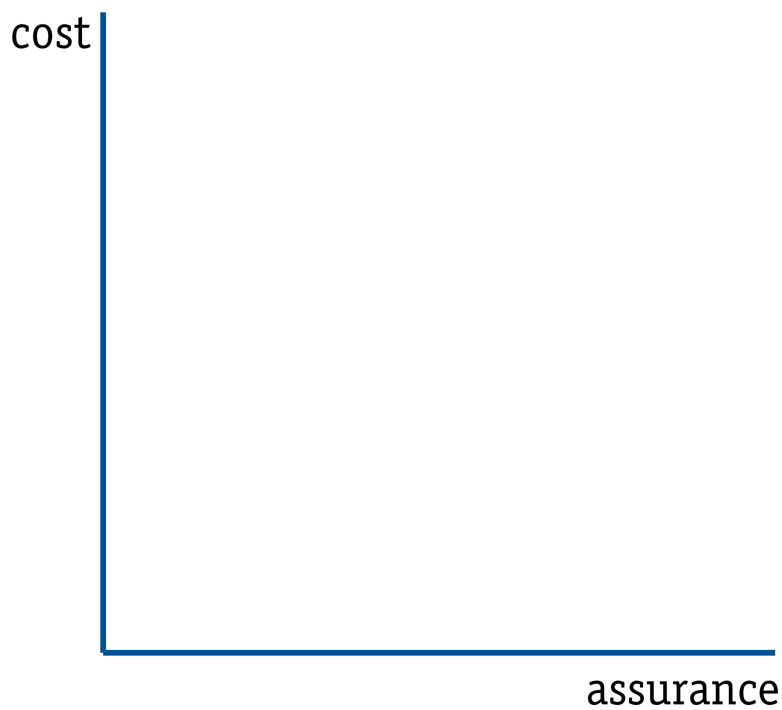
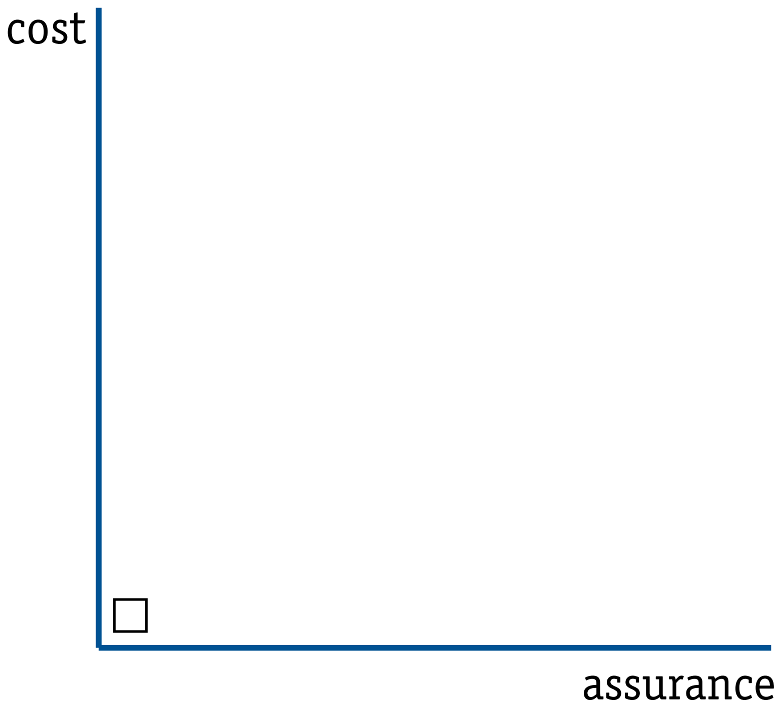


assurance/cost tradeoffs

□hacking



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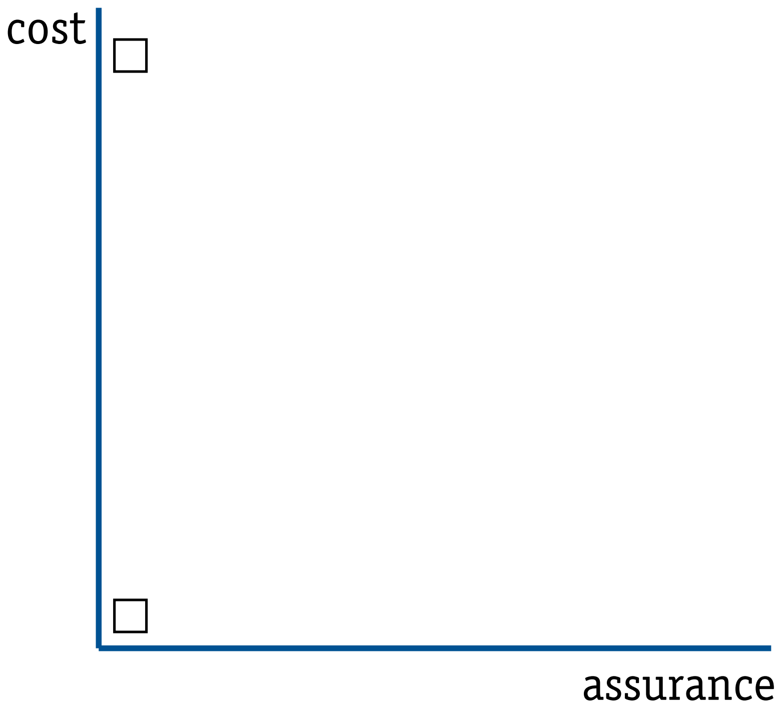

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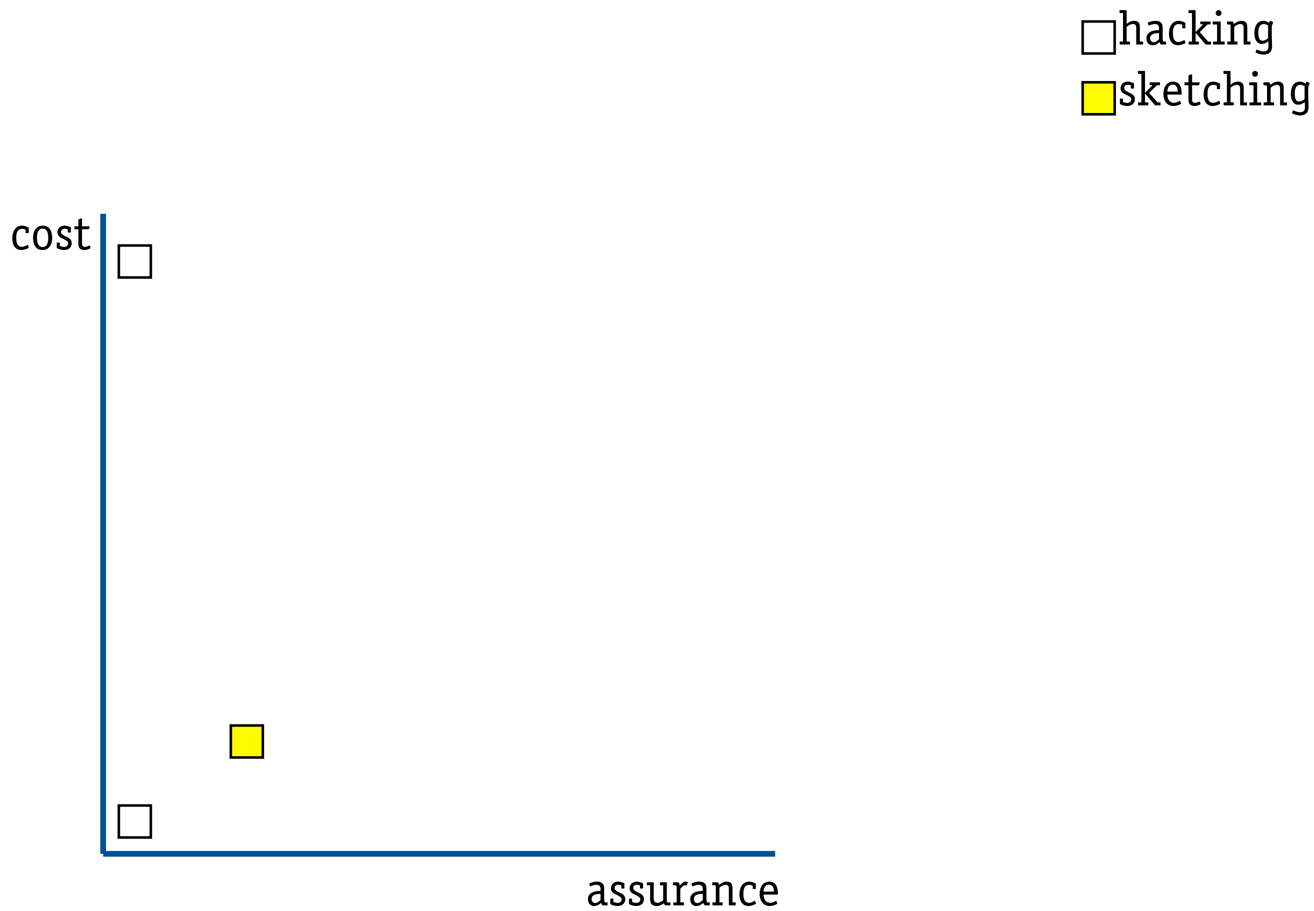

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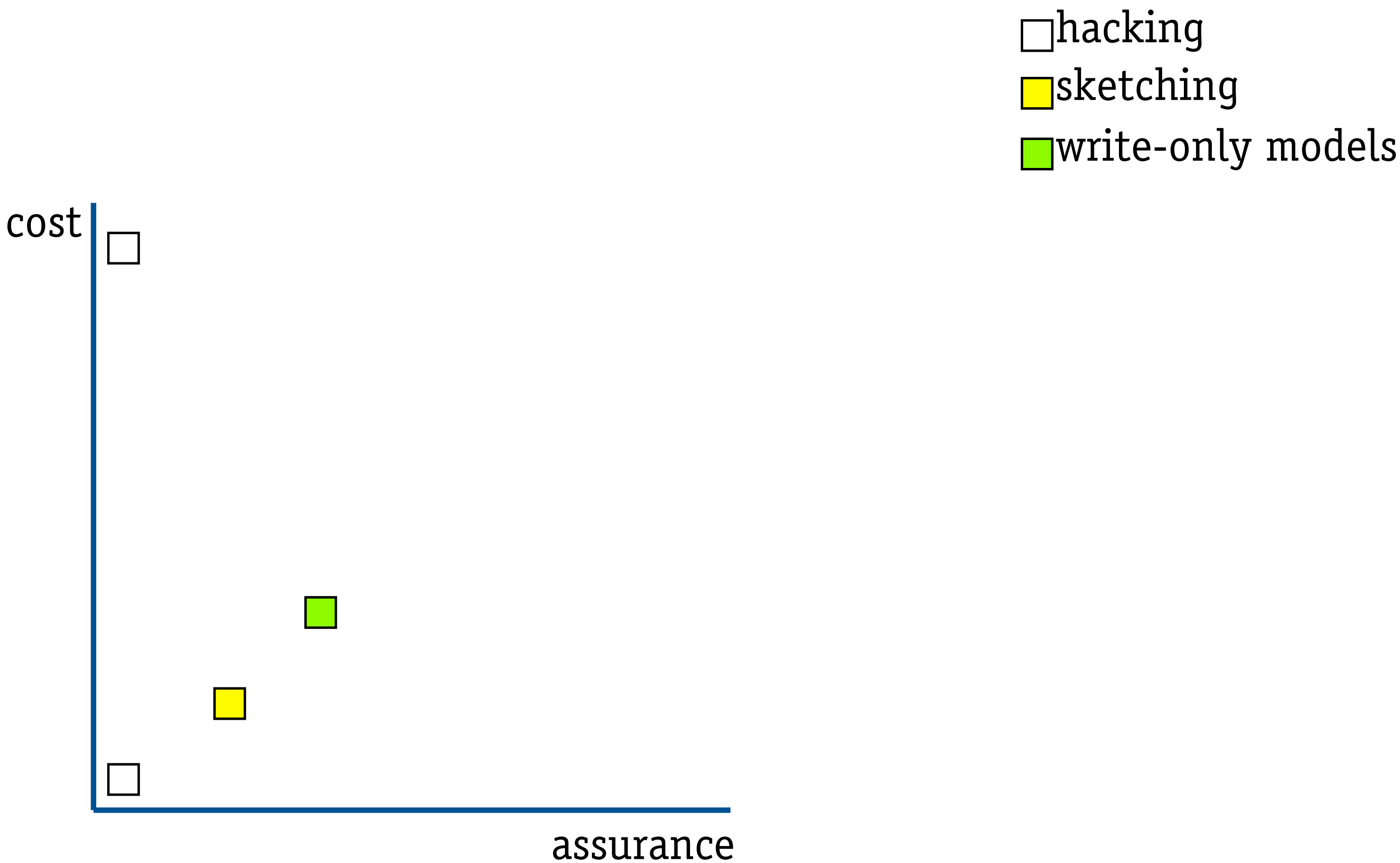

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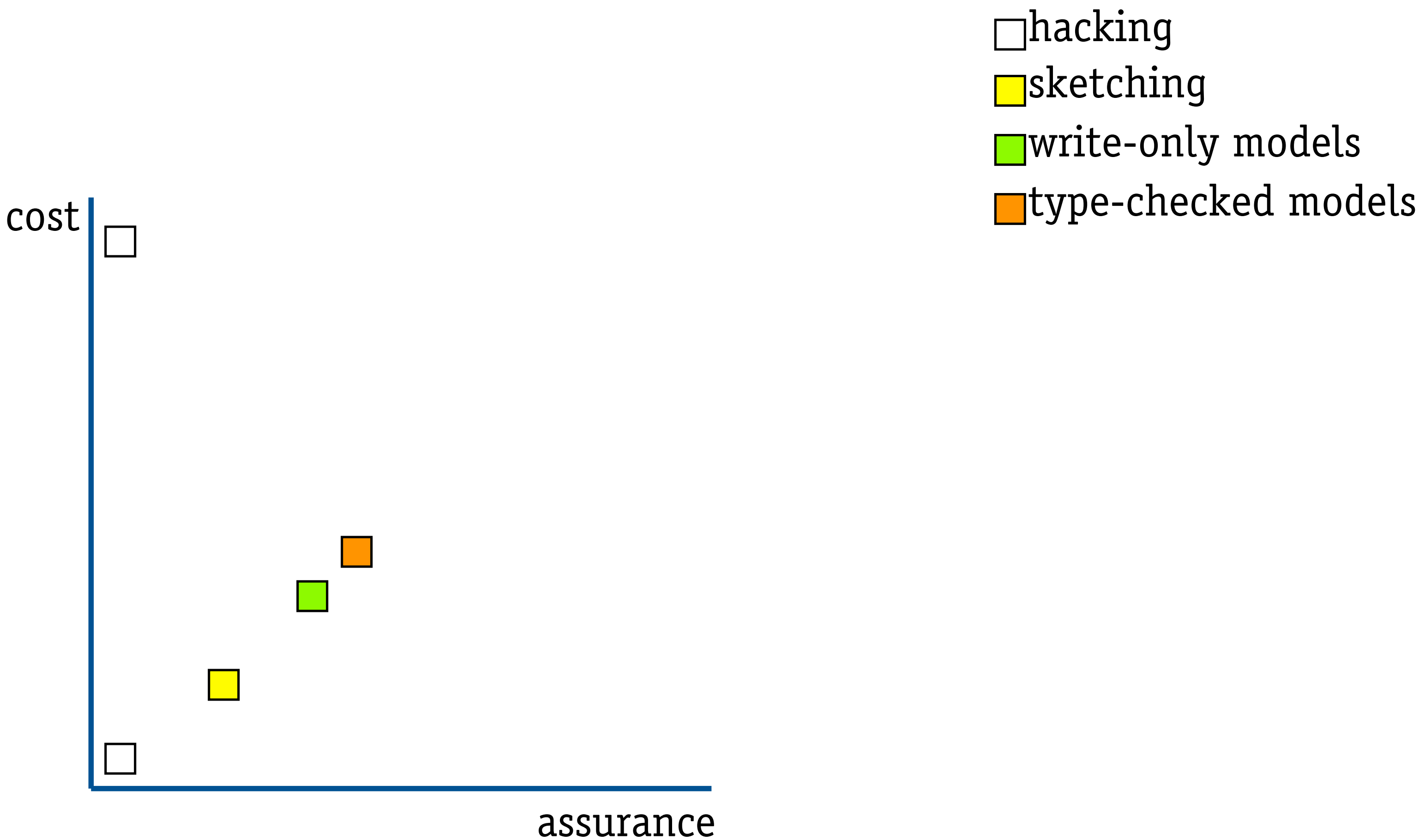

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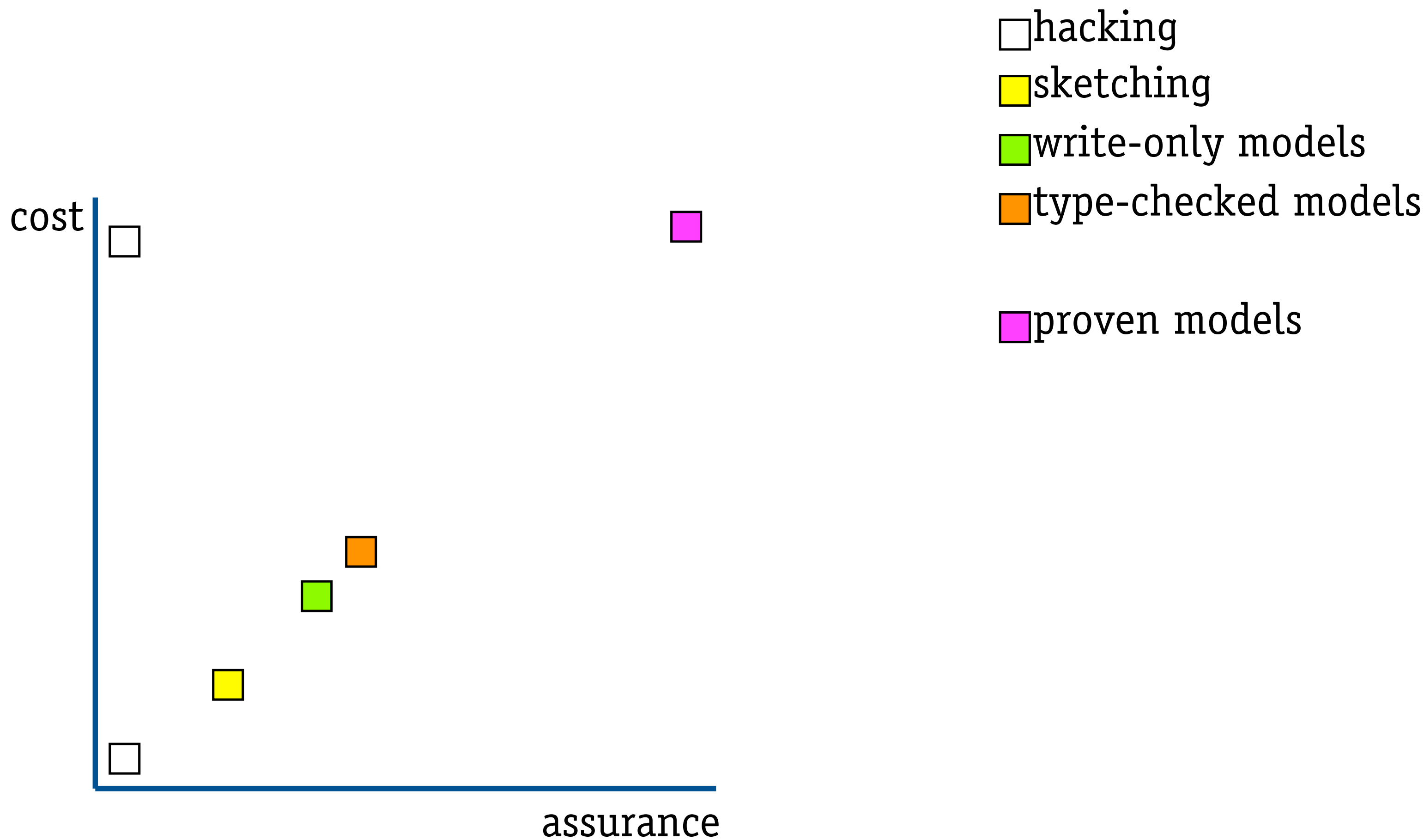

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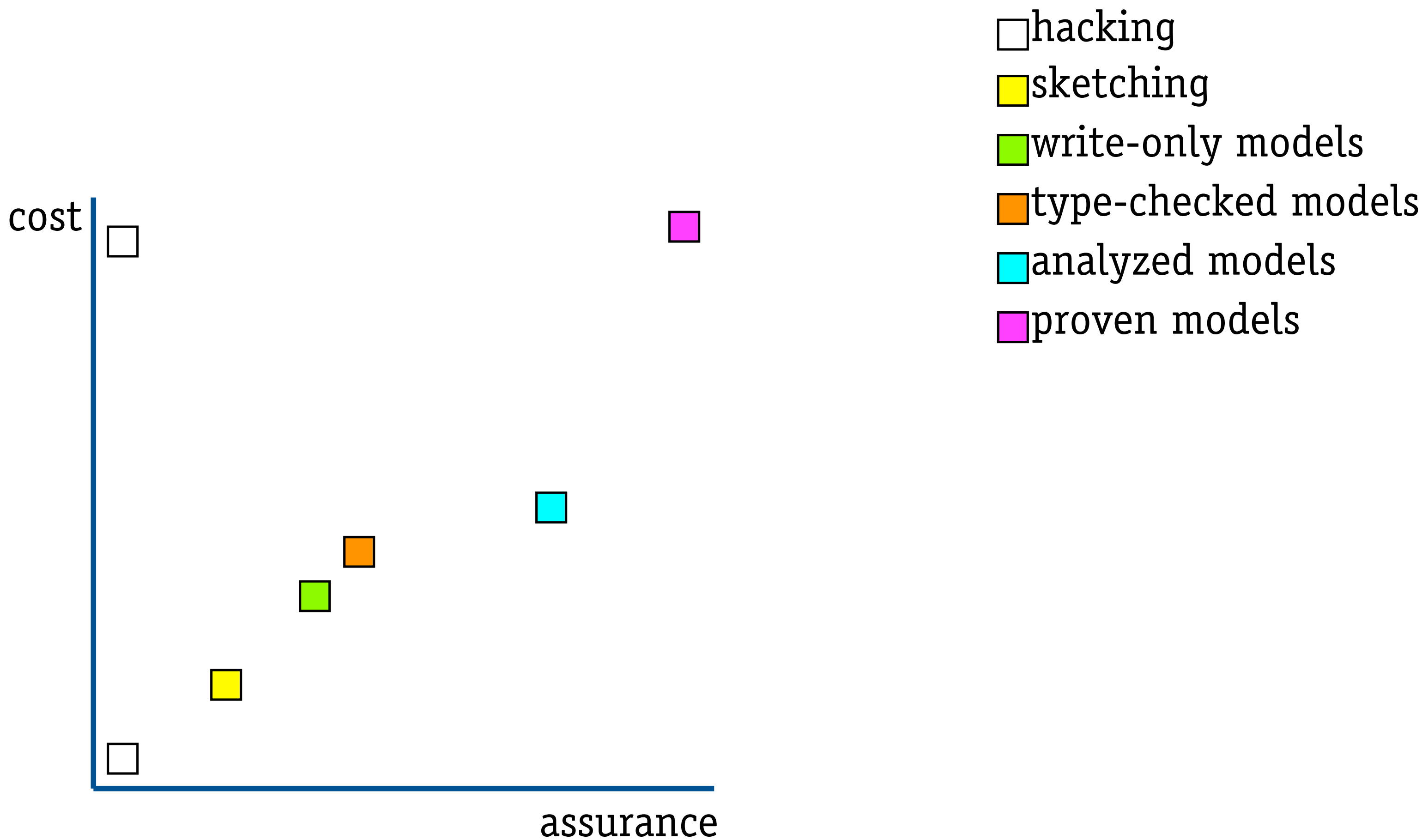

lightweight formal methods

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language must be

- small and simple
- well defined
- expressive enough

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user doesn't want to
- provide test cases
- invent lemmas

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a notation inspired by Z

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- familiar logical quantifiers
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- declarative logic, not state machines

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what to look out for

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the language

- › all structure by relations
- › composites by higher-arity
- › entirely first order
- › familiar syntax by puns

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the analysis

› as in Z, everything's a formula
› tool tries all small tests within a "scope"
› model itself is unbounded

a first alloy model

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```
module email
sig Name, Addr {}
assert A {
  all friends, spammers: set Name, addr: Name -> Addr |
    (friends - spammers).addr = friends.addr - spammers.addr
  }
check A for 3
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a first alloy model

introduces sets of atoms **Name** and **Addr**

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an assertion to be checked

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set difference

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relational image

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a command the tool executes

analysis by constraint solving

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some f, s: set N, a: N -> A | not (f-s).a = f.a - s.a
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- then skolemize away quantifiers
  `not (f-s).a = f.a - s.a`
- and now solve for constants
  `f = {N0, N1} , s = {N1}, a = {N0->A1, N1->A1}`

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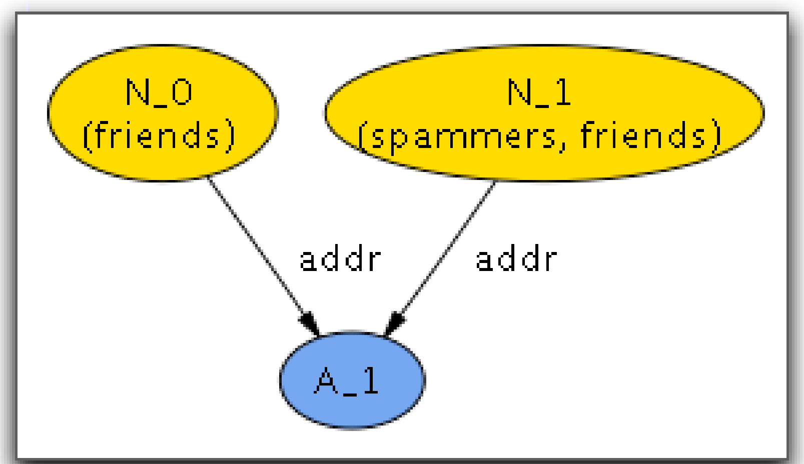

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alloy’s analysis is refutation
- look for a counterexample
- consider all assignments of values to constants
- user selects scope (here, 3 names and 3 addrs)

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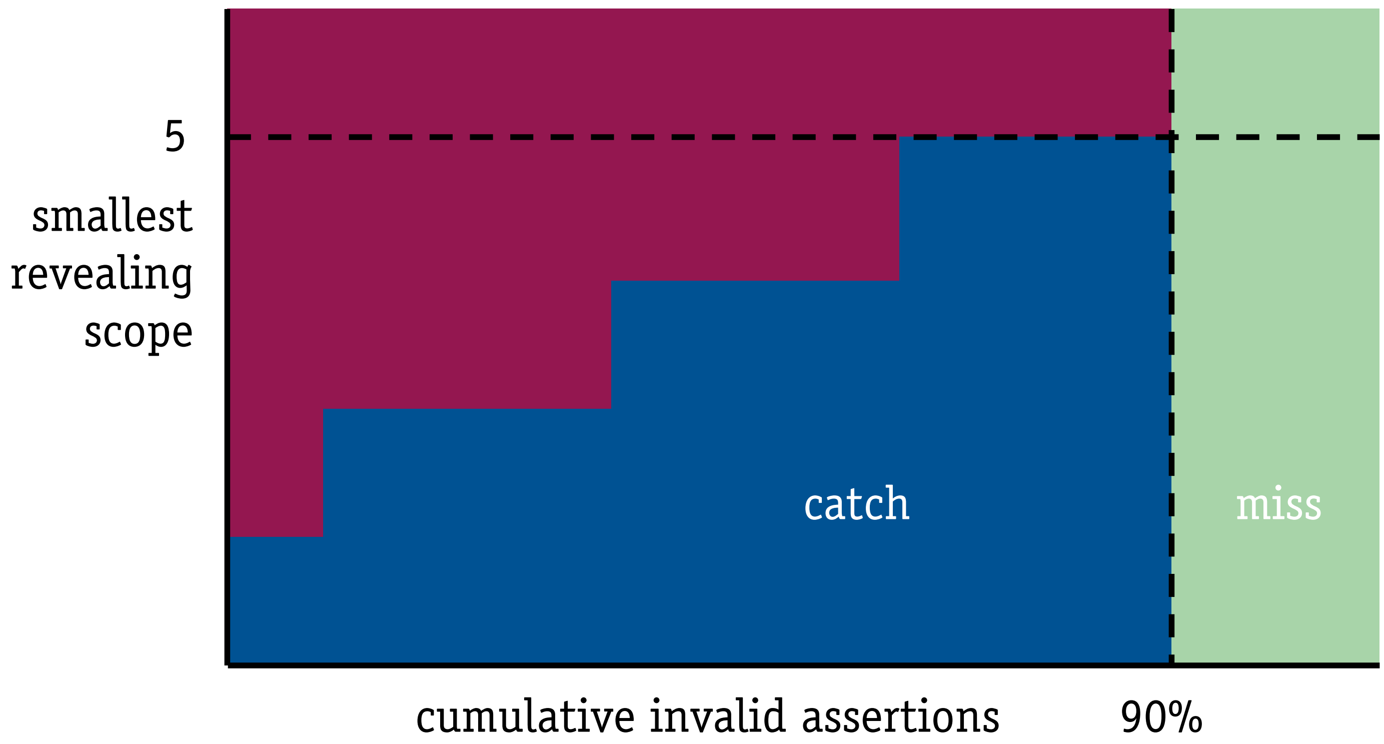
properties of models
- usually flawed, especially in early stages
- many bugs, even subtle ones, have small counterexamples

‘all small tests’



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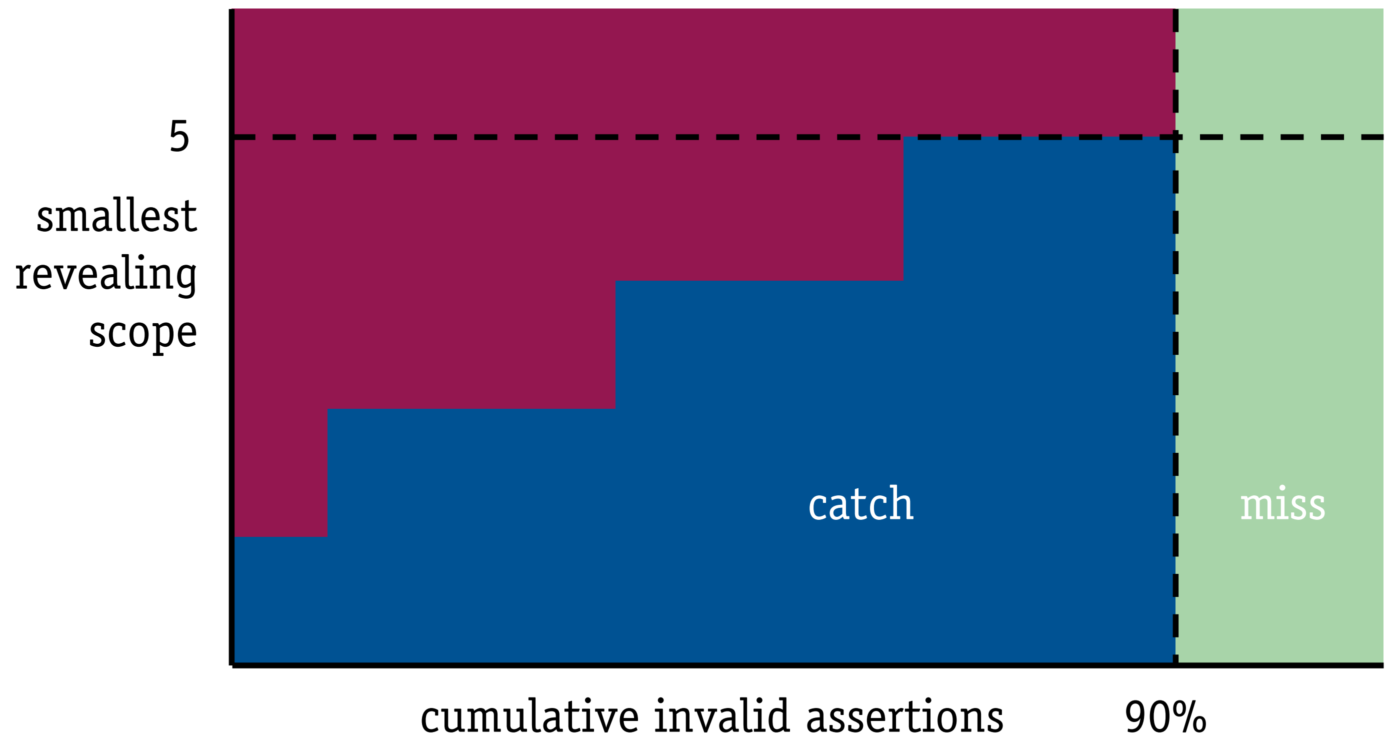

consequences

› sound: no false alarms
› incomplete: can’t prove anything

simulating an operation

simulating an operation

```
module email
sig Name, Addr {}
fun add (addr, addr': Name -> Addr, n: Name, a: Addr) {
  addr' = addr + (n -> a)
  }
run add for 2
```

simulating an operation

declares a parameterized formula

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forms a tuple

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set union

simulating an operation

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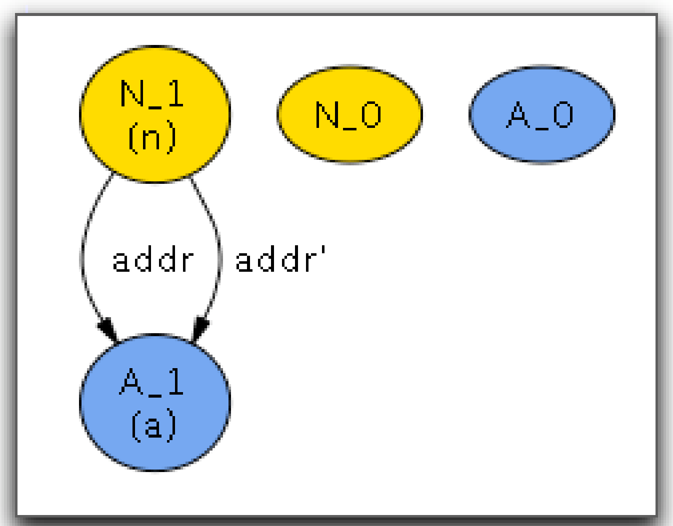

alloy semantics

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all values are relations

- {(a),(b)} is a set
- {(a)} is a scalar
- {(a,b)} is a tuple

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higher-order values

› can't be represented directly

AddrBook = $\mathbb{P}(\mathbb{P}($Name □ Addr$))$

› can often be represented with higher-arity

AddrBook -> Name -> Addr

expressions

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expressions are made from variables and

- set operators

  `p + q, p - q, p & q`

- relational operators

  `p.q, p->q, *p, ^p, ~p`

  $[\![p\ .\ q]\!] = \{(p_1, \ldots p_{n-1}, q_2, \ldots q_m) \mid$

  $(p_1, \ldots p_n) \in [\![p]\!] \wedge (q_1, \ldots q_m) \in [\![q]\!] \wedge p_n = q_1\}$

  $p \texttt{->} q = \{(p_1, \ldots p_n, q_1, \ldots q_m) \mid (p_1, \ldots p_n) \in [\![p]\!] \wedge (q_1, \ldots q_m) \in [\![q]\!]\}$

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puns

for scalars a, b, sets S, T and relations p, q

a -> b is a tuple; S -> T is a relation

S.p is image; p.q is join

formulas

formulas

e **in** e′	e is a subset of e′
not F	
F **and** G	
F **or** G	
F **=>** G	
{ F G }	implicit conjunction
all x: X | F	
some x: X | F	
one x: X | F	there is exactly one x such that F
sole x: X | F	there is at most one x such that F
no x: X | F	
no e	there is no tuple in e; e is empty
some e	there is some tuple in e; e is non-empty
sole e	there is at most one tuple in e

fields

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declares a ternary relation on **AddrBook**, **Name**, **Addr**

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an instance

```
map = {AB2->N2->A2}
b = AB2, b’ = AB2
a = A2, n = N2
```

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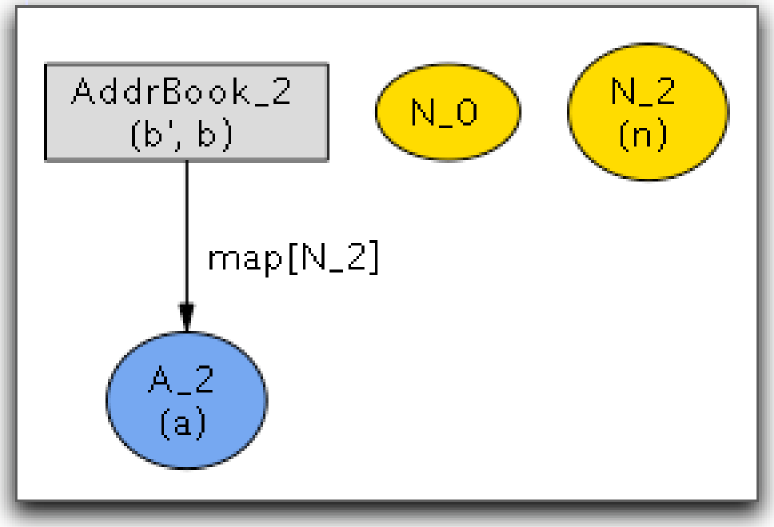

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projection (a visualization technique)

to show ternary relations

- › index the arcs
- › or project a type

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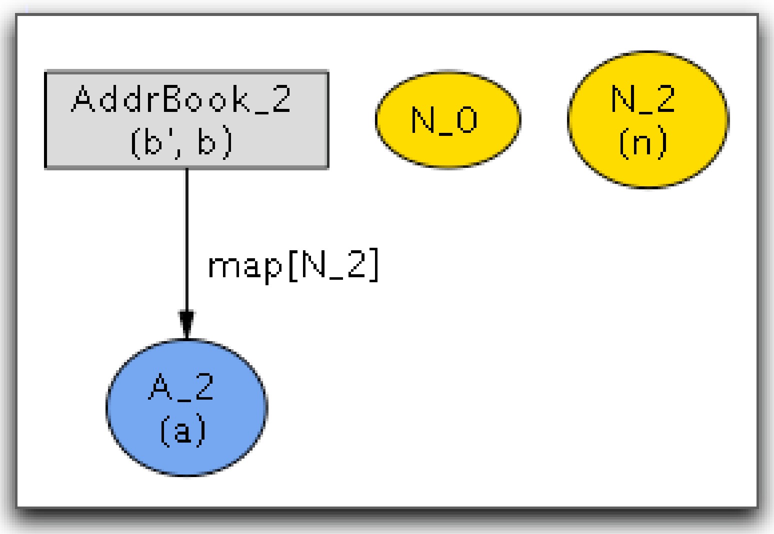

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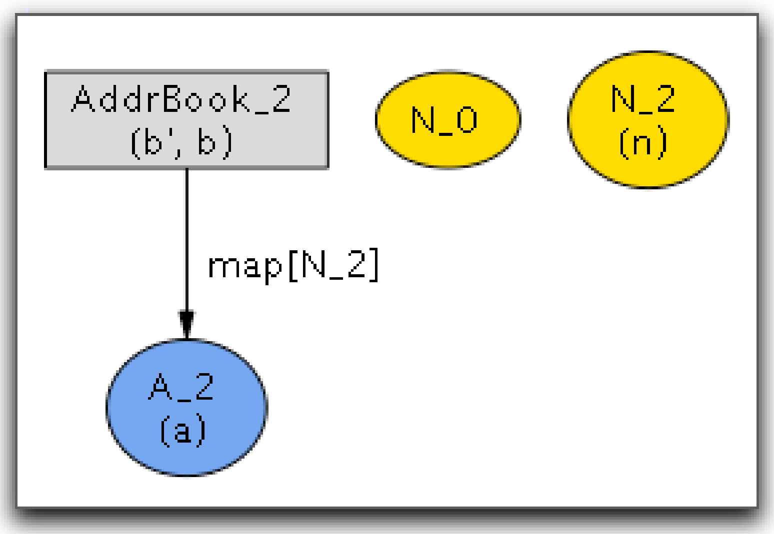

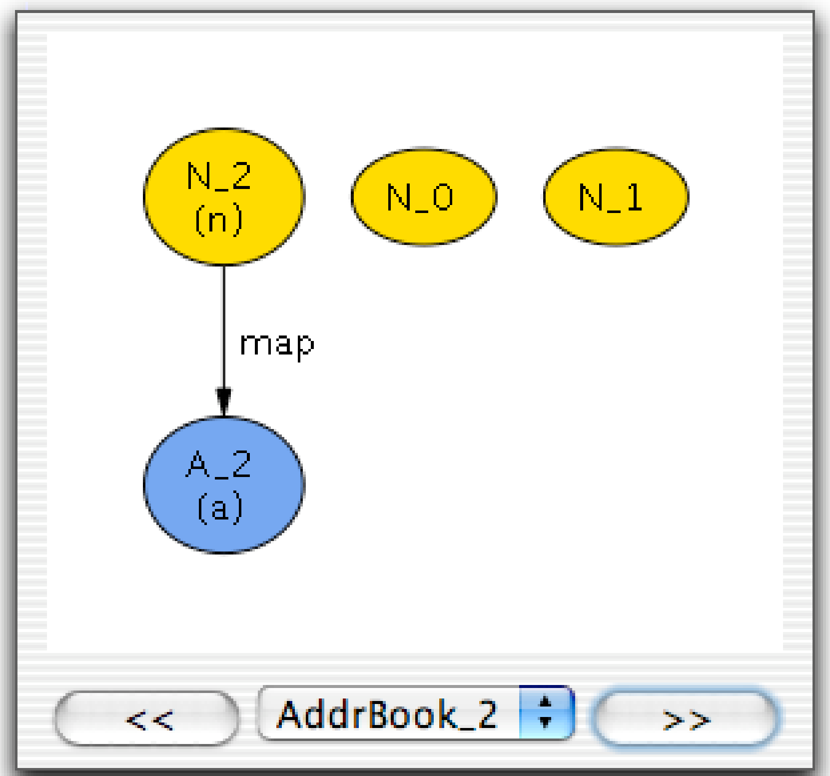

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some conjectures

equivalent to **n.(b.map)**

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assert addLocal {all b,b′: AddrBook, n,n′: Name, a: Addr |
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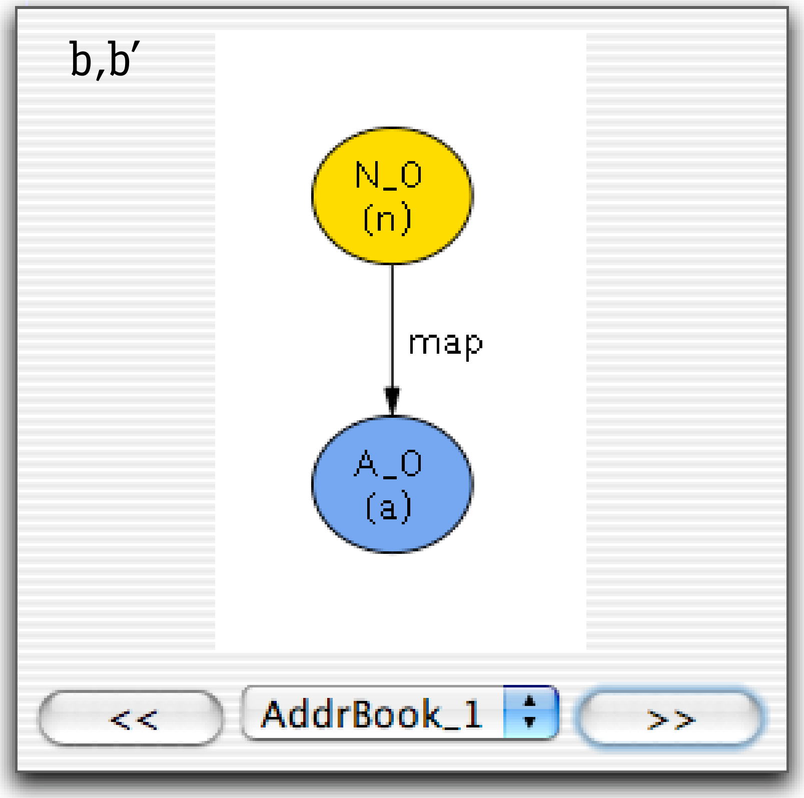

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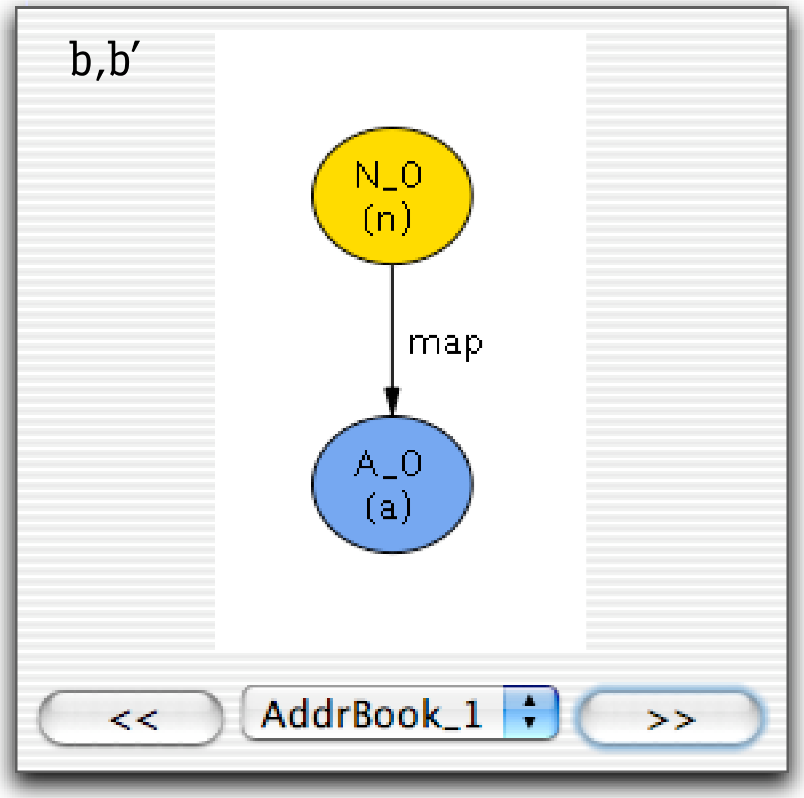

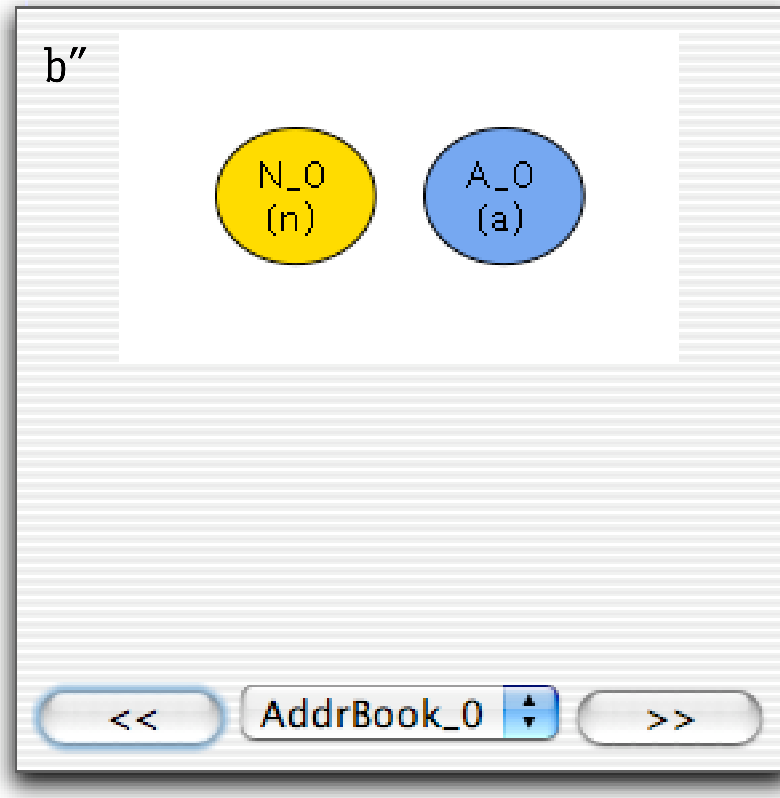

subsignatures

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```
module email
sig Target
part sig Addr, Name extends Target {}
part sig Alias, Group extends Name {}
sig AddrBook {
  map: Name -> Target
  }
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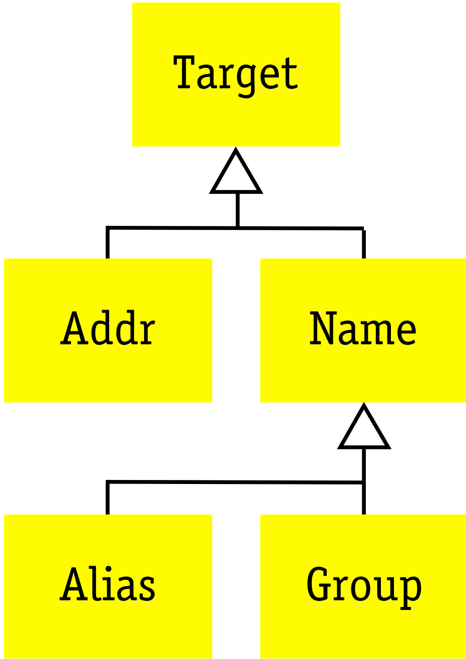

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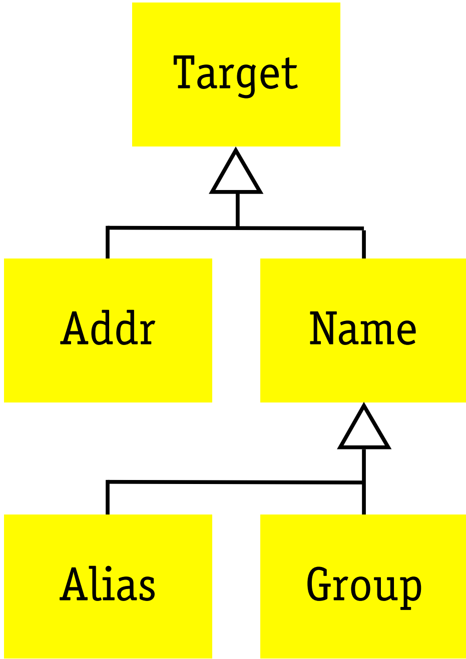


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fun lookup (b: AddrBook, n: Name): set Addr {
  result = n.^(b.map) & Addr }
```

counterexample

```
assert addLocal {all b,b′: AddrBook, n,n′: Name, a: Addr |
  add (b,b′,n,a) and n != n′ => lookup (b,n′) = lookup (b′,n′) }
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counterexample

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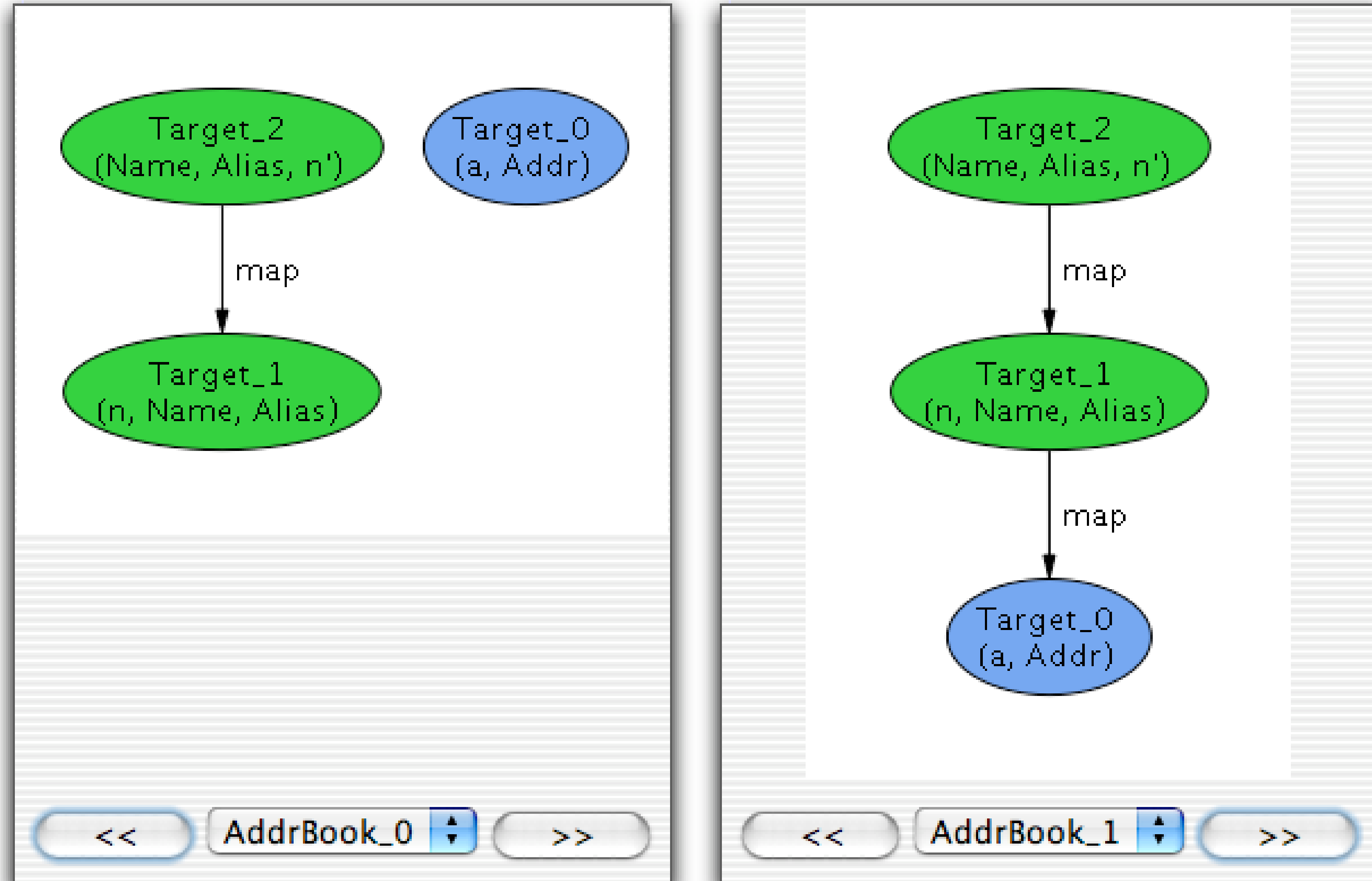

fields of subsignatures

```
module email
sig Host, Target {}
disj sig Name extends Target {}
disj sig Addr extends Target {host: Host}
part sig Alias, Group extends Name {}
sig AddrBook {
  map: Name -> Target
  }{all a: Alias | sole map[a]}
fun getHosts (b: AddrBook, n: Name): set Hosts {
  result = n.^(b.map).host }
```

fields of subsignatures

defines field **host** such that **no t.host** if **t !in Addr**

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```

applies **host** to set of Target; no need to write **(expr & Addr).host**

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  result = n.^(b.map).host }
```

signature fact: **all this: AddrBook ...** implicit

fields of subsignatures

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no ...

- partial functions, undefinedness, third logical value
- type casts

flexible declarations

```
module email
sig Target {}
part sig Addr, Name extends Target {}
part sig Alias, Group extends Name {}
sig AddrBook {names: set Name, map: names ->+ Target}
```

flexible declarations

decl says **names** is domain of **map**

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  result = n.^(b.map) & Addr }
fun add (b, b': AddrBook, n: Name, a: Target) {
  a in Addr or some lookup(b,a)
  b'.map = b.map + n->a}
fun del (b, b': AddrBook, n: Name) {b'.map = b.map - n->Addr}
```

traces

traces

```
module email
```

traces

```
module email
open std/ord
```

traces

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fact traces {
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fact traces {
  all b: AddrBook - Ord[AddrBook].last | let b' = OrdNext(b) |
```

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fact traces {
  all b: AddrBook - Ord[AddrBook].last | let b' = OrdNext(b) |
    some n: Name, a: Target | add (b, b', n, a) or del (b, b', n)
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assert lookupYields {all b: AddrBook, n: b.names | some lookup(b,n)}
```

counterexample

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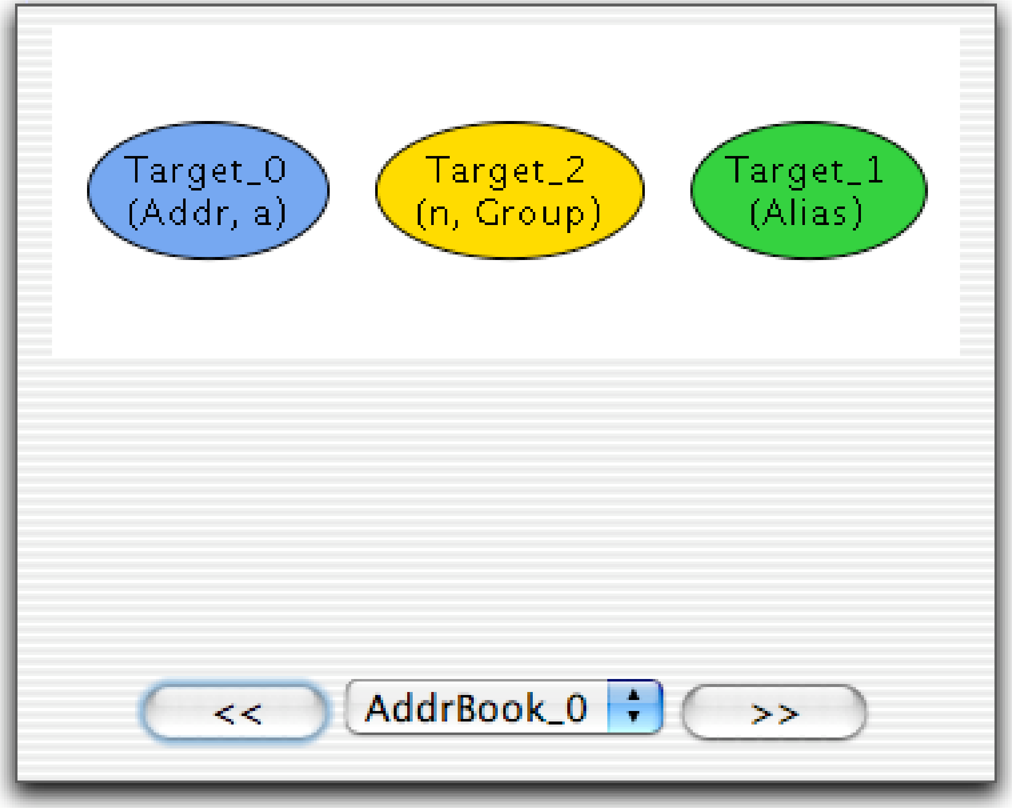

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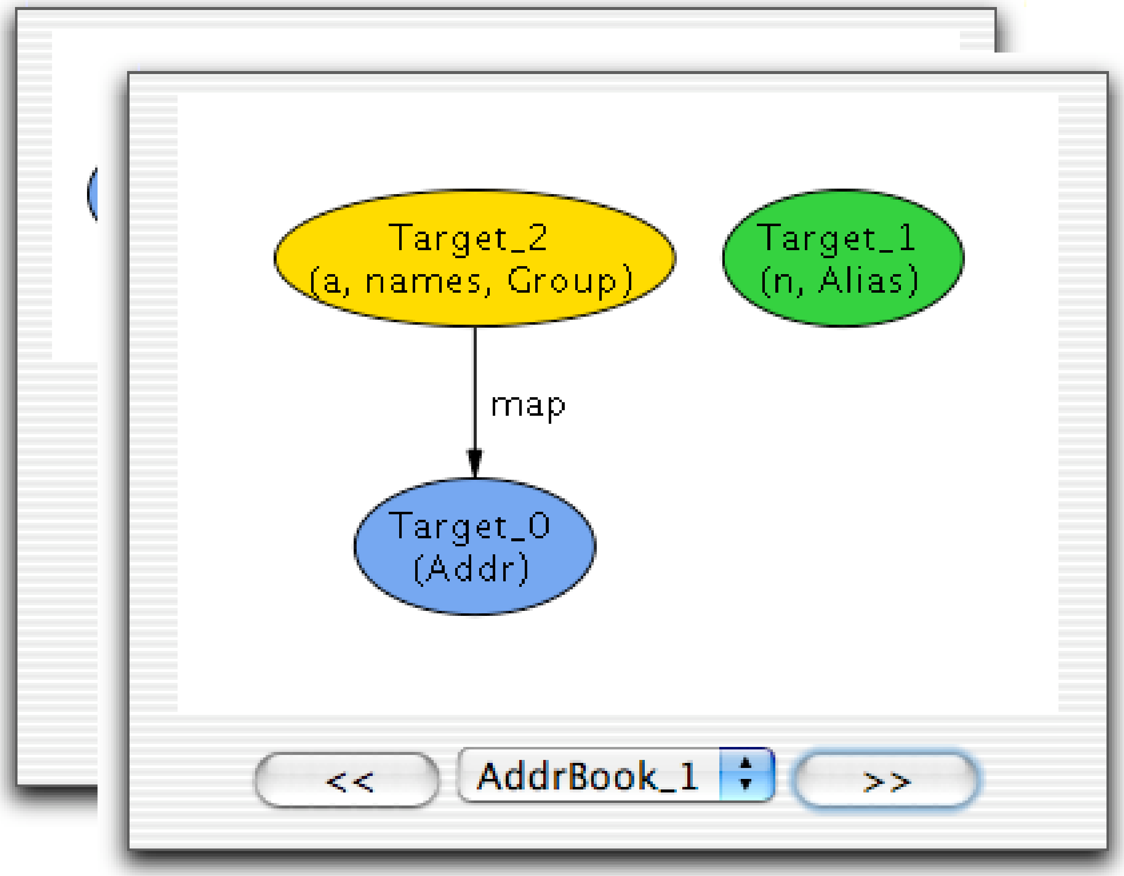

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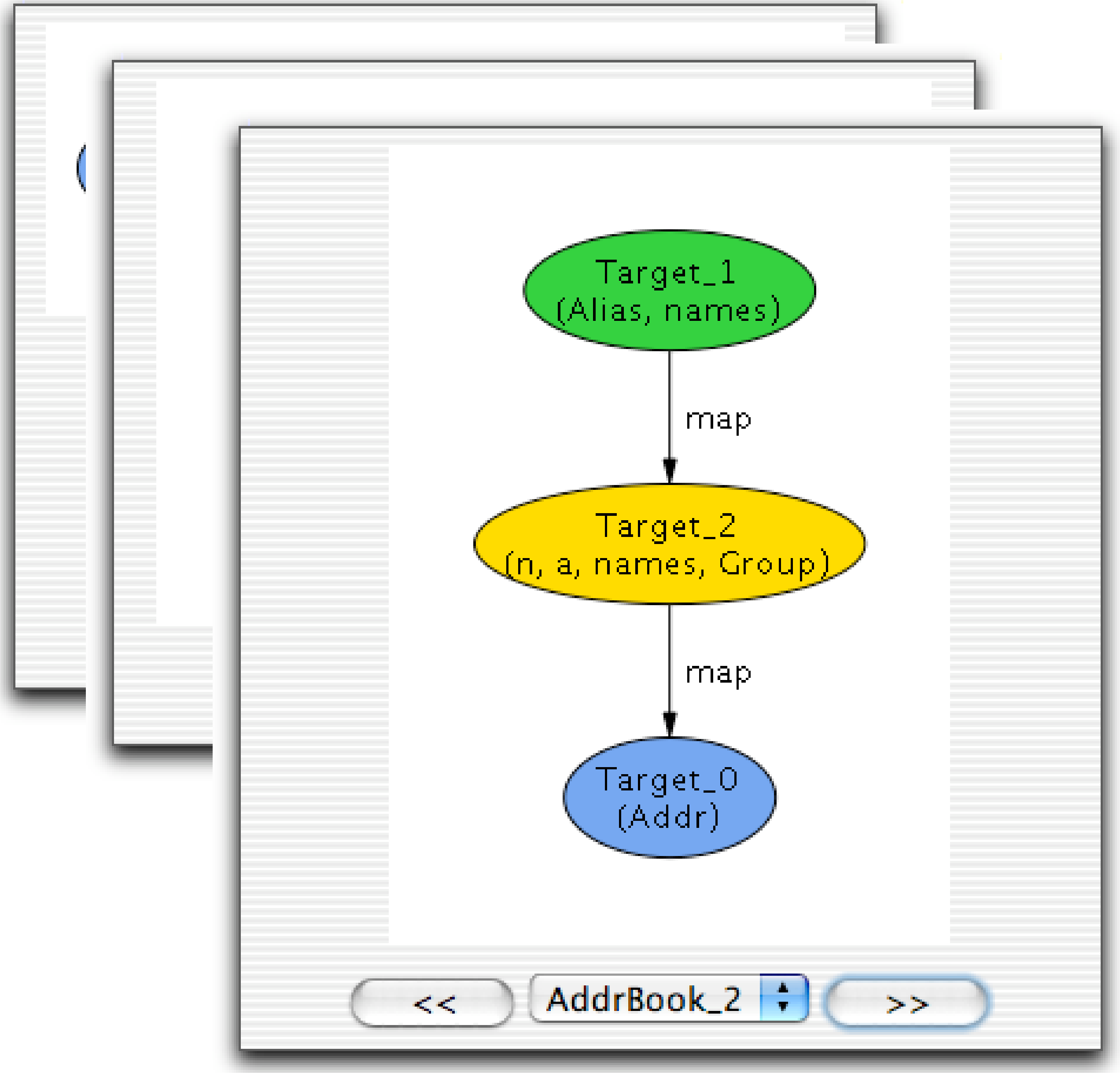

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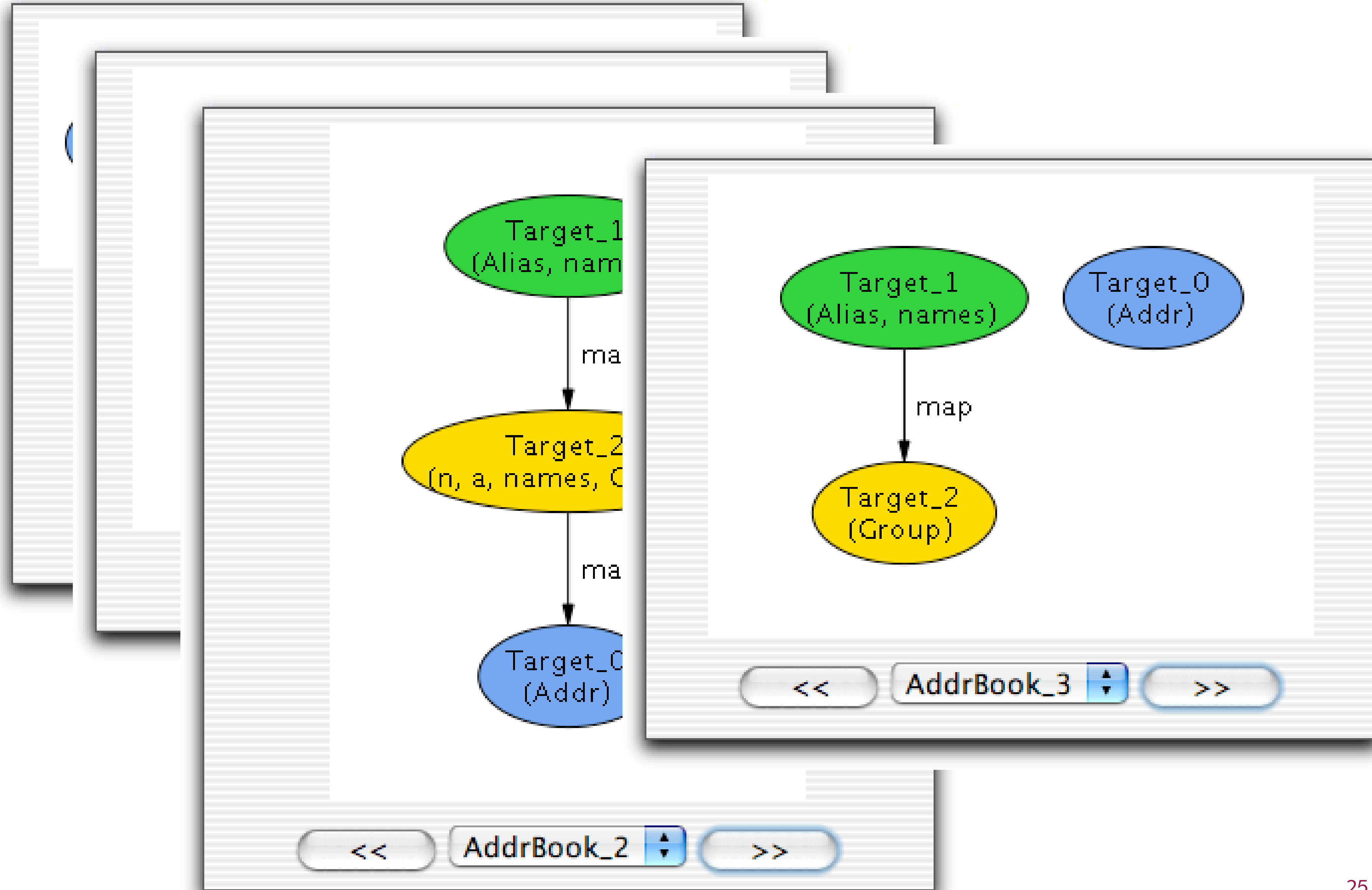

what you've seen

what you've seen

language

- first-order encoding

  r: A -> B looks like r □ ℙ(A□B) but means r □ A□B

  instead of AddrBook = ℙ(ℙ(Name□Addr))

    define map: AddrBook -> Name -> Addr

- simple and uniform syntax

  navigational dot, rich declarations

  explicit parameterization

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› simple and uniform syntax

navigational dot, rich declarations

explicit parameterization

analysis

› executable and declarative

› no ad hoc constraint on language

› no test cases

not seen: modelling idioms

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› schema extension

```
sig AddrBook' extends AddrBook {cache: Name -> Addr}
```

not seen: modelling idioms

- schema extension

  ```
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  ```
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  sig State {obj: Ref -> Obj}
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› asynchronous processes

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sig Process {state: Time ->! State}
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not seen: modelling idioms

- schema extension
  sig AddrBook′ **extends** AddrBook {cache: Name -> Addr}
- object-oriented heap
  sig State {obj: Ref -> Obj}
- asynchronous processes
  sig Process {state: Time ->! State}
- explicit events
  sig Event {t: Time}
  sig AddEvent **extends** Event {n: Name, a: Addr}

not seen: analysis idioms

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› refactoring

```
fun lookup (b: AddrBook, n: Name): set Target {...}
fun lookup' (b: AddrBook, n: Name): set Target {...}
assert same {all b: AddrBook, n: Name | lookup(b,n) = lookup(b',n)
```

not seen: analysis idioms

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```

- abstraction

```
fun abstract {c: ConcreteState, a: AbstractState) {…}
fun opC (c, c’: ConcreteState) {…}
fun opA (a, a’: AbstractState) {…}
assert refines {all a, a’: AbstractState, c, c’: ConcreteState |
  opC(c,c’) and abstract(c,a) and abstract(c’,a’) => opA(a,a’) }
```

not seen: analysis idioms

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fun lookup (b: AddrBook, n: Name): set Target {...}
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```

- machine diameter

```
fun noRepeats {no disj b, b': AddrBook | b.map = b'.map}
-- when noRepeats is unsatisfiable, trace is long enough
```

how analyzer works

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space is huge

- in scope of 5, each relation has 2^{25} possible values
- 10 relations gives 2^{250} possible assignments

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SAT to the rescue

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- fastest solvers (Chaff, Berkmin) can handle
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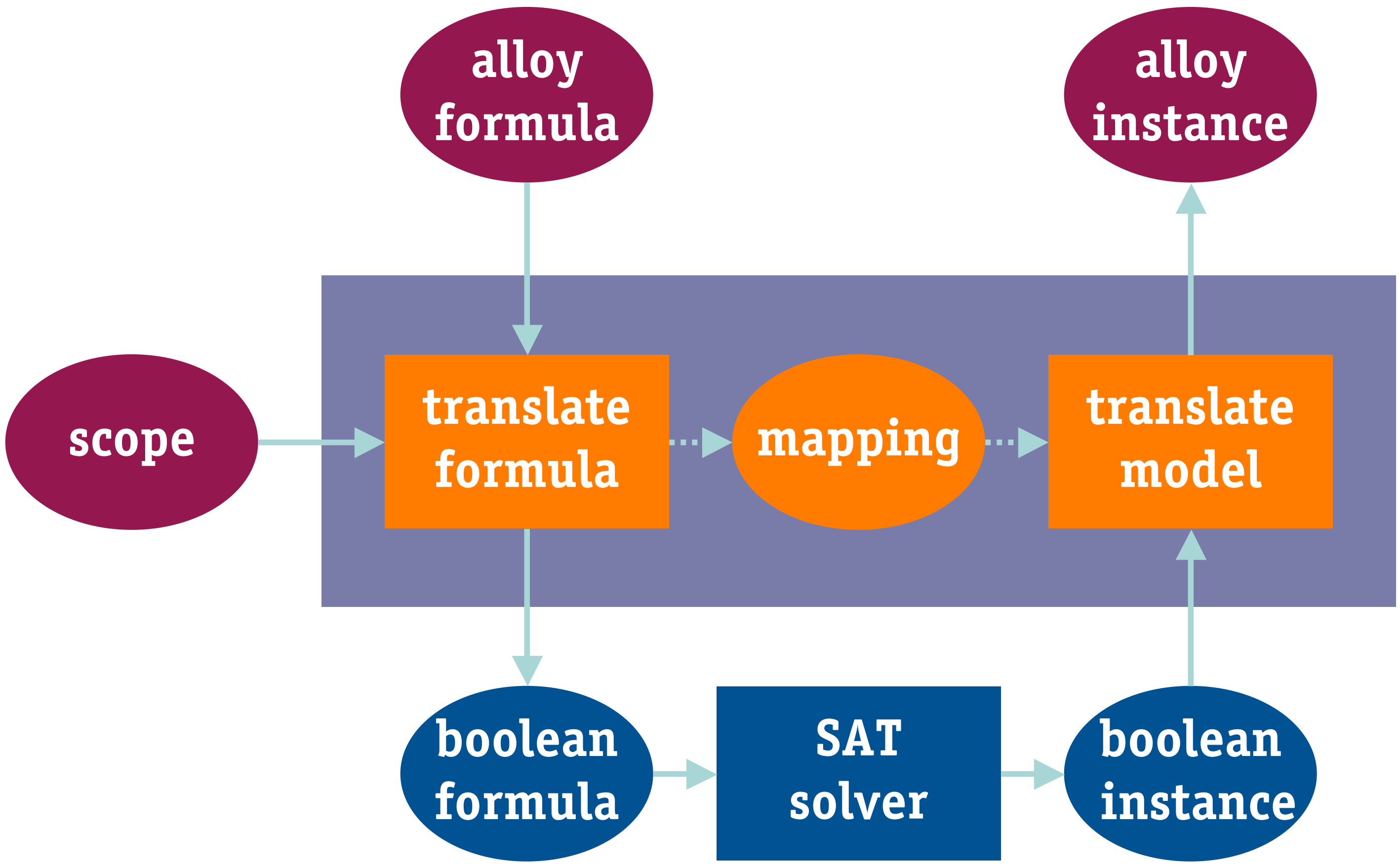
translating to SAT
- an instance is a graph
- for space of instances,
  label arcs with boolean variables

analyzer architecture



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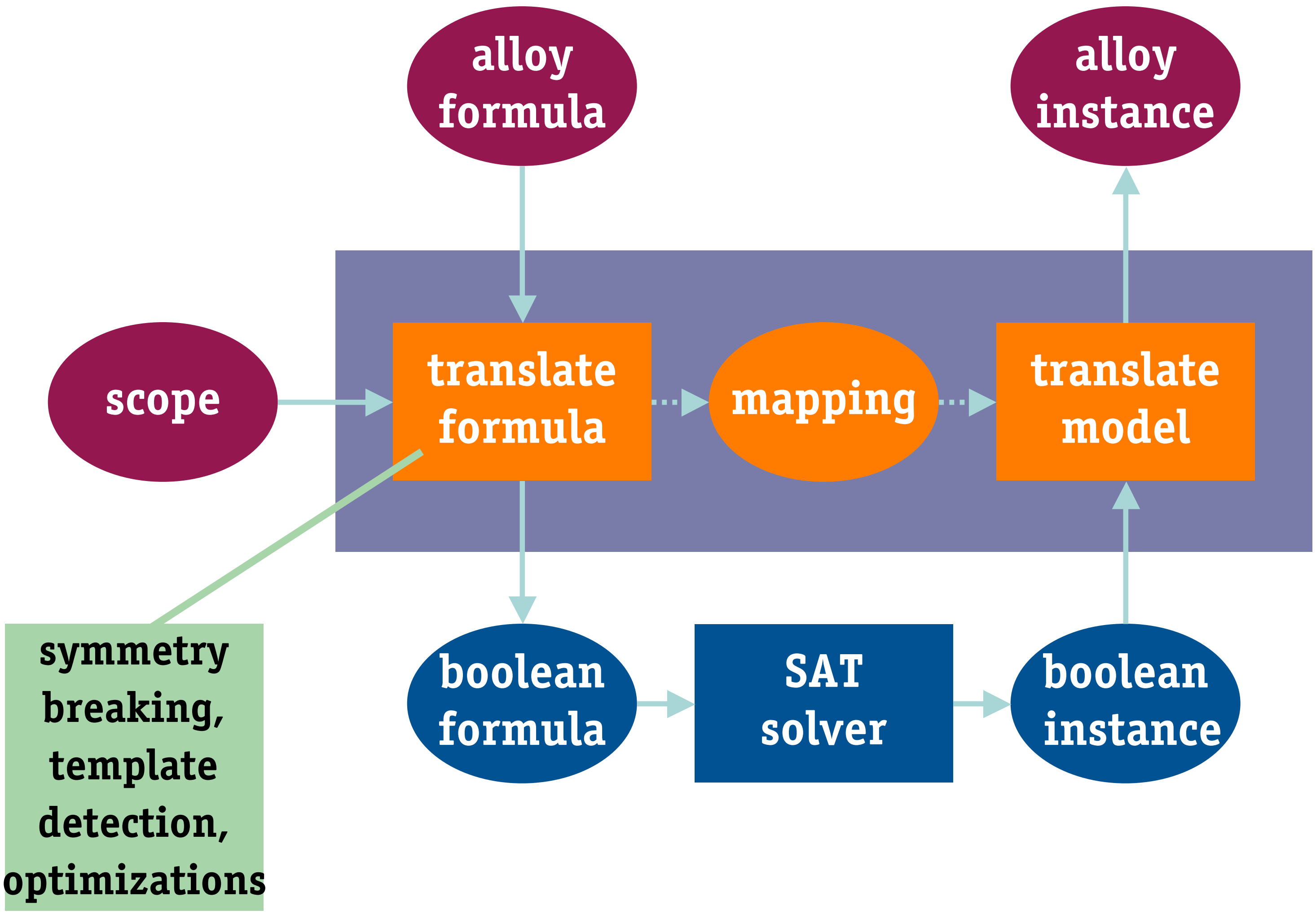

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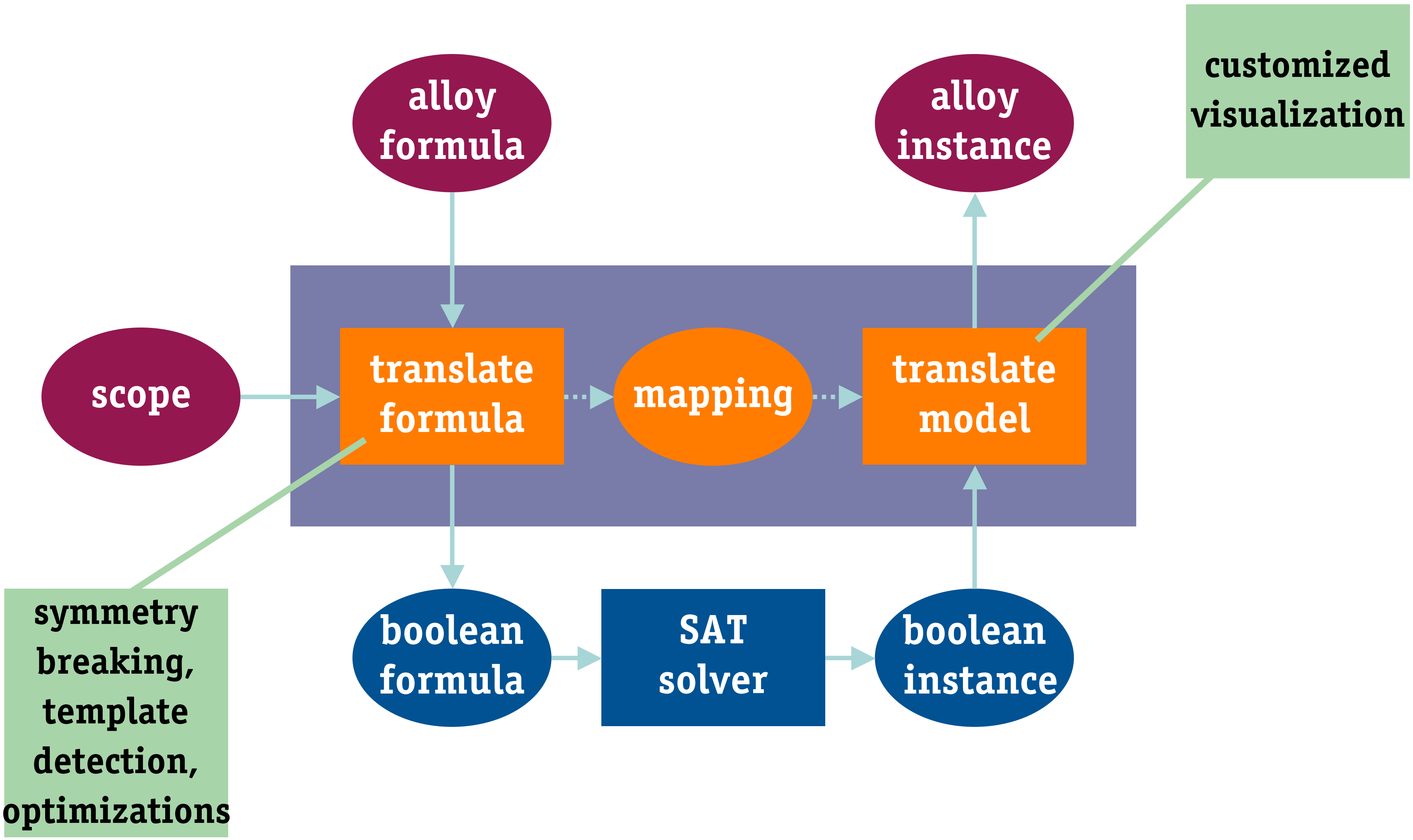
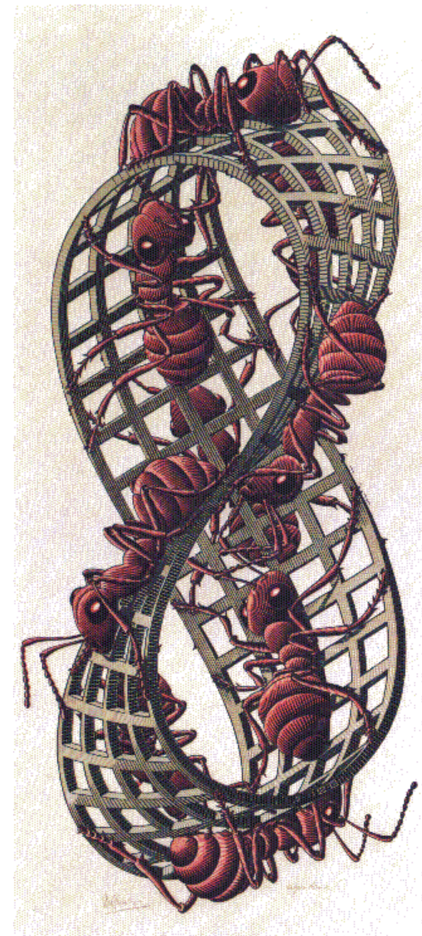


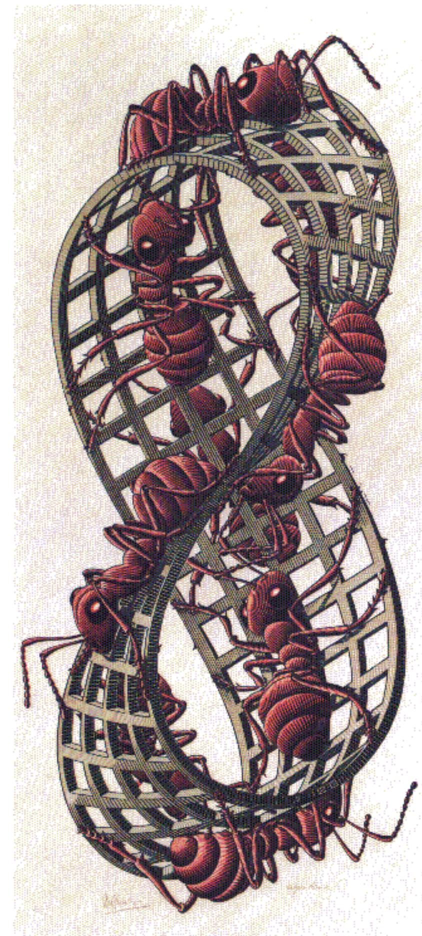
experience: general



experience: general

amazing number of flaws

- blatant and subtle
- in every model



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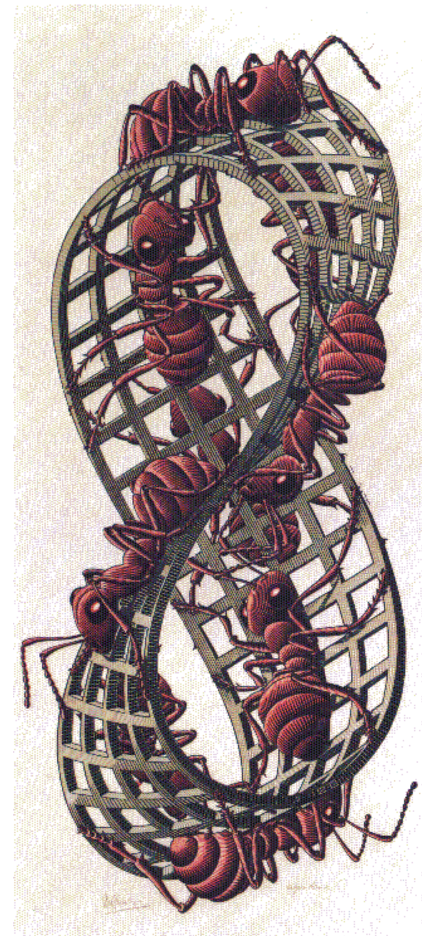
- sense of confidence
- compelling and fun



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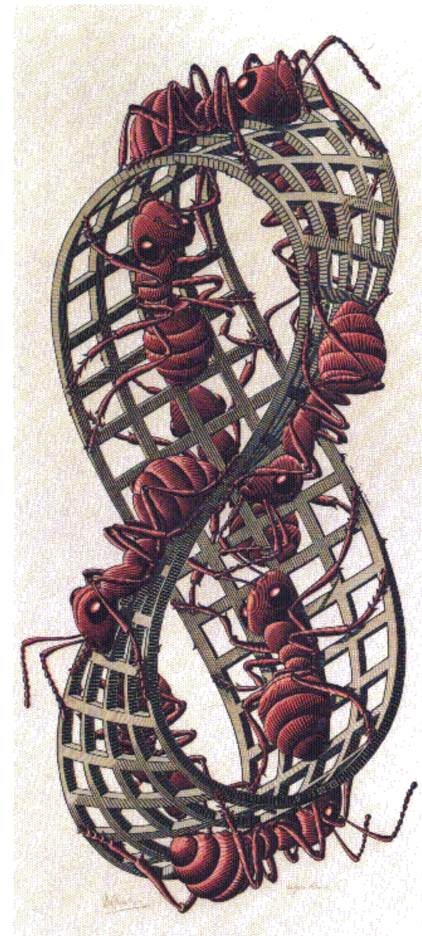
bad things
- encourages hacking
- over confidence



experience: design analyses

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about 20 small case studies completed

- Key management (Taghdiri)
- Chord peer-to-peer storage (Wee)
- Firewire leader election (Jackson)
- Intentional Naming (Khurshid)
- Query Interface in COM (Sullivan)
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typically

› a few hundred lines of Alloy
› longest analysis time: 10 mins to 1 hour

experience: education

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helps teach modelling

- › abstract descriptions, concrete cases
- › very close to standard first-order logic

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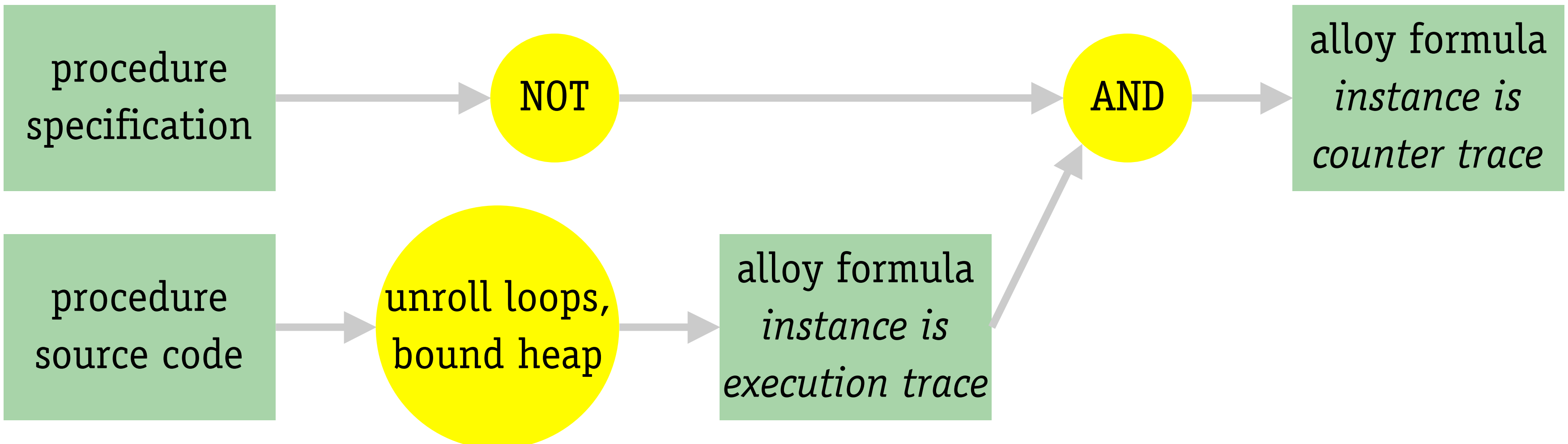
how long?
- undergraduate with no formal methods background
  can build small models in 2 weeks

applications: code analysis



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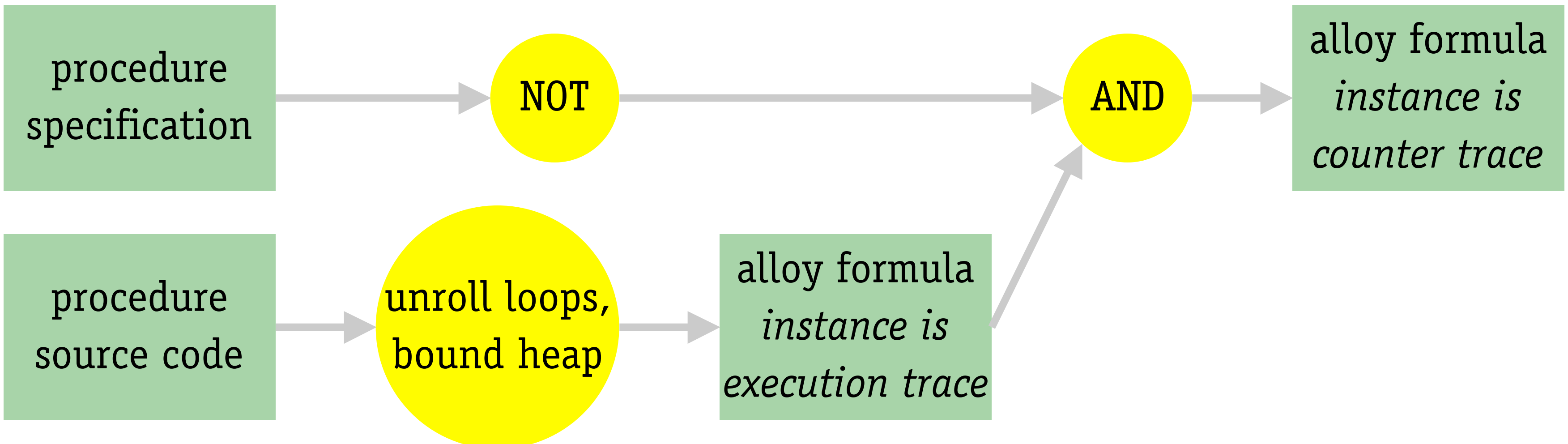

applied to small, complex algorithms

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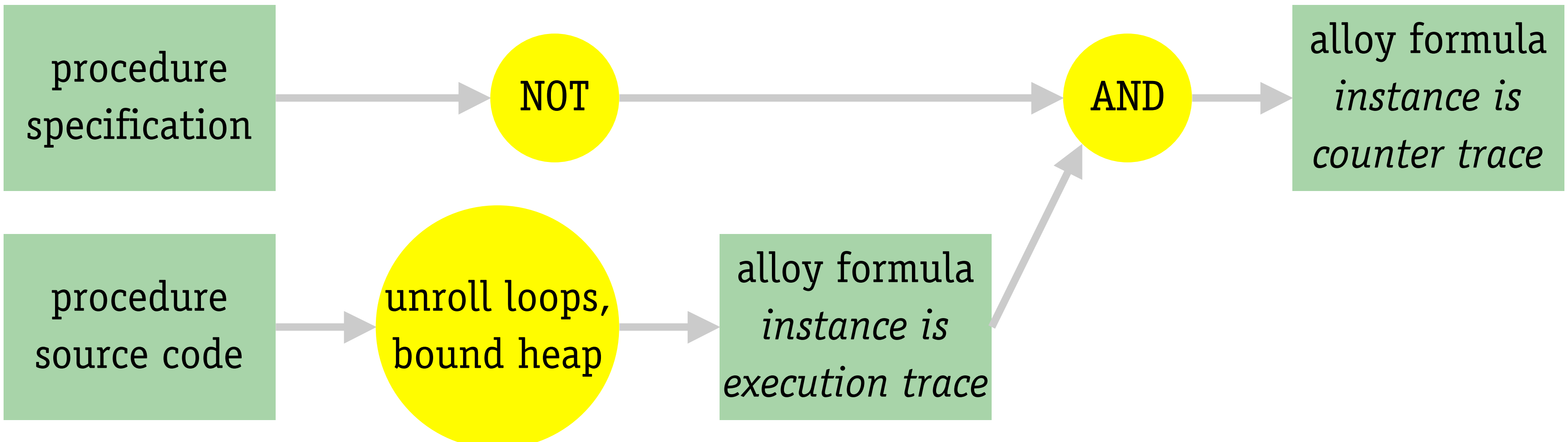

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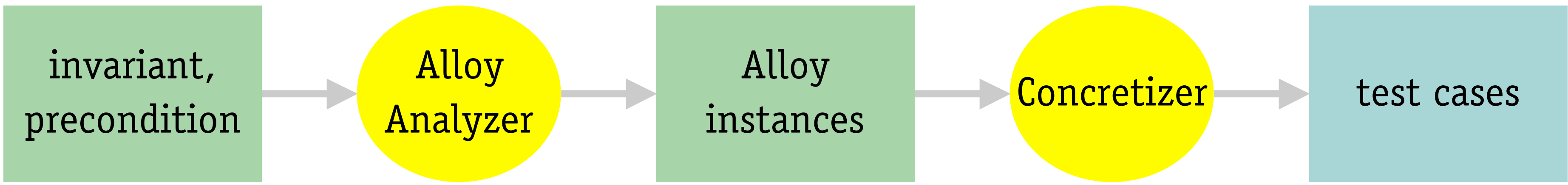
Mandana Vaziri's doctoral thesis

applications: test case generation



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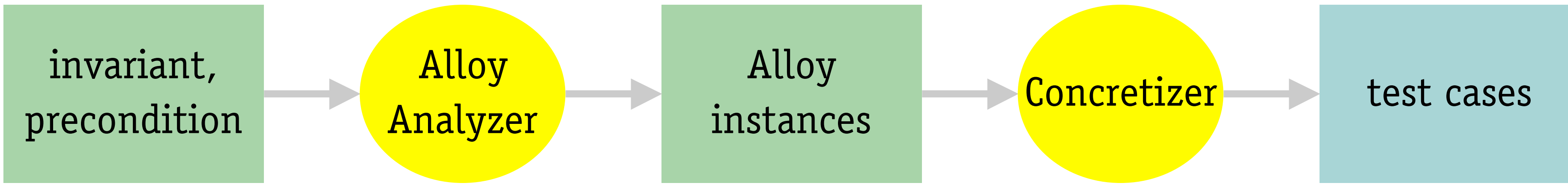

why?

- easier to write invariant than test cases
- all test cases within scope give better coverage
- symmetry breaking gives good quality quite

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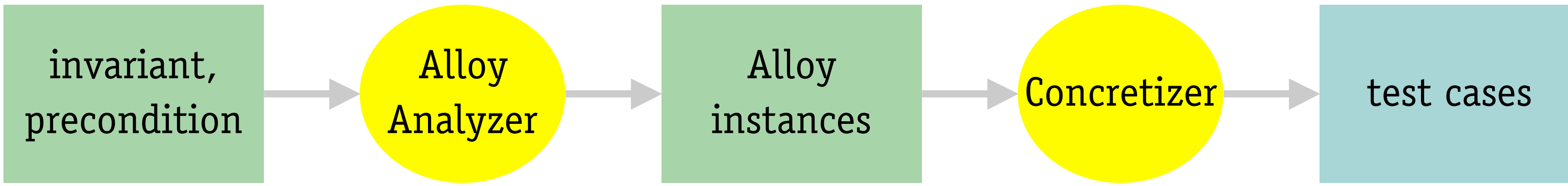

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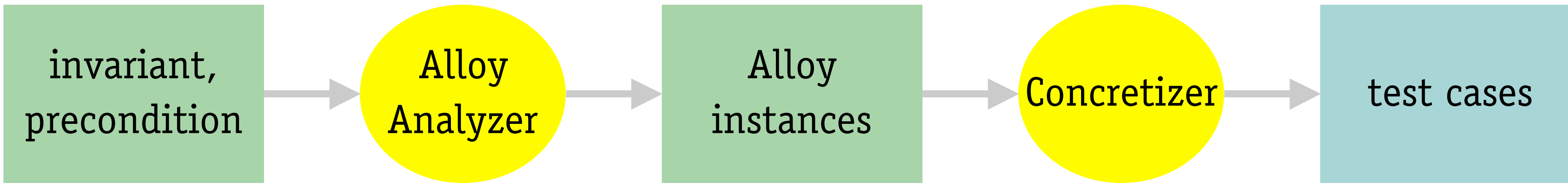

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Sarfraz Khurshid's doctoral thesis

research challenges

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scalability: dancing around the intractability tarpit

› circuit minimization

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model extraction
- looking at how to extract models from code

for more information …

alloy.mit.edu

- downloads for windows, unix, macintosh
- courses, talks, case studies, papers