micromodels of software

declarative modelling and analysis with Alloy

lecture 4: the Alloy language

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the Alloy language

syntactic constructs
- for packaging formulas
- for incremental description of structures

design principles
- remain first-order
- keep it simple (no partial functions, no subtypes)
- idiom-free (like PVS, ACL2)

look out for …

key constructs

- signatures and functions

puns

- signatures like Java’s classes
- dot like Java’s dereferencing dot
- functions like PL functions

what’s missing

- composites
- subtyping
- state machines

no notion of execution at all

gross structure

- signature
  - declares set & relations
- fact
  - global constraint
- function
  - parameterized constraint
- assertion
  - theorem to check
- command
  - run function
  - check assertion

```
module Memory

sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}

fact {all m: Memory, a: Addr | sole m.data[a]}

fun Write (m,m': Memory,a: Addr,d: Data) {
  m'.data = m.data ++ a->d
  }

assert WriteWorks {
  all m,m': Memory, a: Addr, d: Data |
    Write (m,m',a,d) => m.data[a] = d
  }

run Write
check WriteWorks
```

basic signature decls

introduces set and basic type

```
sig s {}
```

set s with basic type (S, say)

set is immutable

› a global variable with one value

examples

```
sig Memory {}
sig Addr {}
sig Data {}
sig FileSystem {}
sig BirthdayBook {}
```

extension decls

introduces subset of existing set

sig t **extends** s {}   *t subset of s*

subset is immutable

- a global variable with one value

examples

sig Cache **extends** Memory {}

sig Man **extends** Person {}

sig Dir **extends** FileSysObj {}

disjointness

can mark subsigs as mutually disjoint

```
disj sig t1 extends s {}        t1 and t2 disjoint ...
disj sig t2 extends s {}
sig t3 extends s {}             ... but not nec t3
```

examples

```
sig Memory {}
disj sig Cache extends Memory {}
disj sig MainMemory extends Memory {}

sig Person {}
part sig Man, Woman extends Person {}
sig Employee extends Person {}
```

singletons

constrains set to be a singleton

static sig s {} *s has one tuple*

examples

static part sig Red, Green, Blue **extends** Colour {}

static sig Root **extends** Directory {}

fields

introduces a relation

sig s {}
sig t {f: s}

f is relation of type (T,S)
with left set t and right set s

relation is immutable

› a global variable with one value

example

sig Date {}
sig Person {birthday: Date}
fact {**some disj** p,q: Person | p.birthday = q.birthday}

birthday has type (PERSON,DATE)

overloading

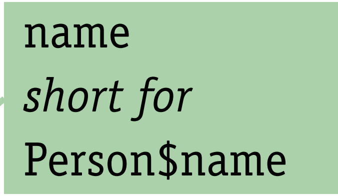
resolved on left type

sig s {f: u}   *s$f has type (S,U)*

sig t {f: u}   *t$f has type (T,U)*

example

sig Name {}
sig City {name: Name}
sig Person {name: Name}
fact {**no disj** p,q: Person | p.name = q.name}



name
short for
Person$name

higher-arity fields

any expression can be used in field decl

```
sig s {f: e}
```

examples

```
sig Memory {data: Addr ->? Data}

sig FileSysObj {}
sig FileSystem {
  disj files, dirs: set FileSysObj,
  root: dirs,
  parent: (files + dirs - root) ->! dirs
  }

sig BirthdayBook {records: Person ->? Date}
```

data has type
(MEMORY,ADDR,DATA)

all m: Memory |
m.data : Addr ->? Data

parent has type
(FILESYSTEM,FILESYSOBJ,FILESYSOBJ)

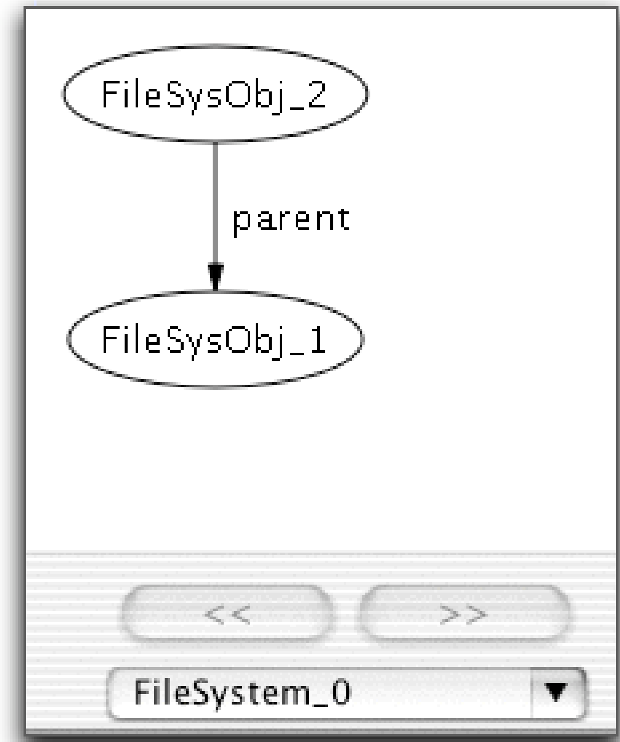
all s: FileSystem |
s.parent:(s.files+s.dirs-s.root)->! s.dirs

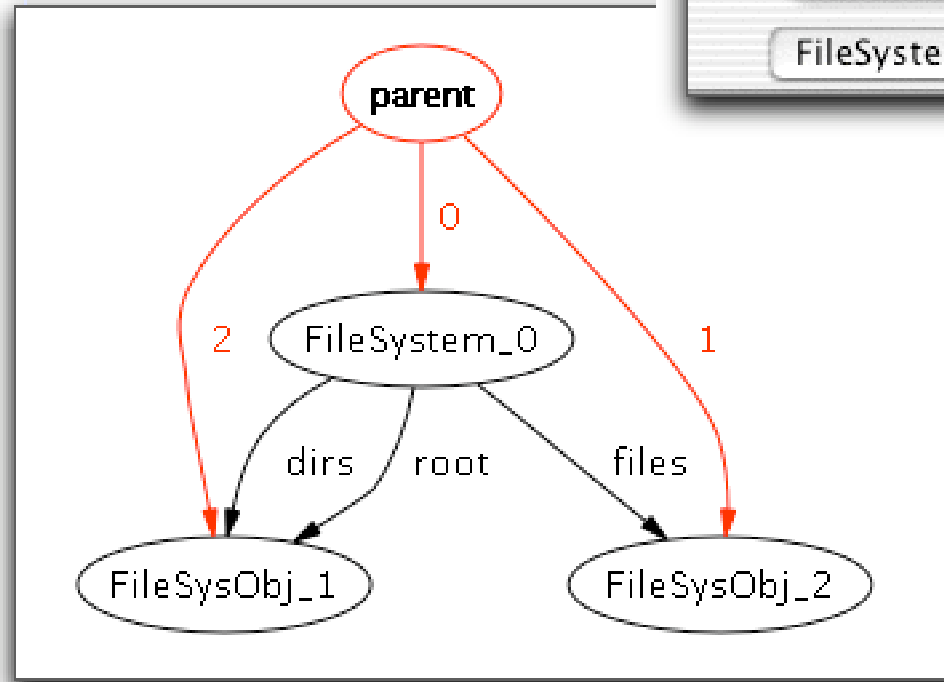
records has type
(BIRTHDAYBOOK,PERSON,DATE)

all bb: BirthdayBook |
bb.records: Person ->? Date

show me…

```
sig FileSysObj {}
sig FileSystem {
  disj files, dirs: set FileSysObj,
  root: dirs,
  parent: (files+dirs-root) ->! dirs
  }
fun showMe () {
  some FileSystem
  }
run showMe
```





subsignature fields

fields of subsignatures

- associated with basic type
- constrained to have subsig as left set

sig s {}
sig t **extends** s {f: u}   *f has type (S,U) and left set t*

example

sig Memory {data: Addr ->? Data}
sig Cache **extends** Memory {dirty: **set** Addr}

dirty has type (MEMORY,ADDR)

dirty in Cache -> Addr

out of domain application

if m is not in Memory, m.cache is **empty**

with

implicit deref

with e | F

is like F with each expression e′
whose left type matches the type of e
replaced by e′.e

example

```
sig FileSystem {
  disj files, dirs: set FileSysObj,
  root: dirs,
  parent: (files+dirs-root) ->! dirs
  }
fact {all fs: FileSystem | with fs | files in root.*~parent
```

short for
fs.files in fs::root.*~fs::parent

signature facts

signature fact has implicit *all* and *with*

```
sig s {...} {F}
  short for
sig s {...}
fact {all x: s | with x | F}
```

example

```
sig FileSystem {
  disj files, dirs: set FileSysObj,
  root: dirs,
  parent: (files+dirs-root) ->! dirs
  } {files + dirs in root.*~parent}
```

short for
all this: FileSystem | **with this** |
files **in** root.*~parent

functions

function

- a parameterized constraint
- ‘invocation’ is a formula
- meaning is just inlining/substitution (but recursion now supported)

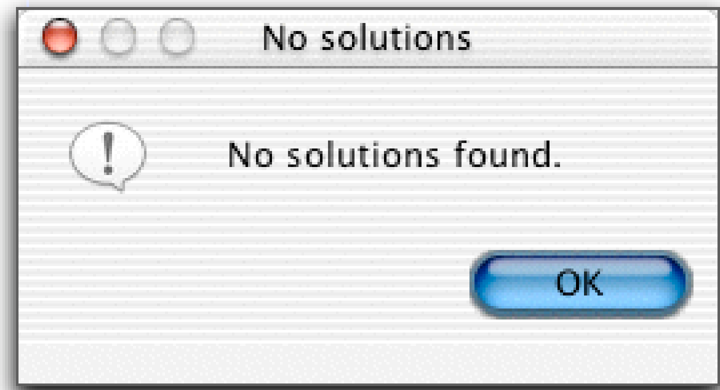
example

```
fun Write (m,m': Memory,a: Addr,d: Data) {
  m'.data = m.data ++ a->d }

assert WriteWorks {
  all m,m': Memory, a: Addr, d: Data |
   Write (m,m',a,d) => m'.data[a] = d }
```

short for
m′.data = m.data ++ a->d

checking assertions



form

check *Assertion [scope]*

scope

for K **but** Number Type, …
for Number Type , …

find any value of
m, m′, a, d, data, Memory, Addr, Data
such that #Addr≤3 ,#Data≤3, ,#Memory≤2
and **Write (m,m',a,d) && m.data[a] != d**

example

sig Memory {data: Addr ->? Data}

assert WriteWorks {
all m,m': Memory, a: Addr, d: Data |
Write (m,m',a,d) => m′.data[a] = d }

check WriteWorks **for** 3 **but** 2 Memory

running functions

form

run *Function* [*scope*]

example

fun Write (m,m': Memory,a: Addr,d: Data) {
  m'.data = m.data ++ a->d }
run Write **for** 2

find any value of
m, m′, a, d, data, Memory, Addr, Data
such that #Addr, #Data, #Memory≤2
and **Write (m,m',a,d)**

simple memory model

```
module Memory
sig Memory {data: Addr ->? Data}
sig Addr {}
sig Data {}
fun Write (m, m': Memory, a: Addr, d: Data) {
  m'.data = m.data ++ a->d
  }
fun Read (m: Memory, a: Addr, d: Data) {
  d = m.data[a]
  }
assert ReadWrite {
  all m,m': Memory, a: Addr, d,d': Data |
     Write (m,m',a,d) && Read (m',a,d') => d = d'
  }
check ReadWrite
```

cache model

```
sig Cache extends Memory {dirty: set Addr, main: Memory}
fun CacheWrite (c, c': Cache, a: Addr, d: Data) {
  c'.dirty = c.dirty + a
  c'.main = c.main
  Write (c,c',a,d)
  }
fun CacheRead (c, c': Cache, a: Addr, d: Data) {
  a in c.data.Data =>
     (Read (c,a,d) && c' = c),
     (Load (c,c',a) && Read (c',a,d))
  }
fun Load (c, c': Cache, a: Addr) {
  some addrs: set Addr {
     c'.dirty = c.dirty - addrs
     c'.data = (c.data - addrs->Data) ++ a->c.main.data[a]
     c'.main.data = c.main.data ++ c.data & (addrs - c.dirty)->Data
     }
  }
```

even weaker drop rule

web cache

```
sig Server {store: URL ->? Doc}
sig URL {}
sig Date {after: set Date}
sig Doc {expires: Date}

fun Request (u: URL, cache, cache', server: Server, now: Date) {
  cache'.store in (cache + server).store
  cache'.store[u] != server.store[u] =>
    cache.store[u].expires in now.after
  }
```

cache policy:
if a URL is not updated,
it can't have expired

sequencing checks

```
assert CacheReadWrite {
  all c,c',c": Cache, a: Addr, d,d': Data |
    CacheWrite (c,c',a,d) && CacheRead (c',c",a,d') => d = d'
  }
check CacheReadWrite
assert CacheReadWrite2 {
  all c,c',c",c"": Cache, a,a': Addr, d,d',d": Data |
    {
    CacheWrite (c,c',a,d)
    CacheRead (c',c",a',d')
    CacheRead (c",c"",a,d")
    } => d = d"
  }
check CacheReadWrite2 for 3 but 4 Memory
```

a refinement check

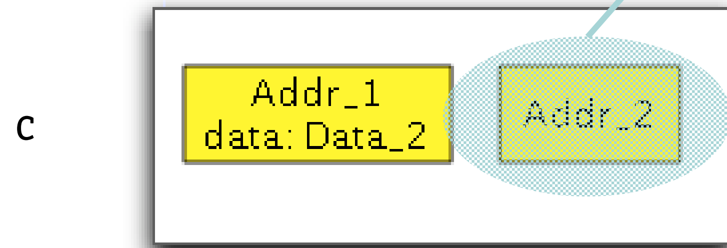
```
fun Abstraction (c: Cache, m: Memory) {
  m.data = c.main.data ++ (c.data & c.dirty->Data)
  }
fun Consistent (c: Cache) {
  c.data - c.dirty->Data in c.main.data
  }
assert CacheReadOK {
  all c, c': Cache, m,m': Memory, a: Addr, d: Data |
     Consistent (c) && CacheRead (c,c',a,d)
     && Abstraction (c,m) && Abstraction (c',m') =>
        Read (m,a,d) && m.data = m’.data && Consistent (c')
  }
check CacheWriteOK for 3 but 4 Memory
```

counterexample

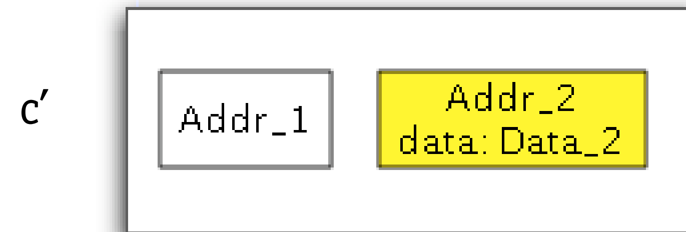
a = Addr_2, d = Data_2

empty line is dirty

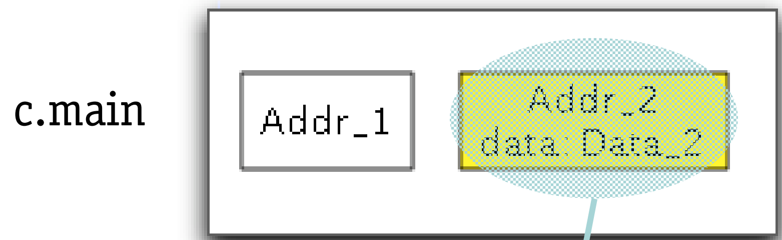
c



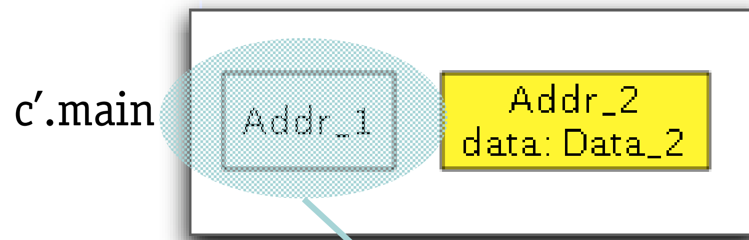
c′



c.main



c′.main



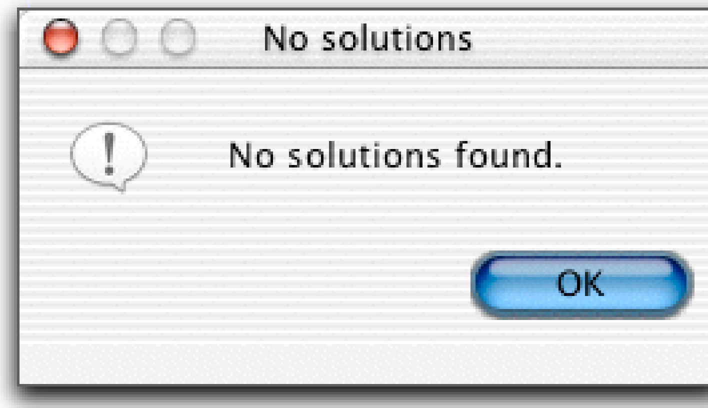
main memory has
dirty line: it's a cache!

dirty line is flushed
but not written through

correction

must write through dirty, not non-dirty lines:

```
fun Load (c, c': Cache, a: Addr) {
  some addrs: set Addr {
    c'.dirty = c.dirty - addrs
    c'.data = (c.data - addrs->Data) ++ a->c.main.data[a]
    c'.main.data = c.main.data ++ c.data & (addrs & c.dirty)->Data
    }
  }
```



other language features

function shorthands

```
fun f (a: A): B {... result...} == fun f (a: A, b: B) {...b...}
```

polymorphism

```
module Lists
sig List [t] {}
sig NonEmptyList [t] extends List[t] {elt: t, next: List[t]}
fun elements [t] (l: List[t]): set t {result = l.*next.elt}
```

modules

```
open Lists
sig Op {}
sig Undo {ops: List[Op]}
```

doing more with less

no composites

- first order, so tractable

no built-in idioms

- more flexible

no separate property language -- allows masking

```
assert {System() => Property1()}
assert {System() && Property1() => Property2()}
```

no subtyping -- no casts

```
dir.contents.contents in File
dir.contents->forall{d | d.oclIsKindOf(Dir) implies
  d.oclAsType(Dir). contents in File}
```

challenges for you

solve Halmos's handshake problem using Alloy

- Alice and Bob invite four couples for dinner.
- When they arrive, they shake hands.
- Nobody shakes their own or spouse's hand.
- After some handshaking, Alice says "Stop shaking hands!", and then asks how many hands each person has shaken. All the answers are different.
- How many hands has Bob shaken?

halmos's handshaking

```
sig Person {
  spouse: Person,
  shaken: set Person}{
    spouse != this
    no (this + spouse) & shaken }
fact {
  univ[Person] in Person
  spouse = ~spouse
  shakes = ~shakes }
fun Halmos () {
  some Alice: Person |
    all disj p1, p2: Person - Alice | # p1.shakes != # p2.shakes }
run Halmos for 10
```