micromodels of software

declarative modelling and analysis with Alloy

lecture 2: a relational logic

Daniel Jackson
MIT Lab for Computer Science
Marktoberdorf, August 2002

the atlantic divide

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American school of formal methods

› emphasis on verification algorithms
› eg, SMV, SPIN, Murphi

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Alloy brings together
- automatic analysis (like SMV)
- logical notation (like Z)

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first order effects

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Alloy is first order

› to allow exhaustive search

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design implications

› no constructors: composites by projection
› no need to distinguish scalars from singleton sets

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design implications

- no constructors: composites by projection
- no need to distinguish scalars from singleton sets

novel features

- no scalars or sets: all expressions are relation-valued
- generalized relational join operator
- finite interpretation

atoms

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structures are built from

› atoms & relations

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atoms are

› indivisible
    can’t be broken into smaller parts
› immutable
    don’t change over time
› uninterpreted
    no built-in properties

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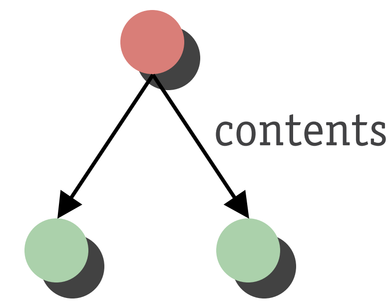
what’s atomic in the real world?

› very little -- a modelling abstraction

atoms



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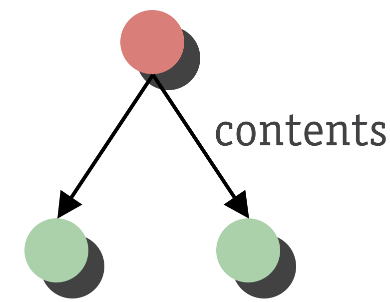

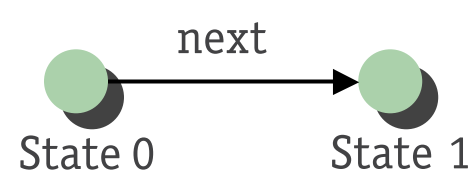

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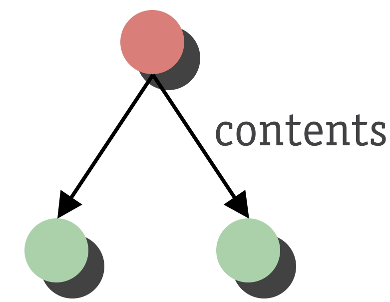

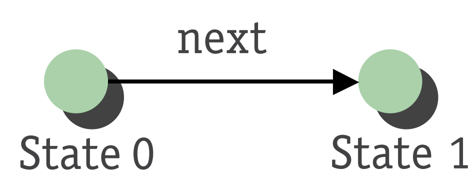

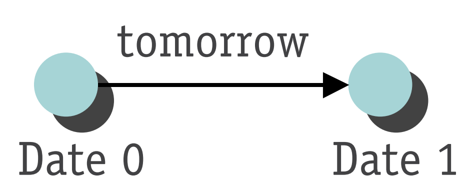

types

types

universe

- contains all atoms
- a finite (but perhaps big) set
- partitioned into basic types, each a set

```
DATE = {JAN1, JAN2, …, DEC31}
PERSON = {ALICE, BOB, CAROL}
STATE = {STATE0, STATE1, STATE2}
FILESYSTEM = {FILESYSTEM0, FILESYSTEM2}
```

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DATE = {JAN1, JAN2, ..., DEC31}
PERSON = {ALICE, BOB, CAROL}
STATE = {STATE0, STATE1, STATE2}
FILESYSTEM = {FILESYSTEM0, FILESYSTEM2}
```

no subtyping, so

- atoms that share properties share a type

```
Employer = {ALICE}
Employee = {BOB, CAROL}
Employer, Employee in PERSON
```

relations

relations

definition

- a tuple is a list of atoms
- a relation is a set of tuples

```
birthday = {(ALICE,MAY1), (BOB,JAN4), (CAROL,DEC9)}
likes = {(ALICE,BOB), (BOB,CAROL), (CAROL,BOB)}
```

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```

typing

- a relation type is a non-empty list of basic types
- if i-th type is T, then i-th atom in each tuple is in T

```
birthday: (PERSON, DATE)
likes: (PERSON, PERSON)
```

relations as tables

relations as tables

can view relation as table

- atoms as entries, tuples as rows
- order of columns matters, but not order of rows
- can have zero rows, but not zero columns
- no blank entries

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example

`birthday = {(ALICE,MAY1), (BOB,JAN4), (CAROL,DEC9)}`

PERSON	DATE
ALICE	MAY1
BOB	JAN4
CAROL	DEC9

dimensions

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arity

- number of columns
- relation of arity k is a k-relation
- unary, binary, ternary for k=1, 2, 3
- finite, >0

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    #p   is an integer expression giving the size of p

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    #p   is an integer expression giving the size of p

homogeneity

- relation of type (T, T, …T) is homogeneous
- else heterogeneous

relations as graphs

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can view 2-relation as graph

- atoms as nodes
- tuples as arcs

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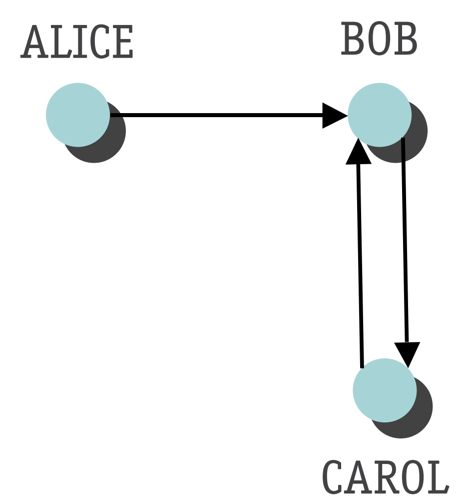

sets and scalars

sets and scalars

sets and scalars

- represented as relations
- set: a unary relation
- scalar: a unary, singleton relation

```
PERSON = {(ALICE), (BOB), (CAROL)}        -- note ()'s!
Employee = {(BOB), (CAROL)}
Employer = {(ALICE)}
Alice = {(ALICE)}
```

sets and scalars

sets and scalars

- represented as relations
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```
PERSON = {(ALICE), (BOB), (CAROL)}        -- note ()'s!
Employee = {(BOB), (CAROL)}
Employer = {(ALICE)}
Alice = {(ALICE)}
```

unlike standard set theory

- no distinction between

  a, (a), {a}, **{(a)}**

ternary relations

ternary relations

for relationships involving 3 atoms

```
salary: [PERSON, COMPANY, SALARY]
salary = {(ALICE,APPLE,$60k), (BOB,BIOGEN,$70k)}
```

ternary relations

for relationships involving 3 atoms

```
salary: [PERSON, COMPANY, SALARY]
salary = {(ALICE,APPLE,$60k), (BOB,BIOGEN,$70k)}
```

for associating binary relations with atoms

```
birthdayRecords: [BIRTHDAYBOOK, PERSON, DATE]
birthdayRecords =
  {(BB0,ALICE,MAY1), (BB0,BOB,JAN4), (BB1,CAROL,DEC9)}
```

left and right

left and right

left and right sets

- left (right) set of p is set of atoms in left-(right-)most column

left and right

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left and right types

- left (right) type of p is the first (last) basic type of p's type

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left and right types

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examples

```
likes = {(ALICE,BOB), (BOB,CAROL), (CAROL,BOB)}
left-set(likes) =  {(ALICE,BOB,CAROL)}
right-set(likes) =  {(BOB,CAROL)}
left-type(likes) = right-type(likes) = PERSON
```

set operators

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standard set operators

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union	p + q	contains tuples of p and tuples of q

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interpretation of +

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- for scalars, makes a set   Alice + Bob

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interpretation of +

- for scalars, makes a set — Alice + Bob
- for sets, makes a new set — Employer + Employee

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interpretation of +

› for scalars, makes a set	Alice + Bob
› for sets, makes a new set	Employer + Employee
› for relations, combines maps	likes + Alice -> Bob

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subset	p in q	q contains every tuple p contains

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subset and equality

subset	p in q	q contains every tuple p contains
equality	p = q	p and q contain same set of tuples

product

product

definition

if p contains (p1,…,pn)

and q contains (q1,…,qm)

then **p -> q** contains (p1,…,pn,q1,…,qm)

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and q contains (q1,…,qm)

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puns

for sets s and t, **s->t** is cartesian product

for scalars a and b, **a->b** is tuple

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for sets s and t, **s->t** is cartesian product

for scalars a and b, **a->b** is tuple

examples

birthday = Alice->May1 + Bob->Jan4 + Carol->Dec9

Employee->Employee in likes

join

join

definition

if p contains (p1,…,pn-1,pn)

and q contains (q1,…,qm)

and pn = q1

then **p . q** contains (p1,…,pn-1,q2,…,qm)

join

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and q contains (q1,…,qm)

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constraints

arity(p) + arity(q) > 2

right-type(p) = left-type(q)

join, examples

join, examples

given

Alice = {(ALICE)}, bb0 = {(BB0)}

likes = {(ALICE,BOB), (BOB,CAROL), (CAROL,BOB)}

birthday = {(ALICE,MAY1), (BOB,JAN4), (CAROL,DEC9)}

birthdayRecords =

{(BB0,ALICE,MAY1), (BB0,BOB,JAN4), (BB1,CAROL,DEC9)}

join, examples

given

Alice = {(ALICE)}, bb0 = {(BB0)}

likes = {(ALICE,BOB), (BOB,CAROL), (CAROL,BOB)}

birthday = {(ALICE,MAY1), (BOB,JAN4), (CAROL,DEC9)}

birthdayRecords =

{(BB0,ALICE,MAY1), (BB0,BOB,JAN4), (BB1,CAROL,DEC9)}

we have

Alice.likes = {(BOB)}; likes.Alice = {}

likes.birthday = {(ALICE,JAN4), (BOB,DEC9),(CAROL,JAN4)}

bb0.birthdayRecords = {(ALICE,MAY1), (BOB,JAN4)}

Alice.(bb0.birthdayRecords) = {(MAY1)}

join, puns

join, puns

puns

for set s and binary relation r, **s.r** is image of s under r

for binary relations p and q, **p.q** is standard join of p and q

for binary relation r of type (S,T),

- **S.r** is right-set of r
- **r.T** is left-set of r

join variants

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for non-binary relations, join is not associative

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3 syntactic variants of join

p.q = p::q = q[p]

binding power: :: most, then ., then []

p.q::r = p.(q.r)

p.q[r] = r.(p.q)

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3 syntactic variants of join

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p.q[r] = r.(p.q)

equivalent expressions

Alice.(bb0.birthdayRecords)

Alice.bb0::birthdayRecords

bb0.birthdayRecords [Alice]

transpose

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for relation r: (S,T)

- **~r** contains (b,a) whenever r contains (a,b)
- **~r** has type (T,S)

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a theorem

- for set s and binary relation r,
  - **r.s = s.~r**

override

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for relations p,q: (S,T)

p++q contains (a,b) whenever

q contains (a,b), or

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given

Alice = {(ALICE)}, March3 = {(MAR3)}

birthday = {(ALICE,MAY1), (BOB,JAN4), (CAROL,DEC9)}

override

for relations p,q: (S,T)

p++q contains (a,b) whenever

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given

Alice = {(ALICE)}, March3 = {(MAR3)}

birthday = {(ALICE,MAY1), (BOB,JAN4), (CAROL,DEC9)}

we have

birthday ++ Alice->March3 =

{(ALICE,MAR3), (BOB,JAN4), (CAROL,DEC9)}

closure

closure

for relation r: (T,T)

^r = r + r.r + r.r.r + r.r.r.r + …

is smallest **transitive** relation p containing r

***r** = iden[T] + r + r.r + r.r.r + r.r.r.r + …

is smallest **reflexive & transitive** relation p containing r

closure

for relation r: (T,T)

^r = r + r.r + r.r.r + r.r.r.r + …

is smallest **transitive** relation p containing r

***r** = iden[T] + r + r.r + r.r.r + r.r.r.r + …

is smallest **reflexive & transitive** relation p containing r

examples

ancestor = ^parent

reaches = *connects

precedes = ^~next

operator types

if …	then …
p,q: (T1,…,Tn)	p+q, p-q, p&q: (T1,…,Tn), p in q
p: (S1,…,Sn), q: (T1, …,Tm), Sn = T1	p.q: (S1,…,Sn-1,T2,…,Tm)
p: (S,T)	~p: (T,S)
p: (T,T)	*p, ^p: (T,T)
p: (T)	iden[p]: (T,T)
p: (T1,…,Tn)	none[p], univ[p]: (T1,…,Tn)

navigation expressions

navigation expressions

from 2-relations and the operators

. + ^ * ~

navigation expressions

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. + ^ * ~

interpret as path-sets

p.q	follow p then q
p+q	follow p or q
^p	follow p once or more
***p**	follow p zero or more times
~p	follow p backwards

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cousin = parent.sibling.~parent

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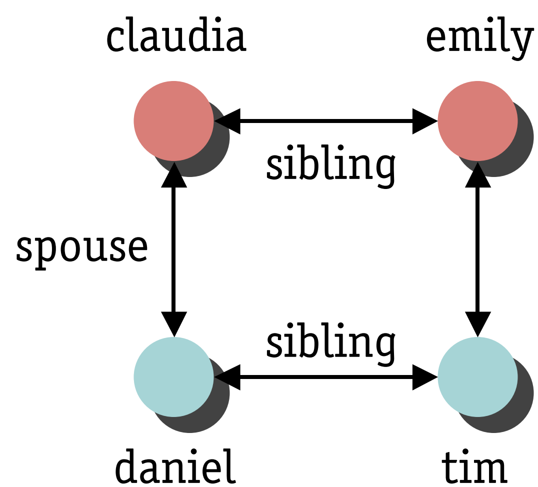

daniel.**spouse.sibling** =
daniel.**sibling.spouse**

a navigation example

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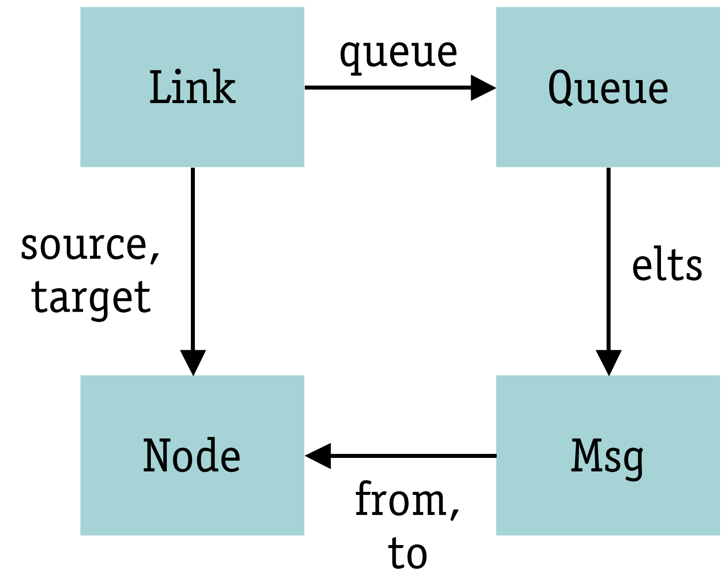
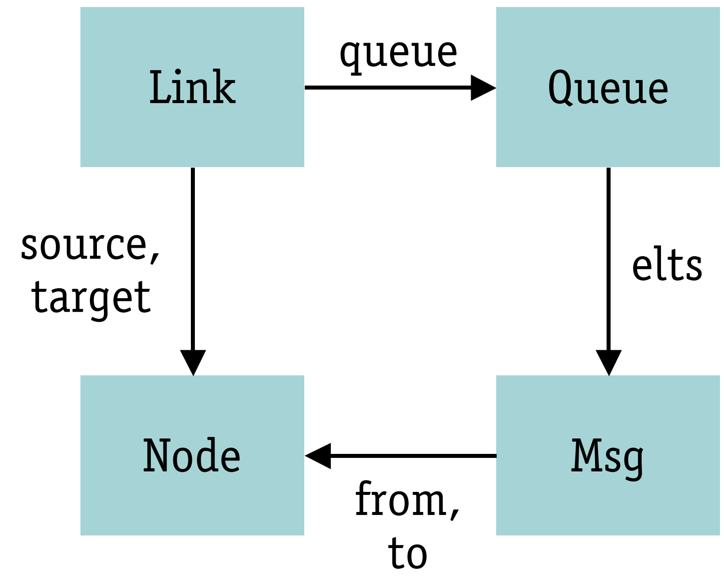


a navigation example

to say

- all messages queued on links emanating from a node have a ‘from’ field of that node



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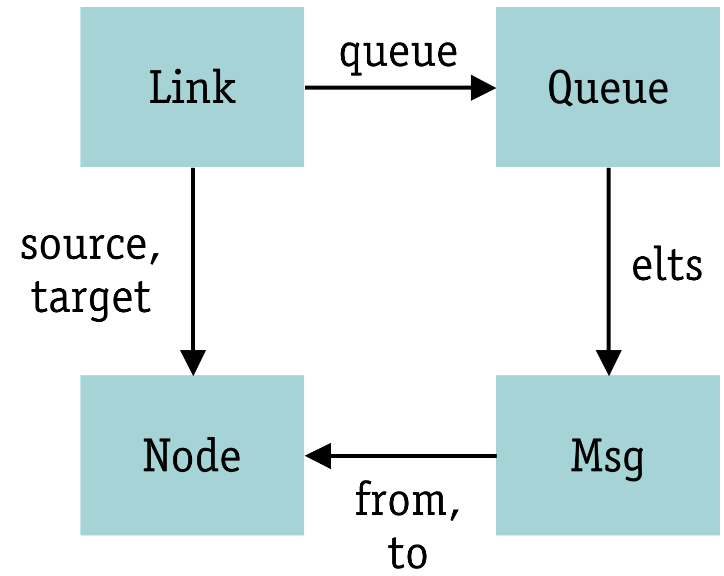

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```
all n: Node | n.~source.queue.elts.from = n
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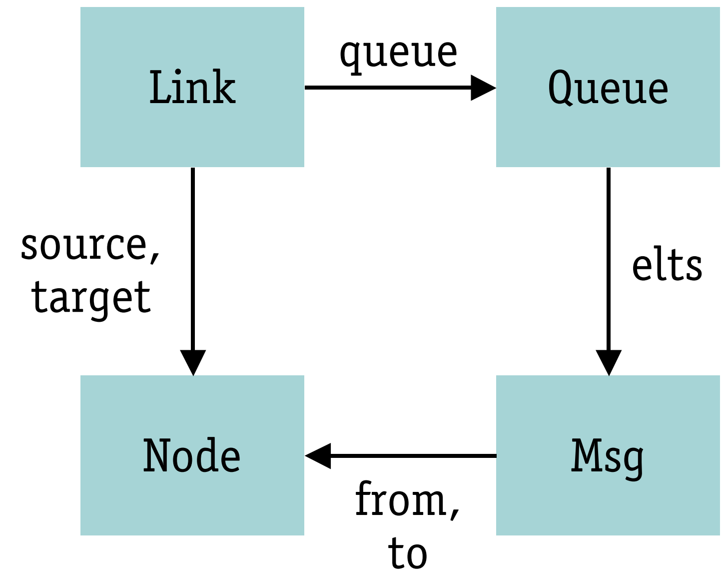

we can write

```
all n: Node | n.~source.queue.elts.from = n
```

or equivalently

```
~source.queue.elts.from in iden[Node]
```

logical operators

logical operators

standard connectives

! F	**not** F	
F **&&** G	F **and** G	**{** F G **}**
F **||** G	F **or** G	
F **=>** G , H	F **implies** G **else** H	
F **<=>** G	F **iff** G	

logical operators

standard connectives

```
! F            not F
F && G         F and G          { F G }
F || G         F or G
F => G , H     F implies G else H
F <=> G        F iff G
```

if-then-else expressions

```
if F then e else e′
```

logical operators

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if-then-else expressions

if F **then** e **else** e′

negated operators

e **!in** e′ , e **!=** e′

set declarations

set declarations

form

var : [**set** | **option**] setexpr

set declarations

form

var : [**set** | **option**] setexpr

meaning

v : e	v **in** e **and** #v = 1
v : **set** e	v **in** e
v : **option** e	v **in** e **and** #v ≤ 1

set declarations

form

var : [**set** | **option**] setexpr

meaning

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examples

p : Person	*p is a scalar in Person*
Employee: set Person	*Employee is a subset of Person*
bb : Person -> Date	*not unary, so no scalar constraint*

set declarations

> same meaning as
> **Employee: ℙ Person**
> in a other languages,
> but first order

form

var : [**set** | **option**] setexpr

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relation declarations

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form

```
var : expr [mult] -> [mult] expr
```

relation declarations

form

```
var : expr [mult] -> [mult] expr
```

multiplicity symbols

- ? zero or one
- ! exactly one
- + one or more

relation declarations

form

var : expr [mult] -> [mult] expr

multiplicity symbols

? zero or one
! exactly one
+ one or more

meaning

r: e0 *m* **->** *n* e1

means r **in** e0 -> e1

and *n e1's for each e0,*
m e0's for each e1

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examples

r: A ->? B

r is a partial function

r: A ->! B

r is a total function

r: A ?->? B

r is an injective

r: A !->! B

r is a bijection

object models

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what is an object model?

- set of declarations drawn as graph
- boxes denote sets, arcs relations
- parentless box has implicit type

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› set of declarations drawn as graph
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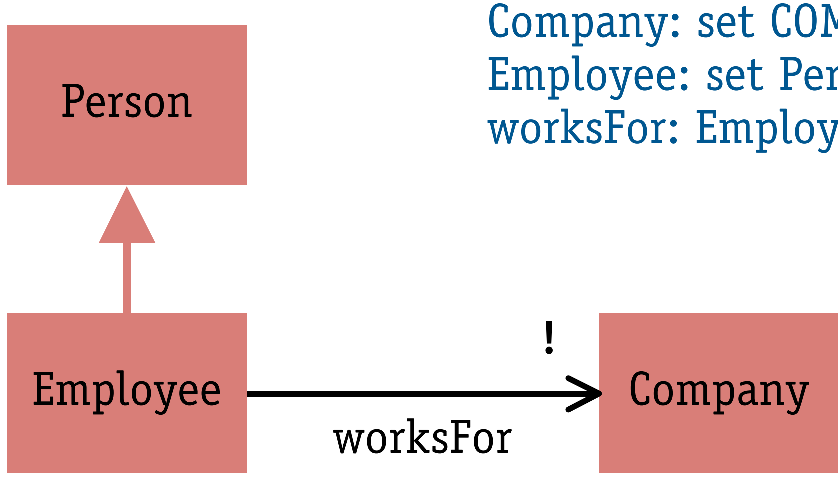
› parentless box has implicit type

```
Person: set PERSON
Company: set COMPANY
Employee: set Person
worksFor: Employee ->! Company
```



comprehensions

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general form

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{ var : setexpr ,... | formula }
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{ v0: e0, v1: e1, ... | F }
  is the relation containing tuples (a0,a1, ...)
  such that F holds when v0 = {(a0)}, v1 = {(a1)}, etc
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example

```
sibling = {a, b: Person | a.parents = b.parents && a != b}
```

quantification

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universal quantification

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example

all a: Person | a **!in** a.parents

other quantifiers

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all x: e | F — F holds for **all** x in e

some x: e | F — F holds for **some** x in e

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note

all v0: e0, v1: e1,… | F *is equivalent to*

all v0: e0 | **all** v1: e1 | … | F

one v0: e0, v1: e1,… | F *is not equivalent to*

one v0: e0 | **one** v1: e1 | … | F

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example

no Man **&** Woman *no person is both a man and a woman*

sample quantifications

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biological constraints

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all p: Person | one p.mother
no p: Person | p in p.parents
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biblical constraints

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one eve: Person | Person in eve.*~mother
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summary: doing more with less

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a->b in r *for* (a, b) □ r *and* a □ b □ r

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intractable — tractable

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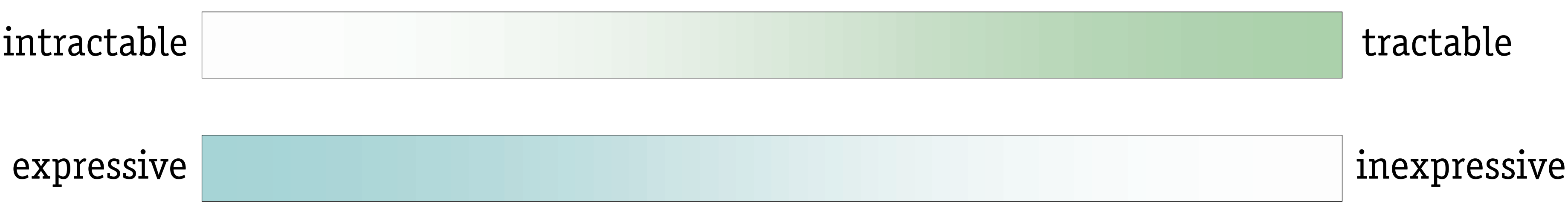

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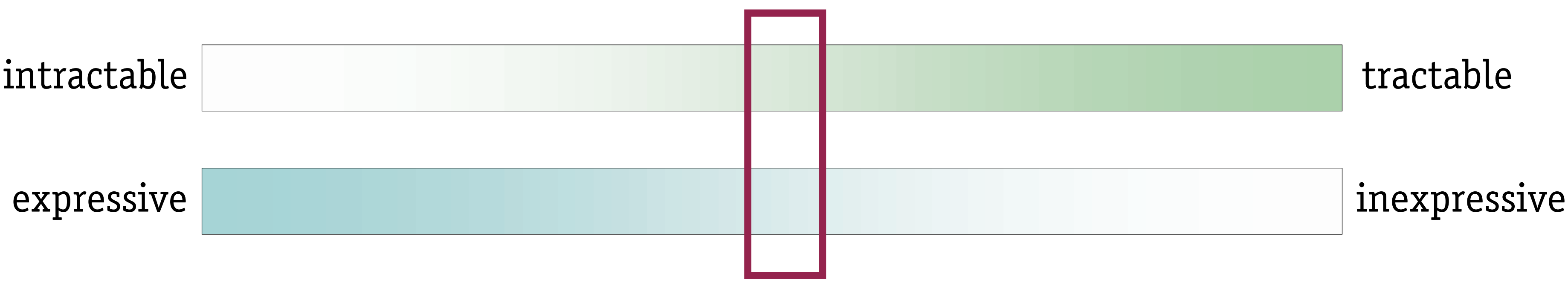

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write a constraint

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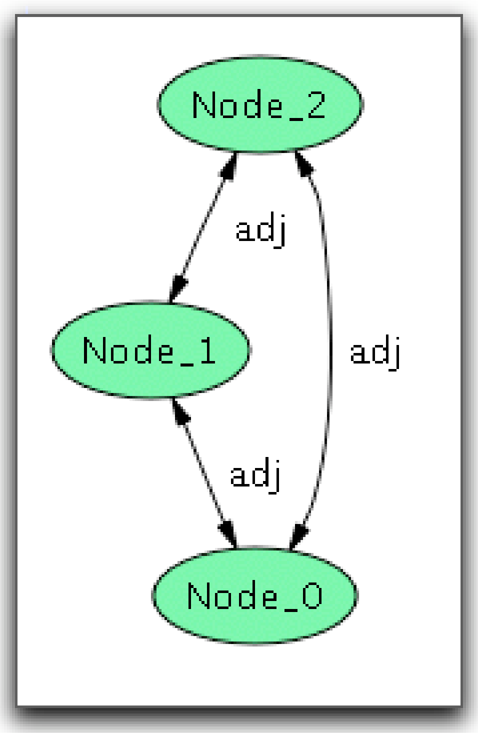

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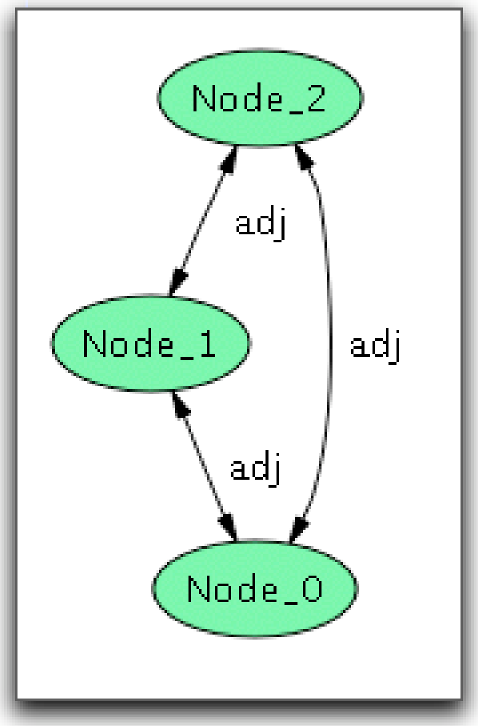
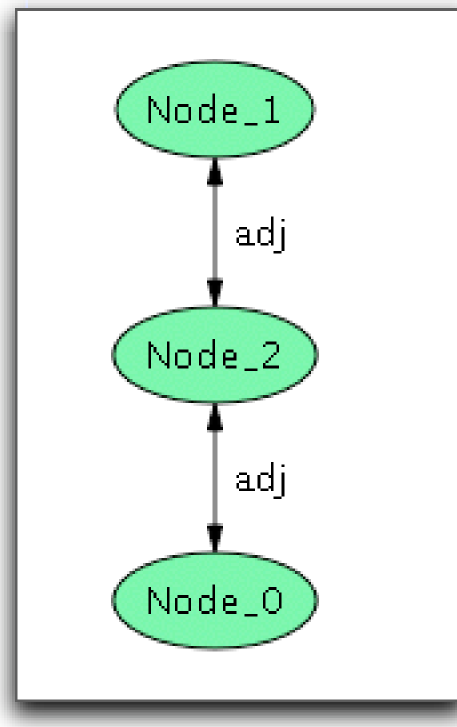




a solution

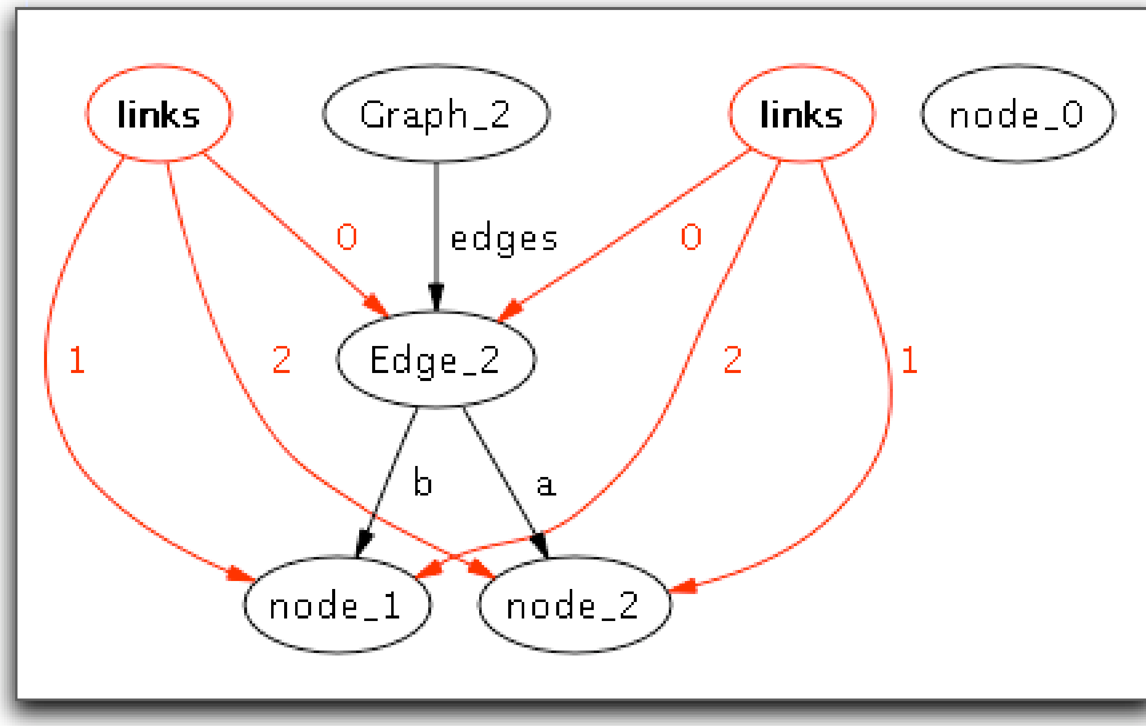
a solution

```
sig Edge[t] {links: t->t}
  {some a,b: t | links = a->b + b->a}

sig Graph[t] {edges: set Edge[t]}

fun Acyclic [t] (g: Graph[t]) {
  no e: Edge[t] |
    let adj = g.edges.links, adj' = (g.edges-e).links |
      *adj = *adj'
  }
```

sample graph



higher-order quantifiers

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general form

quantifier decl ,… | formula

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modified set expressions

all s: **set** S | F	F holds for all s = S′ where S′ in S
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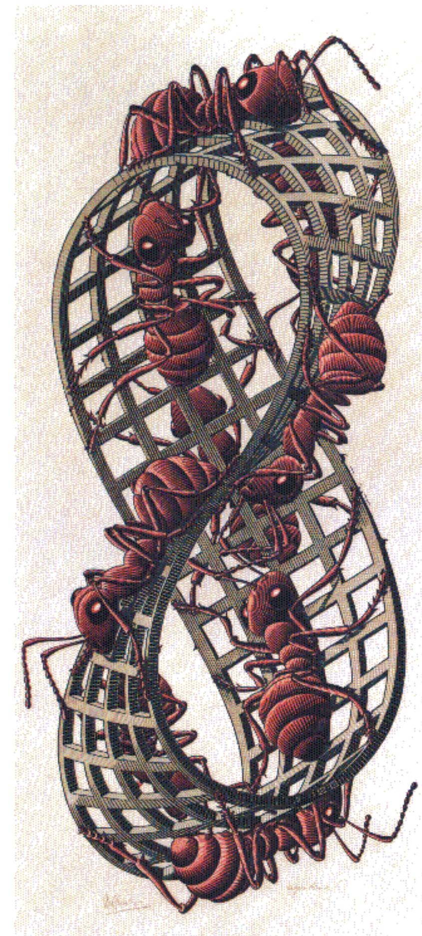
examples

all p, q, r : T -> T | p.(q.r) = (p.q).r

all p : S -> T, s : **set** S | s.p = ~p.s

model checking

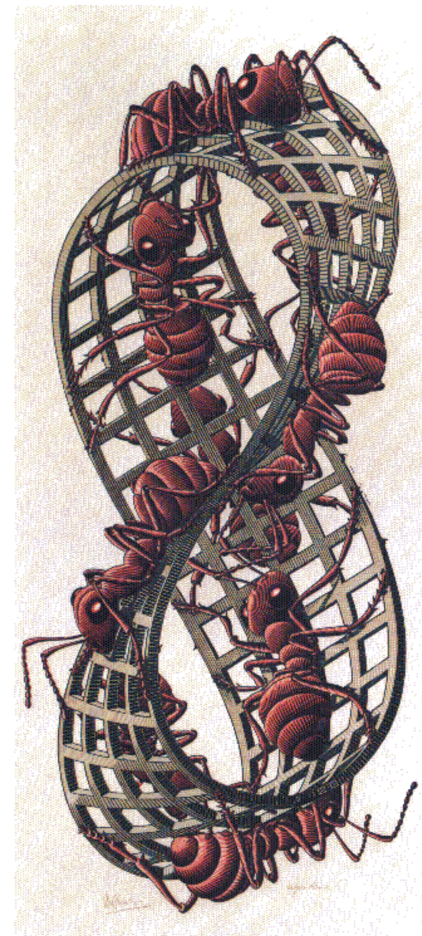


model checking

only low-level datatypes

› must encode in records, arrays

› no transitive closure, etc



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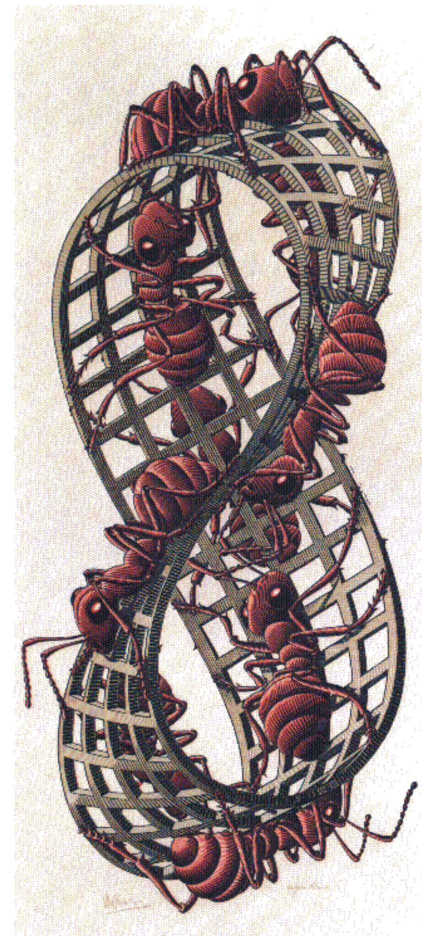
- not suited for abstract schemes
- fixed topology of processes



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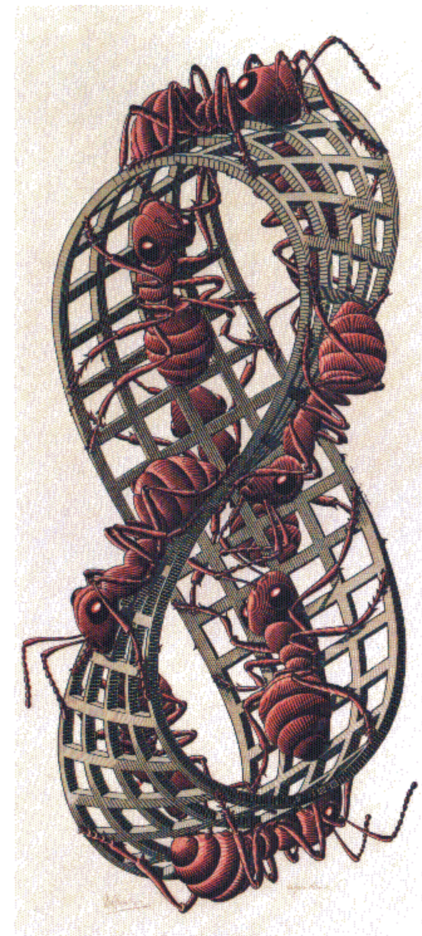

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culture of model checking

- emphasizes finding showstopper flaws
- but in software, essence is incremental modelling
- keep counters, discard model or vice versa?