

**micromodels of software  
declarative modelling  
and analysis with Alloy**

**lecture 4: the Alloy language**

Daniel Jackson

MIT Lab for Computer Science  
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# **the Alloy language**

# the Alloy language

syntactic constructs

- › for packaging formulas
- › for incremental description of structures

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syntactic constructs

- › for packaging formulas
- › for incremental description of structures

design principles

- › remain first-order
- › keep it simple (no partial functions, no subtypes)
- › idiom-free (like PVS, ACL2)

**look out for ...**

# look out for ...

key constructs

- › signatures and functions

# look out for ...

## key constructs

- › signatures and functions

## puns

- › signatures like Java's classes
- › dot like Java's dereferencing dot
- › functions like PL functions

# look out for ...

## key constructs

- › signatures and functions

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- › signatures like Java's classes
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- › functions like PL functions

## what's missing

- › composites
- › subtyping
- › state machines



# look out for ...

## key constructs

- › signatures and functions

## puns

- › signatures like Java's classes
- › dot like Java's dereferencing dot
- › functions like PL functions

## what's missing

- › composites
- › subtyping
- › state machines

no notion of execution at all

# gross structure

```
module Memory
  sig Memory {data: Addr ->? Data}
  sig Addr {}
  sig Data {}
  fact {all m: Memory, a: Addr | sole m.data[a]}
  fun Write (m,m': Memory,a: Addr,d: Data) {
    m'.data = m.data ++ a->d
  }
  assert WriteWorks {
    all m,m': Memory, a: Addr, d: Data |
      Write (m,m',a,d) => m.data[a] = d
  }
  run Write
  check WriteWorks
```

# gross structure

- › signature declares set & relations

```
module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}
fact {all m: Memory, a: Addr | sole m.data[a]}
fun Write (m,m': Memory,a: Addr,d: Data) {
  m'.data = m.data ++ a->d
}
assert WriteWorks {
  all m,m': Memory, a: Addr, d: Data |
    Write (m,m',a,d) => m.data[a] = d
}
run Write
check WriteWorks
```

# gross structure

- › **signature**  
declares set & relations
- › **fact**  
global constraint

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module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
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# gross structure

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declares set & relations
- › **fact**  
global constraint
- › **function**  
parameterized constraint

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module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
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fact {all m: Memory, a: Addr | sole m.data[a]}
fun Write (m,m': Memory,a: Addr,d: Data) {
  m'.data = m.data ++ a->d
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assert WriteWorks {
  all m,m': Memory, a: Addr, d: Data |
    Write (m,m',a,d) => m.data[a] = d
}
run Write
check WriteWorks
```

# gross structure

- › **signature**  
declares set & relations
- › **fact**  
global constraint
- › **function**  
parameterized constraint
- › **assertion**  
theorem to check

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module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}
fact {all m: Memory, a: Addr | sole m.data[a]}
fun Write (m,m': Memory,a: Addr,d: Data) {
  m'.data = m.data ++ a->d
}
assert WriteWorks {
  all m,m': Memory, a: Addr, d: Data |
    Write (m,m',a,d) => m.data[a] = d
}
run Write
check WriteWorks
```

# gross structure

- > **signature**  
declares set & relations
- > **fact**  
global constraint
- > **function**  
parameterized constraint
- > **assertion**  
theorem to check
- > **command**  
run function  
check assertion

```
module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}
fact {all m: Memory, a: Addr | sole m.data[a]}
fun Write (m,m': Memory,a: Addr,d: Data) {
  m'.data = m.data ++ a->d
}
assert WriteWorks {
  all m,m': Memory, a: Addr, d: Data |
    Write (m,m',a,d) => m.data[a] = d
}
run Write
check WriteWorks
```

# basic signature decs



# basic signature decls

introduces set and basic type

**sig** *s* {}

*set s with basic type (S, say)*

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**sig** *s* {}                    *set s with basic type (S, say)*

set is immutable

› a global variable with one value

# basic signature decls

introduces set and basic type

**sig** *s* {}                    *set s with basic type (S, say)*

set is immutable

- › a global variable with one value

examples

**sig** Memory {}

**sig** Addr {}

**sig** Data {}

**sig** FileSystem {}

**sig** BirthdayBook {}

# extension decls

# extension decls

introduces subset of existing set

**sig t extends s {}**      *t subset of s*

# extension decls

introduces subset of existing set

**sig** *t extends s {}*      *t subset of s*

subset is immutable

› a global variable with one value

# extension decls

introduces subset of existing set

**sig** *t* **extends** *s* {}      *t* *subset of s*

subset is immutable

- › a global variable with one value

examples

**sig** Cache **extends** Memory {}

**sig** Man **extends** Person {}

**sig** Dir **extends** FileSysObj {}

# disjointness



# disjointness

can mark subsigs as mutually disjoint

```
disj sig t1 extends s {}           t1 and t2 disjoint ...  
disj sig t2 extends s {}  
sig t3 extends s {}                 ... but not nec t3
```

# disjointness

can mark subsigs as mutually disjoint

```
disj sig t1 extends s {}           t1 and t2 disjoint ...
disj sig t2 extends s {}
sig t3 extends s {}                ... but not nec t3
```

examples

```
sig Memory {}
disj sig Cache extends Memory {}
disj sig MainMemory extends Memory {}

sig Person {}
part sig Man, Woman extends Person {}
sig Employee extends Person {}
```

# singletons

# singletons

constrains set to be a singleton

**static sig s {}**

*s has one tuple*

# singletons

constrains set to be a singleton

**static sig** s {}      *s has one tuple*

examples

**static part sig** Red, Green, Blue **extends** Colour {}

**static sig** Root **extends** Directory {}

# fields

# fields

introduces a relation

**sig**  $s$  {}

**sig**  $t$  {f:  $s$ }

*f is relation of type  $(T,S)$   
with left set  $t$  and right set  $s$*

# fields

introduces a relation

**sig**  $s$  {}

**sig**  $t$  {f:  $s$ }

*f is relation of type (T,S)  
with left set t and right set s*

relation is immutable

- › a global variable with one value



# fields

introduces a relation

```
sig s {}
```

```
sig t {f: s}
```

*f is relation of type (T,S)  
with left set t and right set s*

relation is immutable

- › a global variable with one value

example

```
sig Date {}
```

```
sig Person {birthday: Date}
```

```
fact {some disj p,q: Person | p.birthday = q.birthday}
```

# fields

introduces a relation

**sig** s {}

**sig** t {f: s}

*f is relation of type (T,S)*

*with left set t and right set s*

relation is immutable

- › a global variable with one value

example

**sig** Date {}

**sig** Person {**birthday**: Date}

**fact** {**some disj** p,q: Person | p.birthday = q.birthday}

birthday has type  
(PERSON,DATE)

# overloading

# overloading

resolved on left type

**sig** s {f: u}

*s\$f has type (S, U)*

**sig** t {f: u}

*t\$f has type (T, U)*

# overloading

resolved on left type

<b>sig</b> s {f: u}	<i>s\$f has type (S,U)</i>
<b>sig</b> t {f: u}	<i>t\$f has type (T,U)</i>

example

```
sig Name {}  
sig City {name: Name}  
sig Person {name: Name}  
fact {no disj p,q: Person | p.name = q.name}
```

# overloading

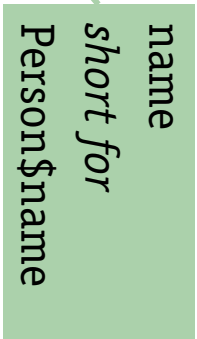
resolved on left type

**sig** s {f: u}                      *s\$f has type (S,U)*  
**sig** t {f: u}                      *t\$f has type (T,U)*

example

**sig** Name {}  
**sig** City {name: Name}  
**sig** Person {name: Name}  
**fact** {**no disj** p,q: Person | p.name = q.name}

name  
short for  
Person\$name



# higher-arity fields

# higher-arity fields

any expression can be used in field decl

```
sig s {f: e}
```



# higher-arity fields

any expression can be used in field decl

```
sig s {f: e}
```

examples

```
sig Memory {data: Addr ->? Data}
```

```
sig FileSysObj {}
```

```
sig FileSystem {
```

```
  disj files, dirs: set FileSysObj,
```

```
  root: dirs,
```

```
  parent: (files + dirs - root) ->! dirs
```

```
}
```

```
sig BirthdayBook {records: Person ->? Date}
```

# higher-arity fields

any expression can be used in field decl

```
sig s {f: e}
```

examples

```
sig Memory {data: Addr ->? Data}
```

```
sig FileSysObj {}
```

```
sig FileSystem {
```

```
  disj files, dirs: set FileSysObj,
```

```
  root: dirs,
```

```
  parent: (files + dirs - root) ->! dirs
```

```
}
```

```
sig BirthdayBook {records: Person ->? Date}
```

data has type  
(MEMORY, ADDR, DATA)  
all m: Memory |  
m.data : Addr ->? Data

# higher-arity fields

any expression can be used in field decl

```
sig s {f: e}
```

examples

```
sig Memory {data: Addr ->? Data}
```

```
sig FileSysObj {}
```

```
sig FileSystem {
```

```
  disj files, dirs: set FileSysObj,
```

```
  root: dirs,
```

```
  parent: (files + dirs - root) ->! dirs  
}
```

```
sig BirthdayBook {records: Person ->? Date}
```

data has type  
(MEMORY, ADDR, DATA)  
all m: Memory |  
m.data : Addr ->? Data

parent has type  
(FILESYSTEM, FILESYSOBJ, FILESYSOBJ)  
all s: FileSystem |  
s.parent:(s.files+s.dirs-s.root)->! s.dirs

# higher-arity fields

any expression can be used in field decl

```
sig s {f: e}
```

examples

```
sig Memory {data: Addr ->? Data}
```

```
sig FileSysObj {}
```

```
sig FileSystem {
```

```
  disj files, dirs: set FileSysObj,
```

```
  root: dirs,
```

```
  parent: (files + dirs - root) ->! dirs  
}
```

```
sig BirthdayBook {records: Person ->? Date}
```

data has type  
(MEMORY, ADDR, DATA)  
all m: Memory |  
m.data : Addr ->? Data

parent has type  
(FILESYSTEM, FILESYSOBJ, FILESYSOBJ)  
all s: FileSystem |  
s.parent:(s.files+s.dirs-s.root)->! s.dirs

records has type  
(BIRTHDAYBOOK, PERSON, DATE)  
all bb: BirthdayBook |  
bb.records: Person ->? Date

**show me...**

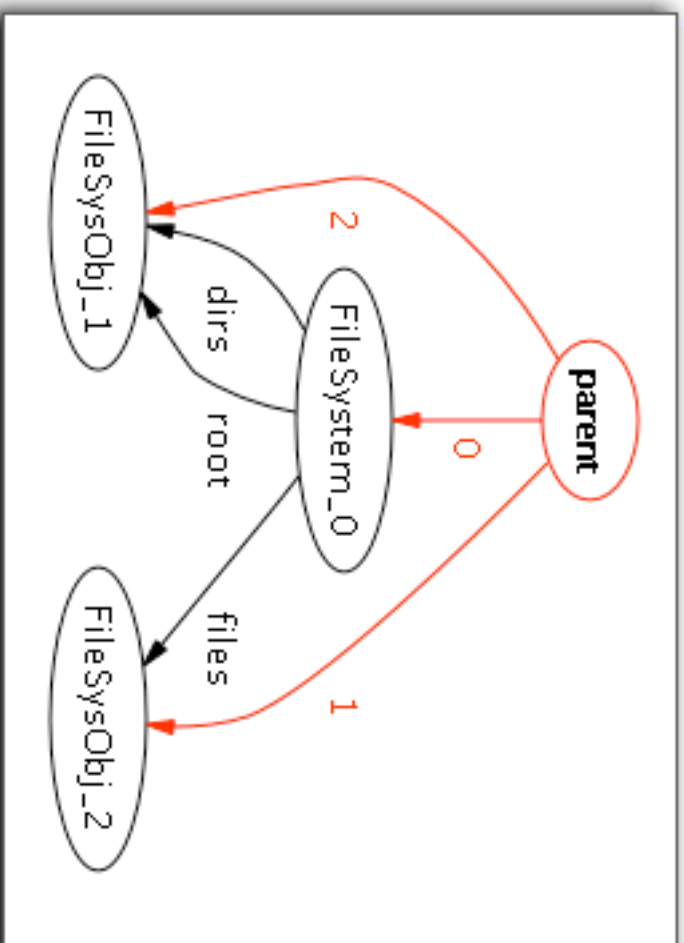
# show me...

```
sig FileSysObj {}  
sig FileSystem {  
  disj files, dirs: set FileSysObj,  
  root: dirs,  
  parent: (files+dirs-root) ->! dirs  
}  
  
fun showMe () {  
  some FileSystem  
}  
  
run showMe
```

# show me...

```
sig FileSysObj {}  
sig FileSystem {  
  disj files, dirs: set FileSysObj,  
  root: dirs,  
  parent: (files+dirs-root) ->! dirs  
}
```

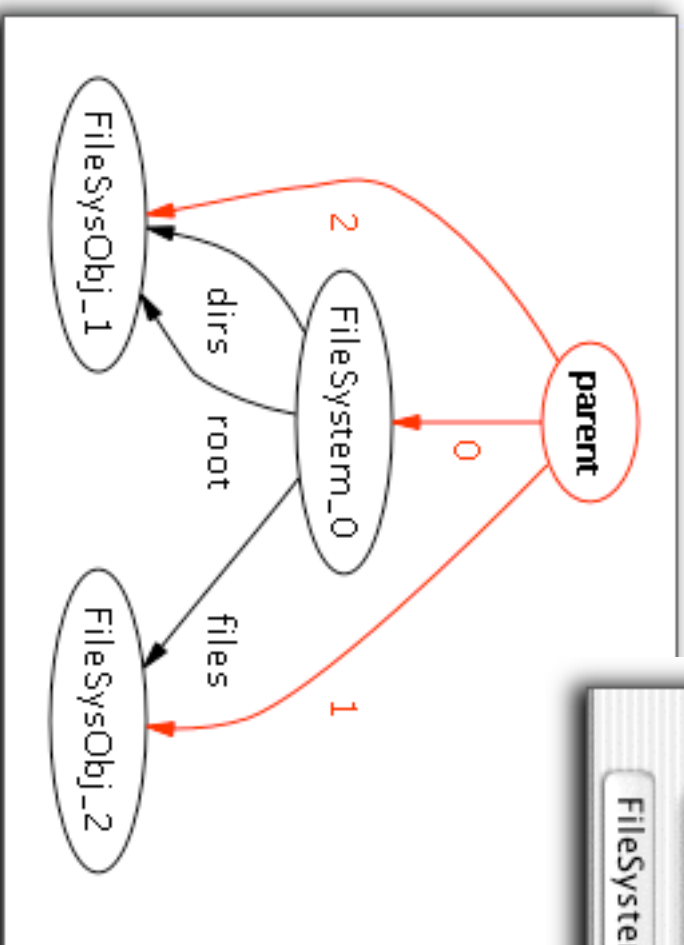
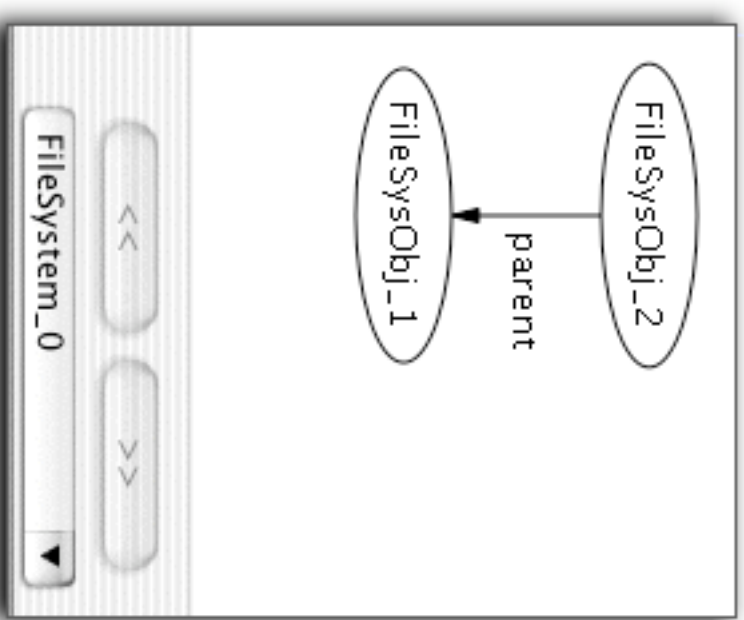
```
fun showMe () {  
  some FileSystem  
}  
run showMe
```



# show me...

```
sig FileSysObj {}  
sig FileSystem {  
  disj files, dirs: set FileSysObj,  
  root: dirs,  
  parent: (files+dirs-root) ->! dirs  
}
```

```
fun showMe () {  
  some FileSystem  
}  
run showMe
```





# subsignature fields

# subsignature fields

fields of subsignatures

- › associated with basic type
- › constrained to have subsig as left set

**sig** *s* {}

**sig** *t* **extends** *s* {*f*: *u*}

*f* has type  $(S, U)$  and left set *t*

# subsignature fields

fields of subsignatures

- › associated with basic type
- › constrained to have subsig as left set

```
sig s {}
```

```
sig t extends s {f: u}    f has type (S, U) and left set t
```

example

```
sig Memory {data: Addr ->? Data}
```

```
sig Cache extends Memory {dirty: set Addr}
```

# subsignature fields

fields of subsignatures

- › associated with basic type
- › constrained to have subsig as left set

```
sig s {}
```

```
sig t extends s {f: u}    f has type (S,U) and left set t
```

example

```
sig Memory {data: Addr ->? Data}
```

```
sig Cache extends Memory {dirty: set Addr}
```

dirty has type  
(MEMORY,ADDR)  
dirty in Cache -> Addr

# subsignature fields

fields of subsignatures

- › associated with basic type
- › constrained to have subsig as left set

```
sig s {}
```

```
sig t extends s {f: u}    f has type (S, U) and left set t
```

example

```
sig Memory {data: Addr ->? Data}
```

```
sig Cache extends Memory {dirty: set Addr}
```

out of domain application

if *m* is not in Memory, *m.cache* is **empty**

dirty has type  
(MEMORY, ADDR)  
dirty in Cache -> Addr

**with**

# with

implicit deref

**with**  $e \mid F$

is like  $F$  with each expression  $e'$   
whose left type matches the type of  $e$   
replaced by  $e'.e$

# with

implicit deref

**with** e | F

is like F with each expression e'  
whose left type matches the type of e  
replaced by e.e

example

```
sig FileSystem {  
  disj files, dirs: set FileSysObj,  
  root: dirs,  
  parent: (files+dirs-root) ->! dirs  
}  
fact {all fs: FileSystem | with fs | files in root.*~parent
```



# with

implicit deref

**with** e | F

is like F with each expression e' whose left type matches the type of e replaced by e.e

example

```
sig FileSystem {  
  disj files, dirs: set FileSysObj,  
  root: dirs,  
  parent: (files+dirs-root) ->! dirs  
}
```

```
fact {all fs: FileSystem | with fs | files in root.*~parent
```

*short for*  
fs.files in fs::root.\*~fs::parent

files in root.\*~parent

# signature facts

# signature facts

signature fact has implicit *all* and *with*

**sig** s {...} {F}

short for

**sig** s {...}

**fact** {all x: s | **with** x | F}

# signature facts

signature fact has implicit *all* and *with*

```
sig s {...} {F}
```

short for

```
sig s {...}
```

```
fact {all x: s | with x | F}
```

example

```
sig FileSystem {  
  disj files, dirs: set FileSysObj,  
  root: dirs,  
  parent: (files+dirs-root) ->! dirs  
} {files + dirs in root.*~parent}
```

# signature facts

signature fact has implicit *all* and *with*

```
sig s {...} {F}
```

short for

```
sig s {...}
```

```
fact {all x: s | with x | F}
```

example

```
sig FileSystem {  
  disj files, dirs: set FileSysObj,  
  root: dirs,  
  parent: (files+dirs-root) ->! dirs  
} {files + dirs in root.*~parent}
```

*short for*  
**all this:** FileSystem | **with this** |  
files **in** root.\*~parent

# functions

# functions

function

- › a parameterized constraint
- › ‘invocation’ is a formula
- › meaning is just inlining/substitution  
(but recursion now supported)

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- › a parameterized constraint
- › ‘invocation’ is a formula
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(but recursion now supported)

example

```
fun Write (m,m': Memory,a: Addr,d: Data) {  
  m'.data = m.data ++ a->d }  
assert WriteWorks {  
  all m,m': Memory, a: Addr, d: Data |  
  Write (m,m',a,d) => m'.data[a] = d }
```



# functions

function

- › a parameterized constraint
- › ‘invocation’ is a formula
- › meaning is just inlining/substitution  
(but recursion now supported)

example

```
fun Write (m,m': Memory,a: Addr,d: Data) {  
  m'.data = m.data ++ a->d }
```

*short for*  
m'.data = m.data ++ a->d

```
assert WriteWorks {  
  all m,m': Memory,a: Addr,d: Data |  
    Write (m,m',a,d) ==> m'.data[a] = d }
```

# checking assertions

# checking assertions

form

**check Assertion [ scope ]**

# checking assertions

form

*check Assertion [ scope ]*

scope

**for** K **but** Number Type, ...

**for** Number Type , ...

# checking assertions

form

**check Assertion** [ *scope* ]

scope

**for** *K* **but** Number Type, ...  
**for** Number Type , ...

example

**sig** Memory {data: Addr ->? Data}

**assert** WriteWorks {

**all** *m,m'*: Memory, *a*: Addr, *d*: Data |

Write (*m,m',a,d*) => *m'.data[a] = d* }

**check** WriteWorks **for** 3 **but** 2 Memory

# checking assertions

form

*check Assertion [ scope ]*

scope

**for** K **but** Number Type, ...  
**for** Number Type , ...

find any value of  
**m, m', a, d, data, Memory, Addr, Data**  
such that #Addr≤3 , #Data≤3, #Memory≤2  
and **Write (m,m',a,d) && m.data[a] != d**

example

**sig** Memory {data: Addr ->? Data}

**assert** WriteWorks {

**all** m,m': Memory, a: Addr, d: Data |

Write (m,m',a,d) => m'.data[a] = d }

*check WriteWorks for 3 but 2 Memory*

# checking assertions

form

*check Assertion [ scope ]*

scope

**for** K **but** Number Type, ...  
**for** Number Type , ...

example

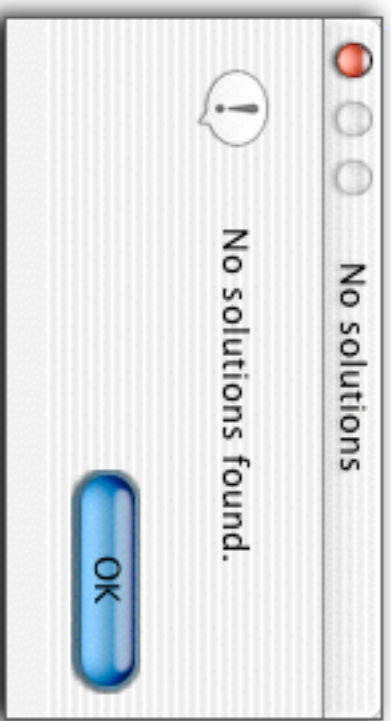
**sig** Memory {data: Addr ->? Data}

**assert** WriteWorks {

**all** m,m': Memory, a: Addr, d: Data |

Write (m,m',a,d) => m'.data[a] = d }

*check WriteWorks for 3 but 2 Memory*



find any value of  
**m, m', a, d, data, Memory, Addr, Data**  
such that #Addr≤3, #Data≤3, #Memory≤2  
and **Write (m,m',a,d) && m.data[a] != d**

# running functions



# running functions

form

**run** *Function* [ *scope* ]

# running functions

form

```
run Function [ scope ]
```

example

```
fun Write (m,m': Memory,a: Addr,d: Data) {  
  m'.data = m.data ++ a->d }  
run Write for 2
```

# running functions

form

```
run Function [ scope ]
```

example

```
fun Write (m,m': Memory,a: Addr,d: Data) {  
  m'.data = m.data ++ a->d }  
run Write for 2
```

run Write for 2

find any value of  
**m, m', a, d, data, Memory, Addr, Data**  
such that #Addr, #Data, #Memory≤2  
and **Write (m,m',a,d)**

# running functions

form

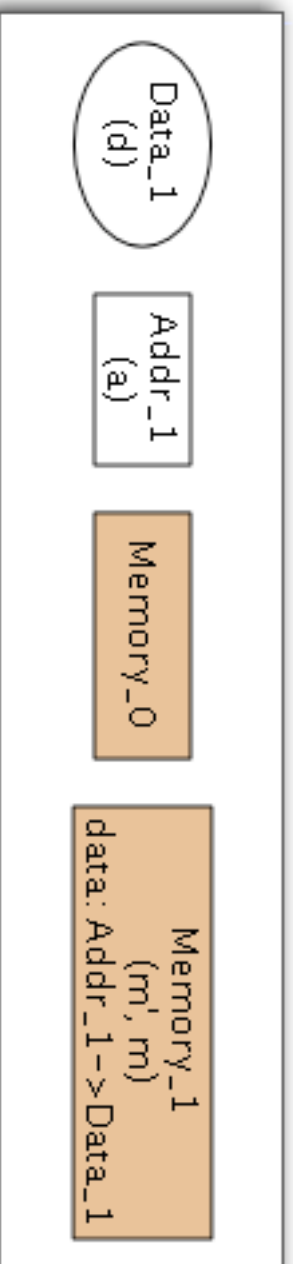
```
run Function [ scope ]
```

example

```
fun Write (m,m': Memory,a: Addr,d: Data) {  
  m'.data = m.data ++ a->d }  
run Write for 2
```

run Write for 2

find any value of  
**m, m', a, d, data, Memory, Addr, Data**  
such that #Addr, #Data, #Memory≤2  
and **Write (m,m',a,d)**



# simple memory model

# simple memory model

**module** Memory

# simple memory model

**module** Memory

**sig** Memory {data: Addr ->? Data}

# simple memory model

**module** Memory

**sig** Memory {data: Addr ->? Data}

**sig** Addr {}



# simple memory model

**module** Memory

**sig** Memory {data: Addr ->? Data}

**sig** Addr {}

**sig** Data {}

# simple memory model

```
module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}
fun Write (m, m': Memory, a: Addr, d: Data) {
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# simple memory model

```
module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}

fun Write (m, m': Memory, a: Addr, d: Data) {
  m'.data = m.data ++ a->d
}

fun Read (m: Memory, a: Addr, d: Data) {
  d = m.data[a]
}
```

# simple memory model

```
module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}

fun Write (m, m': Memory, a: Addr, d: Data) {
  m'.data = m.data ++ a->d
}

fun Read (m: Memory, a: Addr, d: Data) {
  d = m.data[a]
}

assert ReadWrite {
  all m,m': Memory, a: Addr, d,d': Data |
    Write (m,m',a,d) && Read (m',a,d') => d = d'
}
```

# simple memory model

```
module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}

fun Write (m, m': Memory, a: Addr, d: Data) {
  m'.data = m.data ++ a->d
}

fun Read (m: Memory, a: Addr, d: Data) {
  d = m.data[a]
}

assert ReadWrite {
  all m,m': Memory, a: Addr, d,d': Data |
    Write (m,m',a,d) && Read (m',a,d') => d = d'
}

check ReadWrite
```

# cache model

# cache model

**sig** Cache **extends** Memory {dirty: **set** Addr, main: Memory}

# cache model

```
sig Cache extends Memory {dirty: set Addr, main: Memory}
fun CacheWrite (c, c': Cache, a: Addr, d: Data) {
  c'.dirty = c.dirty + a
  c'.main = c.main
  Write (c,c',a,d)
}
```



# cache model

```
sig Cache extends Memory {dirty: set Addr, main: Memory}
fun CacheWrite (c, c': Cache, a: Addr, d: Data) {
  c'.dirty = c.dirty + a
  c'.main = c.main
  Write (c,c',a,d)
}
fun CacheRead (c, c': Cache, a: Addr, d: Data) {
  a in c.data.Data =>
    (Read (c,a,d) && c' = c),
    (Load (c,c',a) && Read (c',a,d))
}
```

# cache model

```
sig Cache extends Memory {dirty: set Addr, main: Memory}
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fun CacheRead (c, c': Cache, a: Addr, d: Data) {
  a in c.data.Data =>
    (Read (c,a,d) && c' = c),
    (Load (c,c',a) && Read (c',a,d))
}

fun Load (c, c': Cache, a: Addr) {
  some addr: set Addr {
    c'.dirty = c.dirty - addr
    c'.data = (c.data - addr->Data) ++ a->c.main.data[a]
    c'.main.data = c.main.data ++ c.data & (addr - c.dirty)->Data
  }
}
```

# **even weaker drop rule**

# even weaker drop rule

web cache

```
sig Server {store: URL ->? Doc}
sig URL {}
sig Date {after: set Date}
sig Doc {expires: Date}

fun Request (u: URL, cache, cache', server: Server, now: Date) {
  cache'.store in (cache + server).store
  cache'.store[u] != server.store[u] =>
    cache.store[u].expires in now.after
}
```

# even weaker drop rule

web cache

```
sig Server {store: URL ->? Doc}  
sig URL {}  
sig Date {after: set Date}  
sig Doc {expires: Date}
```

```
fun Request (u: URL, cache, cache', server: Server, now: Date) {  
  cache'.store in (cache + server).store  
  cache'.store[u] != server.store[u] =>  
    cache.store[u].expires in now.after  
}
```

cache policy:  
if a URL is not updated,  
it can't have expired

# sequencing checks

# sequencing checks

```
assert CacheReadWrite {  
  all c,c':Cache, a: Addr, d,d': Data |  
    CacheWrite (c,c',a,d) && CacheRead (c',c'',a,d') => d = d'  
}
```

# sequencing checks

```
assert CacheReadWrite {  
  all c,c': Cache, a: Addr, d,d': Data |  
    CacheWrite (c,c',a,d) && CacheRead (c',c'',a,d') => d = d'  
}  
check CacheReadWrite
```



# sequencing checks

```
assert CacheReadWrite {  
  all c,c': Cache, a: Addr, d,d': Data |  
    CacheWrite (c,c',a,d) && CacheRead (c',c",a,d') => d = d'  
}  
check CacheReadWrite  
assert CacheReadWrite2 {  
  all c,c',c",c"" : Cache, a,a': Addr, d,d',d"": Data |  
    {  
      CacheWrite (c,c',a,d)  
      CacheRead (c',c",a',d')  
      CacheRead (c",c""",a,d")  
    } => d = d"  
}
```

# sequencing checks

```
assert CacheReadWrite {  
  all c,c': Cache, a: Addr, d,d': Data |  
    CacheWrite (c,c',a,d) && CacheRead (c',c",a,d') => d = d'  
}
```

**check** CacheReadWrite

```
assert CacheReadWrite2 {  
  all c,c',c",c"" : Cache, a,a': Addr, d,d',d"": Data |  
    {  
      CacheWrite (c,c',a,d)  
      CacheRead (c',c",a',d')  
      CacheRead (c",c""",a,d")  
    } => d = d"  
}
```

**check** CacheReadWrite2 **for** 3 **but** 4 Memory

# a refinement check

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```
fun Abstraction (c: Cache, m: Memory) {  
  m.data = c.main.data ++ (c.data & c.dirty->Data)  
}
```

# a refinement check

```
fun Abstraction (c: Cache, m: Memory) {  
    m.data = c.main.data ++ (c.data & c.dirty->Data)  
}  
  
fun Consistent (c: Cache) {  
    c.data - c.dirty->Data in c.main.data  
}
```

# a refinement check

```
fun Abstraction (c: Cache, m: Memory) {  
  m.data = c.main.data ++ (c.data & c.dirty->Data)  
}  
  
fun Consistent (c: Cache) {  
  c.data - c.dirty->Data in c.main.data  
}  
  
assert CacheReadOK {  
  all c, c': Cache, m, m': Memory, a: Addr, d: Data |  
    Consistent (c) && CacheRead (c,c',a,d)  
    && Abstraction (c,m) && Abstraction (c',m') =>  
      Read (m,a,d) && m.data = m'.data && Consistent (c')  
}
```

# a refinement check

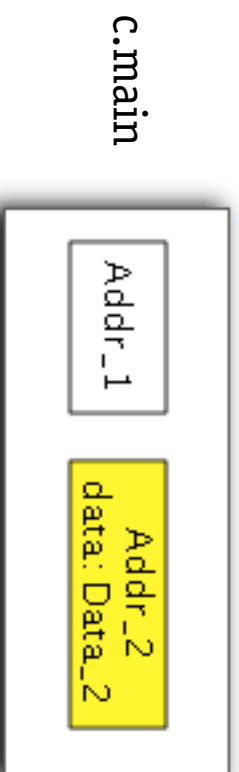
```
fun Abstraction (c: Cache, m: Memory) {  
  m.data = c.main.data ++ (c.data & c.dirty->Data)  
}  
  
fun Consistent (c: Cache) {  
  c.data - c.dirty->Data in c.main.data  
}  
  
assert CacheReadOK {  
  all c, c': Cache, m, m': Memory, a: Addr, d: Data |  
    Consistent (c) && CacheRead (c,c',a,d)  
    && Abstraction (c,m) && Abstraction (c',m') =>  
      Read (m,a,d) && m.data = m'.data && Consistent (c')  
}  
  
check CacheWriteOK for 3 but 4 Memory
```

# counterexample



# counterexample

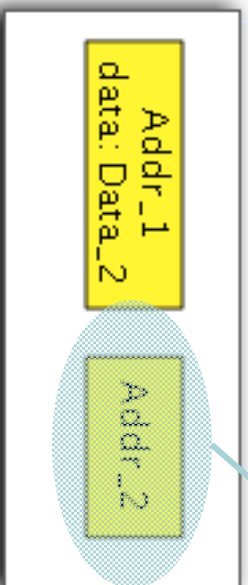
a = Addr\_2, d = Data\_2



# counterexample

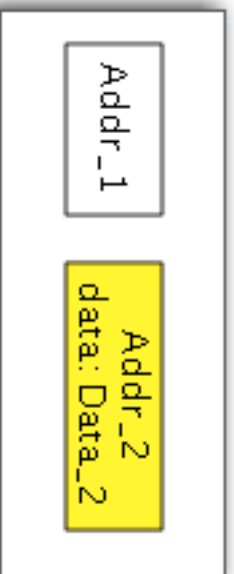
a = Addr\_2, d = Data\_2

c



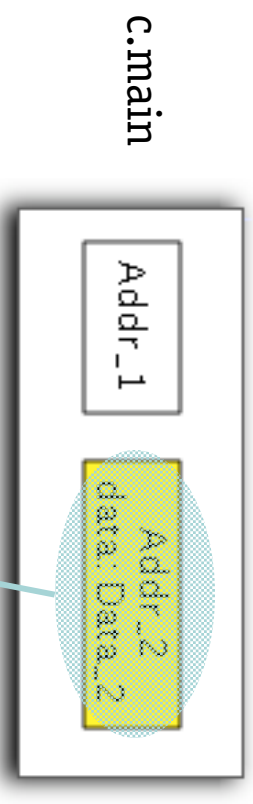
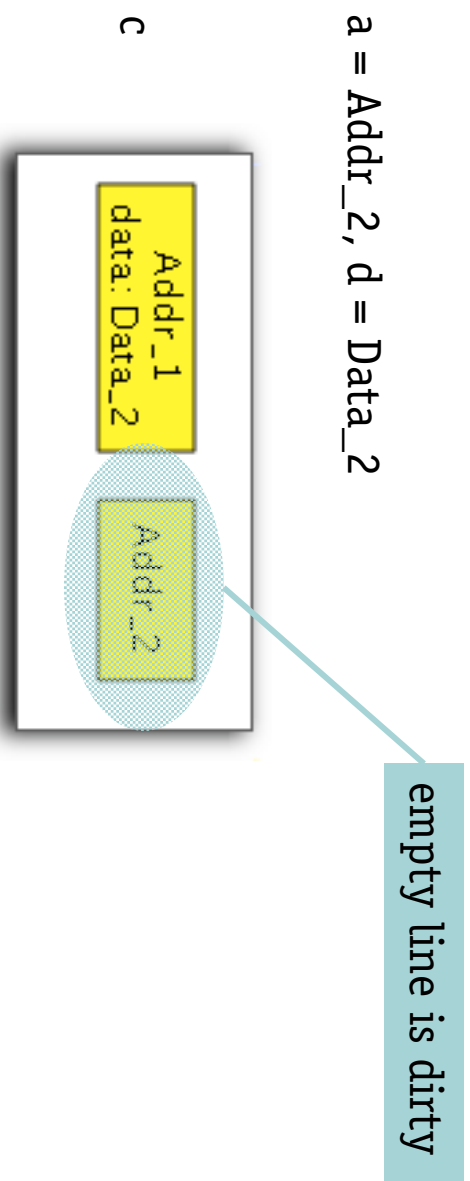
empty line is dirty

c.main



# counterexample

a = Addr\_2, d = Data\_2

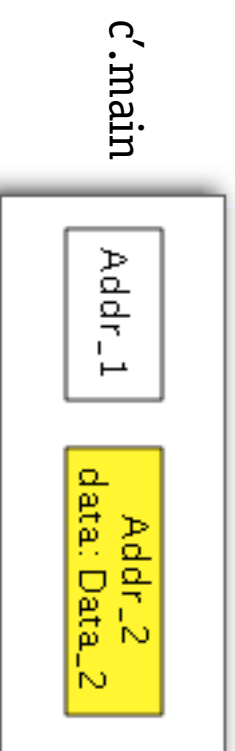
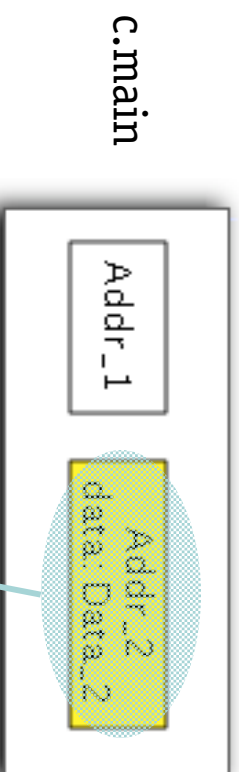
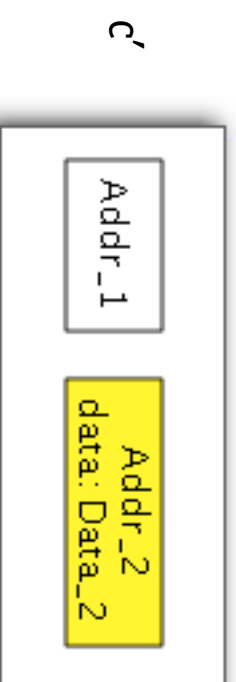
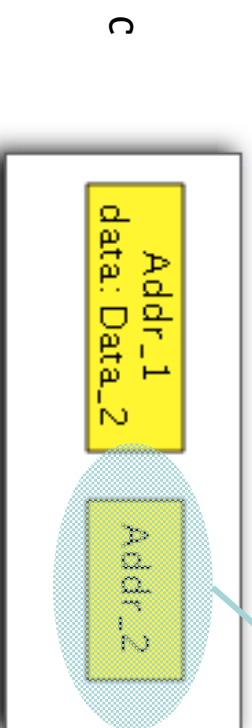


main memory has  
dirty line: it's a cache!

# counterexample

a = Addr\_2, d = Data\_2

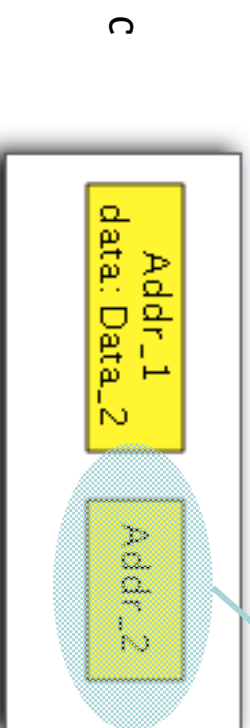
empty line is dirty



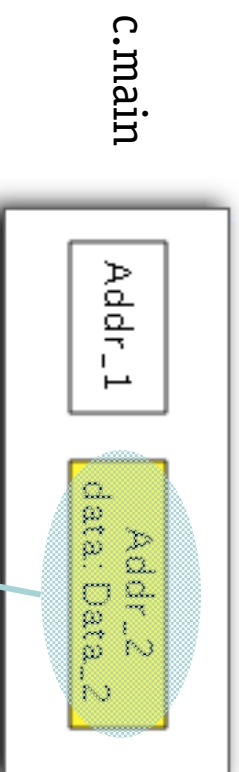
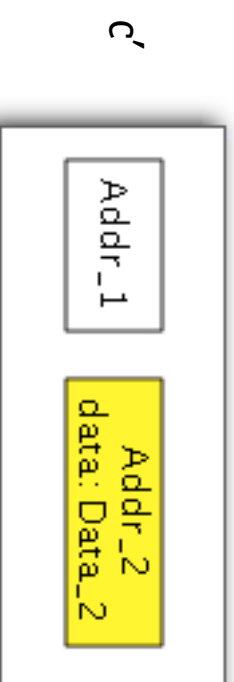
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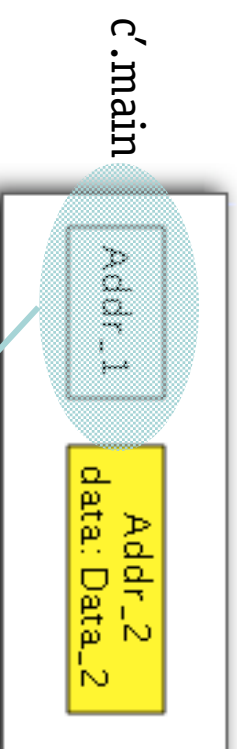
a = Addr\_2, d = Data\_2



empty line is dirty



main memory has  
dirty line: it's a cache!



dirty line is flushed  
but not written through

# correction

# correction

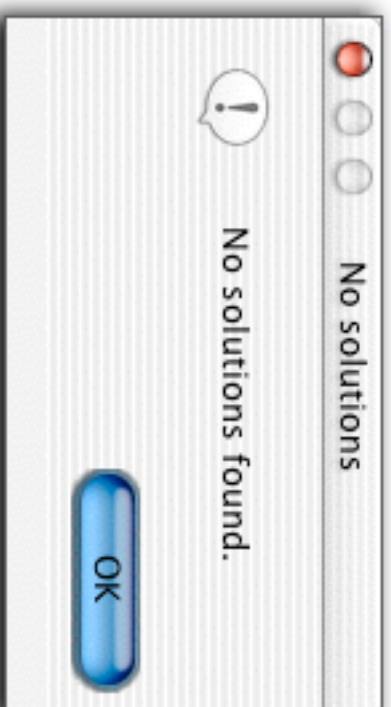
must write through dirty, not non-dirty lines:

```
fun Load (c, c': Cache, a: Addr) {  
  some addr: set Addr {  
    c'.dirty = c.dirty - addr  
    c'.data = (c.data - addr->Data) ++ a->c.main.data[a]  
    c'.main.data = c.main.data ++ c.data & (addr & c.dirty)->Data  
  }  
}
```

# correction

must write through dirty, not non-dirty lines:

```
fun Load (c, c': Cache, a: Addr) {  
  some addr: set Addr {  
    c'.dirty = c.dirty - addr  
    c'.data = (c.data - addr->Data) ++ a->c.main.data[a]  
    c'.main.data = c.main.data ++ c.data & (addr & c.dirty)->Data  
  }  
}
```





# other language features

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function shorthands

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fun f (a: A): B {... result...} == fun f (a: A, b: B) {...b...}
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polymorphism

```
module Lists
```

```
sig List [t] {}
```

```
sig NonEmptyList [t] extends List[t] {elt: t, next: List[t]}
```

```
fun elements [t] (l: List[t]): set t {result = l.*next.elt}
```

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```

```
fun elements [t] (l: List[t]): set t {result = l.*next.elt}
```

modules

```
open Lists
```

```
sig Op {}
```

```
sig Undo {ops: List[Op]}
```

**doing more with less**

# doing more with less

no composites

› first order, so tractable

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no composites

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no built-in idioms

› more flexible

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no separate property language -- allows masking

```
assert {System()} => Property1() }
```

```
assert {System()} && Property1() => Property2() }
```



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› first order, so tractable

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no separate property language -- allows masking

```
assert {System()} => Property1() }
```

```
assert {System()} && Property1() => Property2() }
```

no subtyping -- no casts

```
dir.contents.contents in File
```

```
dir.contents->forall{d | d.oclIsKindOf(Dir) implies
```

```
  d.oclAsType(Dir).contents in File }
```

# challenges for you

# challenges for you

solve Halmos's handshake problem using Alloy

Alice and Bob invite four couples for dinner.

When they arrive, they shake hands.

Nobody shakes their own or spouse's hand.

After some handshaking, Alice says "Stop shaking hands!", and then asks how many hands each person has shaken. All the answers are different.

How many hands has Bob shaken?

# **halmos's handshaking**

# halmos's handshaking

```
sig Person {
  spouse: Person,
  shaken: set Person}{
  spouse != this
  no (this + spouse) & shaken }
fact {
  univ[Person] in Person
  spouse = ~spouse
  shakes = ~shakes }
fun Halmos () {
  some Alice: Person |
    all disj p1, p2: Person - Alice | # p1.shakes != # p2.shakes }
run Halmos for 10
```