micromodels of software
declarative modelling
and analysis with Alloy

lecture 1: introduction

Daniel Jackson
MIT Lab for Computer Science
Marktoberdorf, August 2002

lightweight models

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a foundation for robust, useable programs

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elements
- small & simple notations
- partial models & analyses
- full automation

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a foundation for robust, useable programs

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- small & simple notations
- partial models & analyses
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focus on risky aspects

- hard to get right, or to check
- structure-determining
- high cost of failure

what assurance costs

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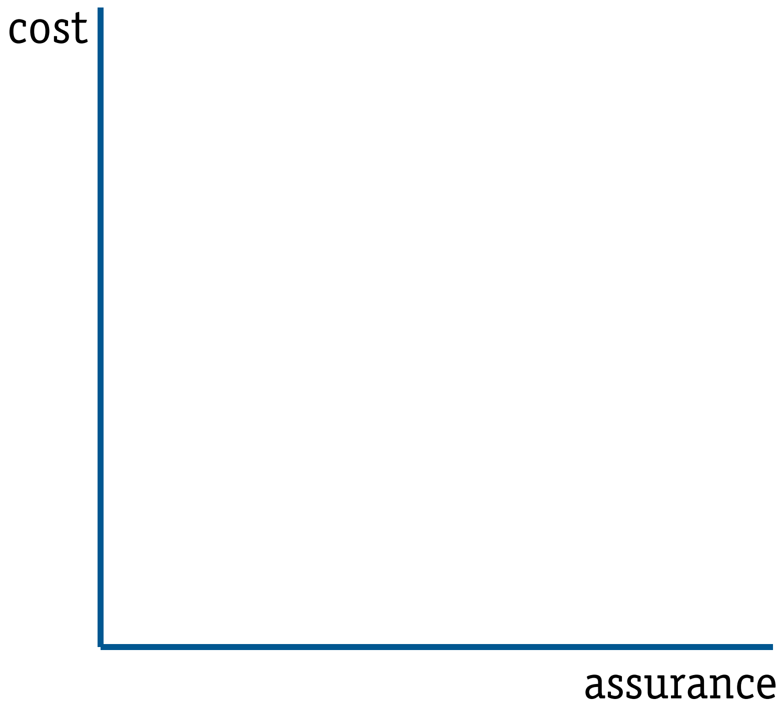
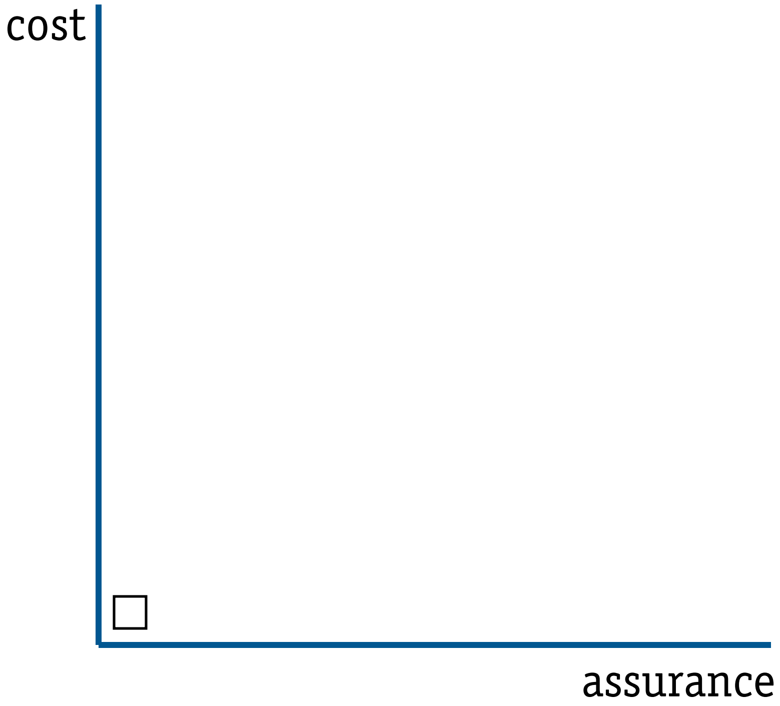


what assurance costs

□hacking



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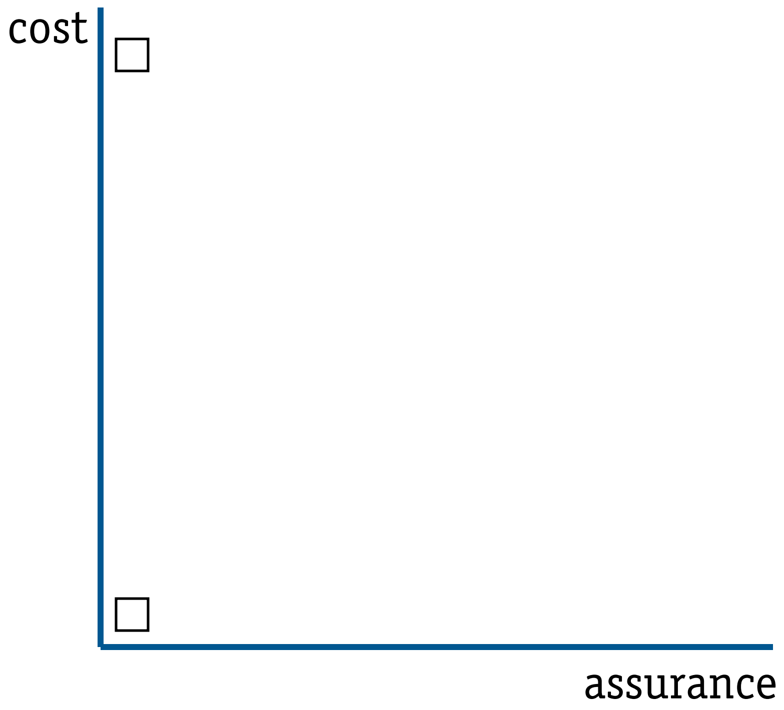

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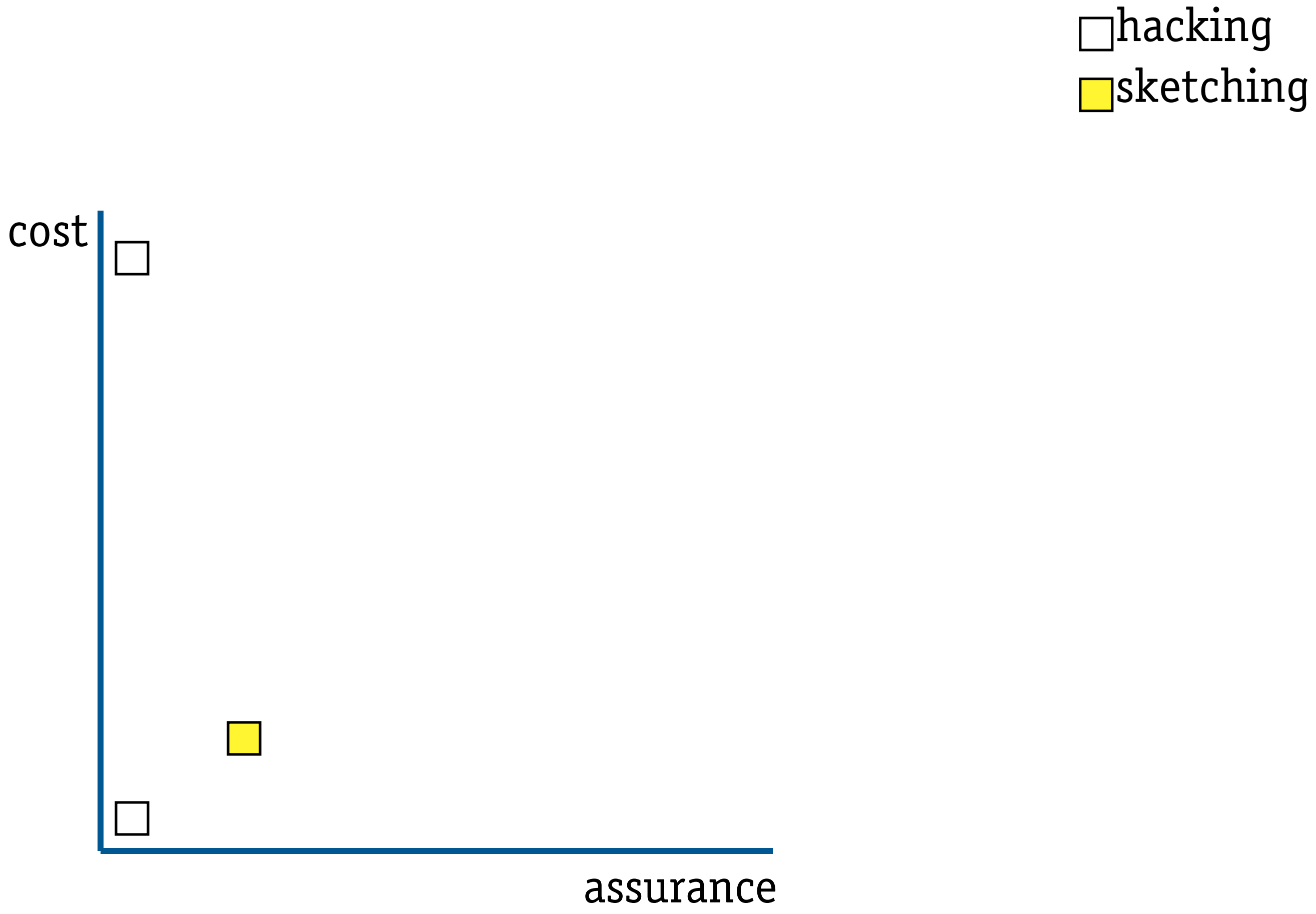

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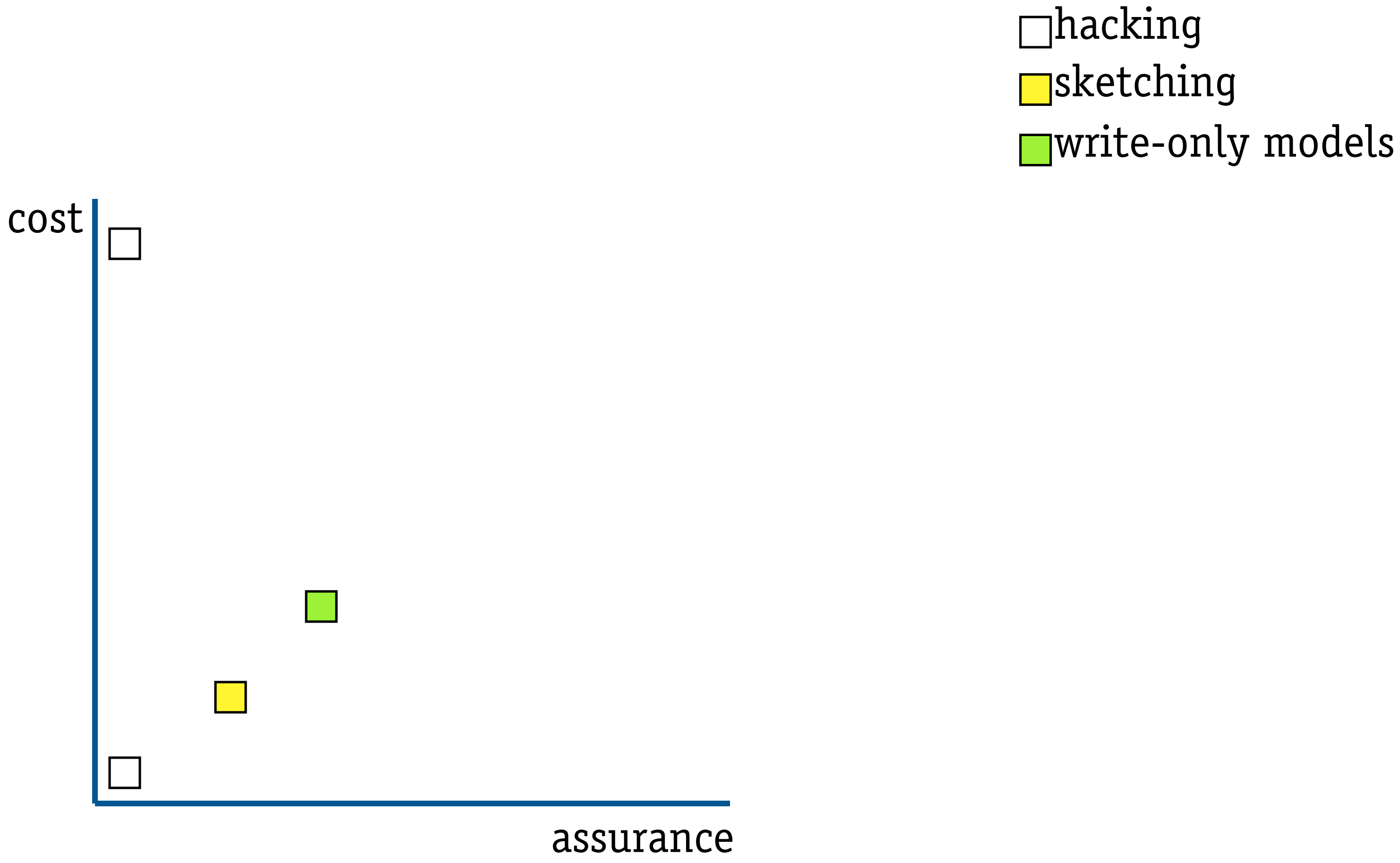

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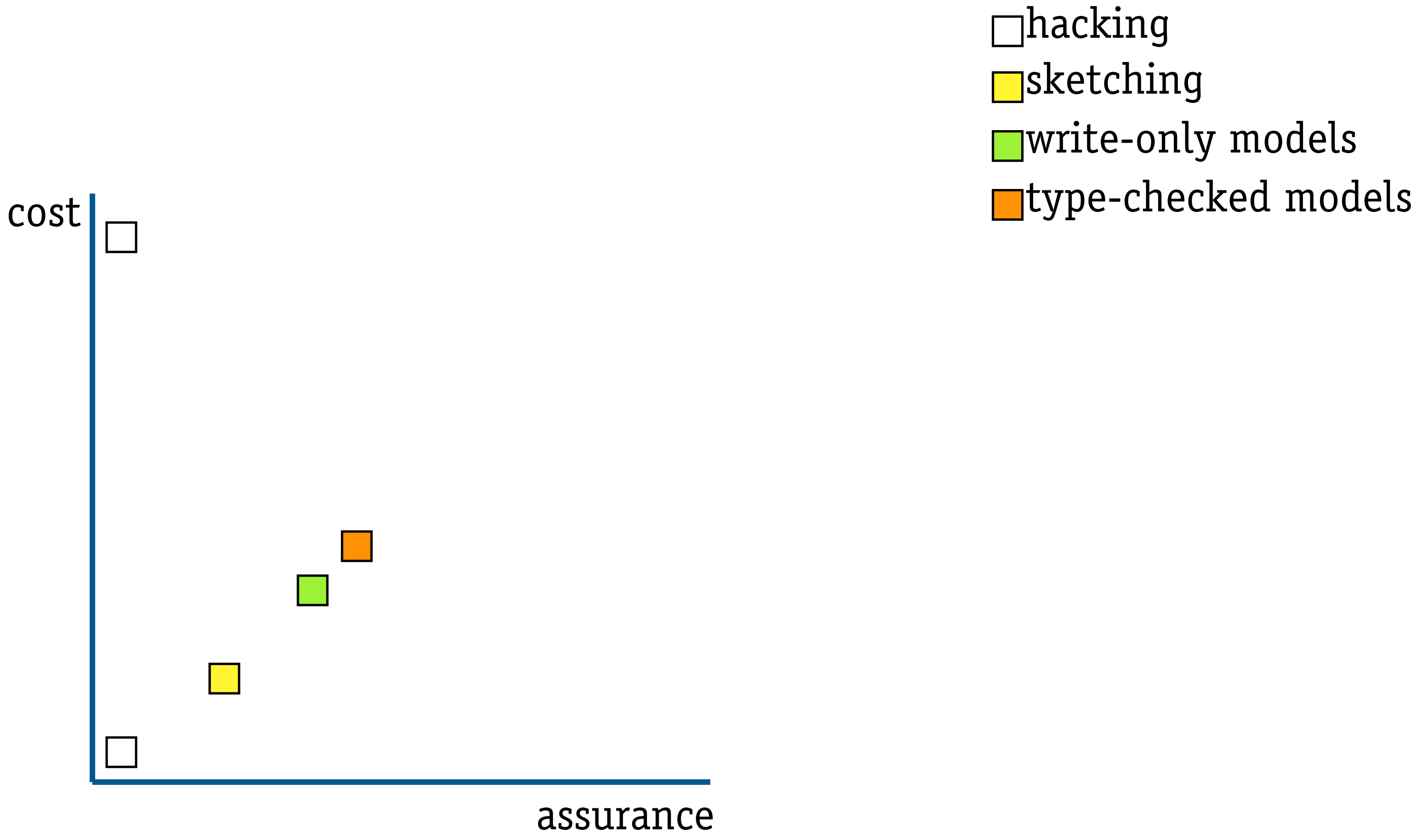

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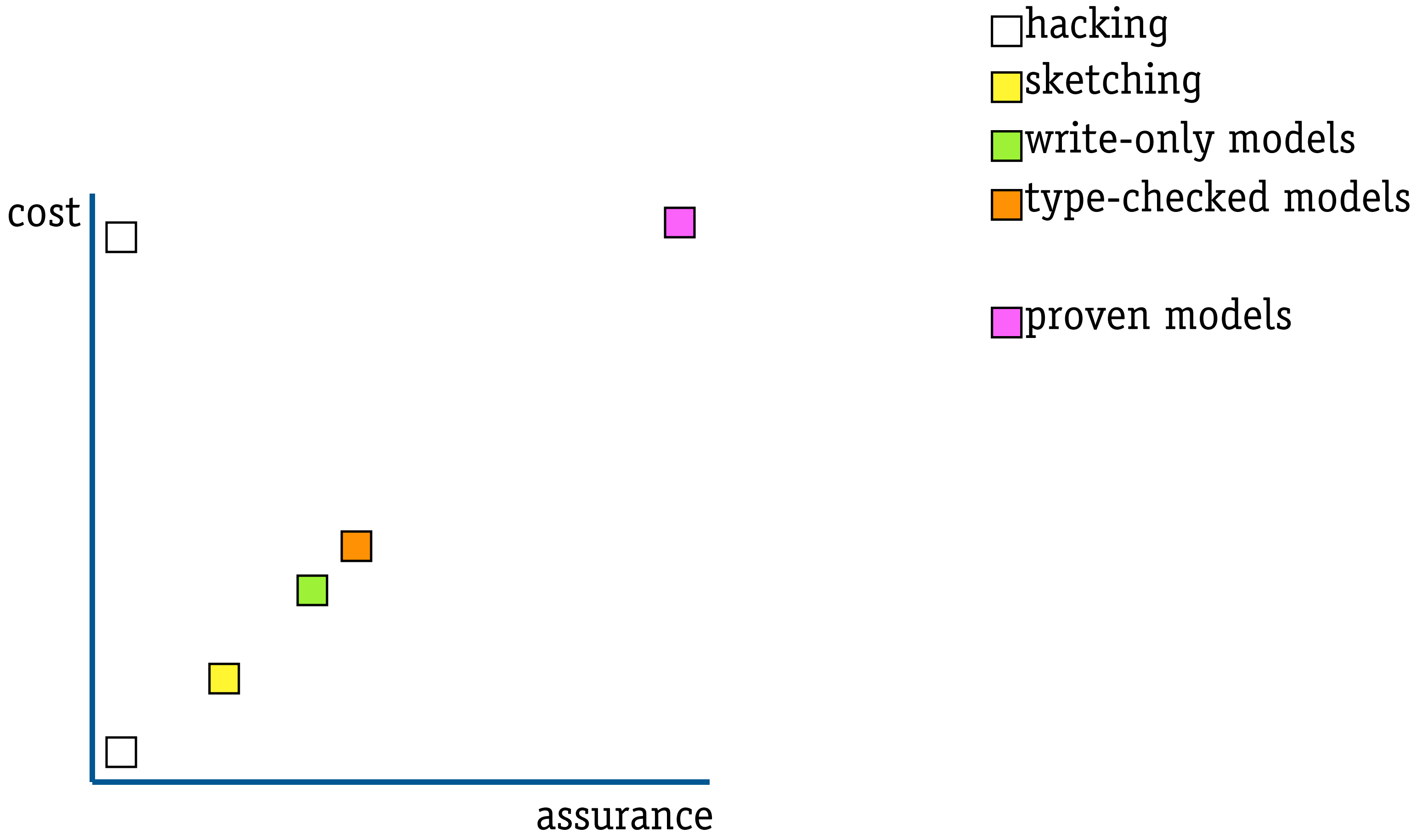

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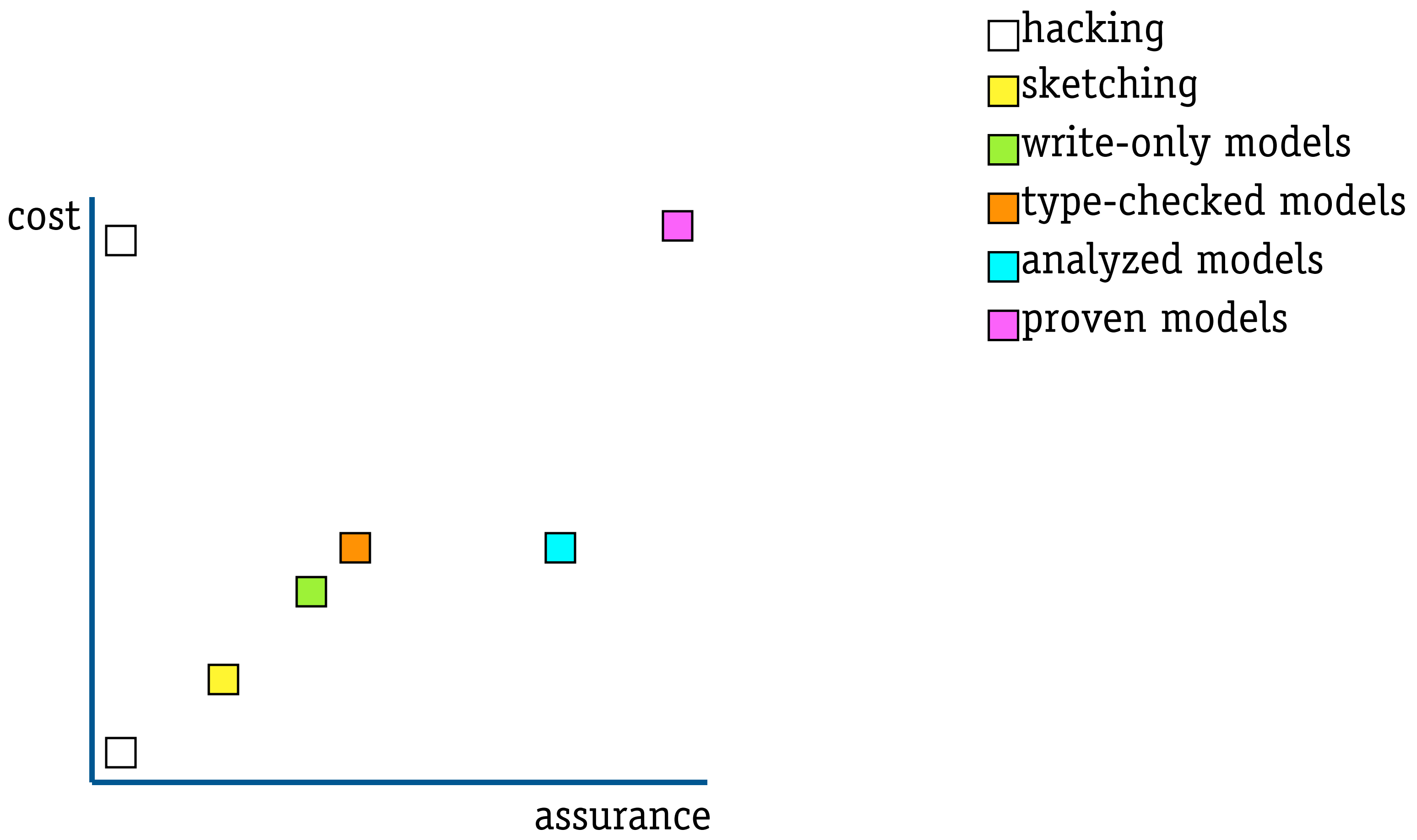

my work in marktoberdorf context

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computation, not interaction

- complementary to Harel & Pnueli
- relational, not algebraic (cf. Tarlecki and Meseguer)
- underlying idioms due to Hoare, Woodcock et al

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role of mathematics
- only way to make things simple
- semantics in terms of sets, and SAT

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started this in 1994, and have had some successes
but much less mature than ACL2, PVS, Statemate, etc

features of Alloy

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structural

- express complex structure, static and dynamic
- with just a few powerful operators

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analyzable

- simulation & checking
- fully automatic

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structure is everywhere

- highway systems, postal routes, company organizations, library catalogues, address books, phone networks, …

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structure is becoming more pervasive

- self-assembling software (eg, Observer pattern)
- memory gets cheaper: address books in every phone

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tool researchers have neglected structure

- one traffic light is a state machine, but a city’s lights are a net

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There is no problem in computer science that cannot be solved by introducing another level of indirection, but that usually reveals new problems *--David Wheeler*

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Sys meets Prop:  Sys => Prop
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why less is more
- less constrained system means implementation freedom
- less constrained environment means greater safety

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‘write-only’ models

› useful if precise enough
› but missed opportunity (and wishful thinking)

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tool-assisted modelling

- simulate and check incrementally
- catch errors early, develop confidence
- optimize for failing case: most of my examples will be wrong

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tool-assisted modelling
- simulate and check incrementally
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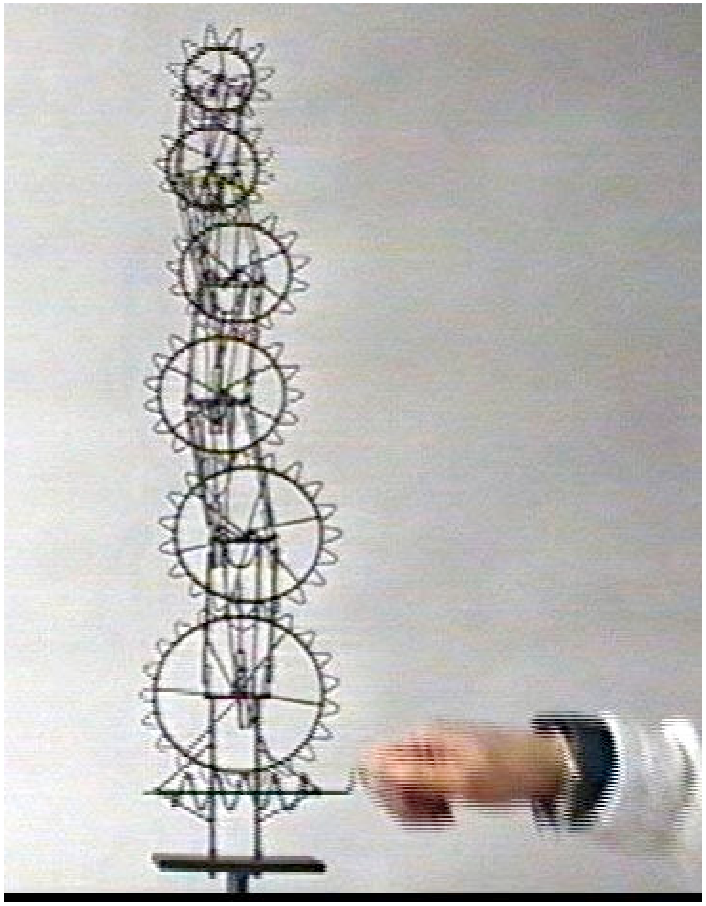
Alloy’s analysis
- fully automatic, with no user intervention
- concrete: generates samples & counterexamples
- like testing, sound but not complete
- unlike testing, billions cases/second

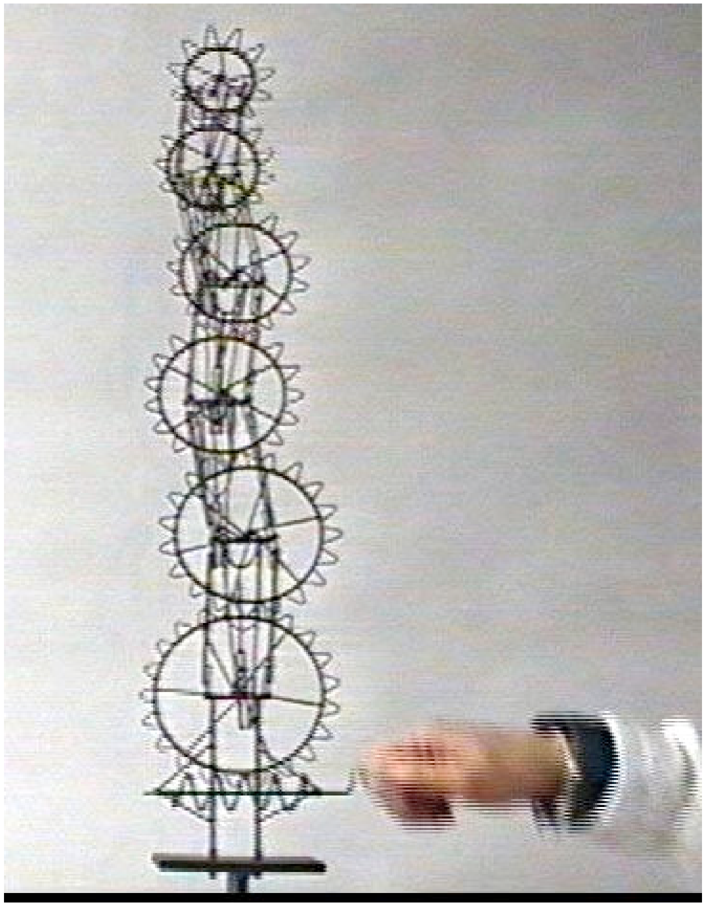
declarative & executable?



Small Tower of 6 Gears, Arthur Ganson

declarative & executable?

traditionally
- declarative XOR executable
- good arguments for both



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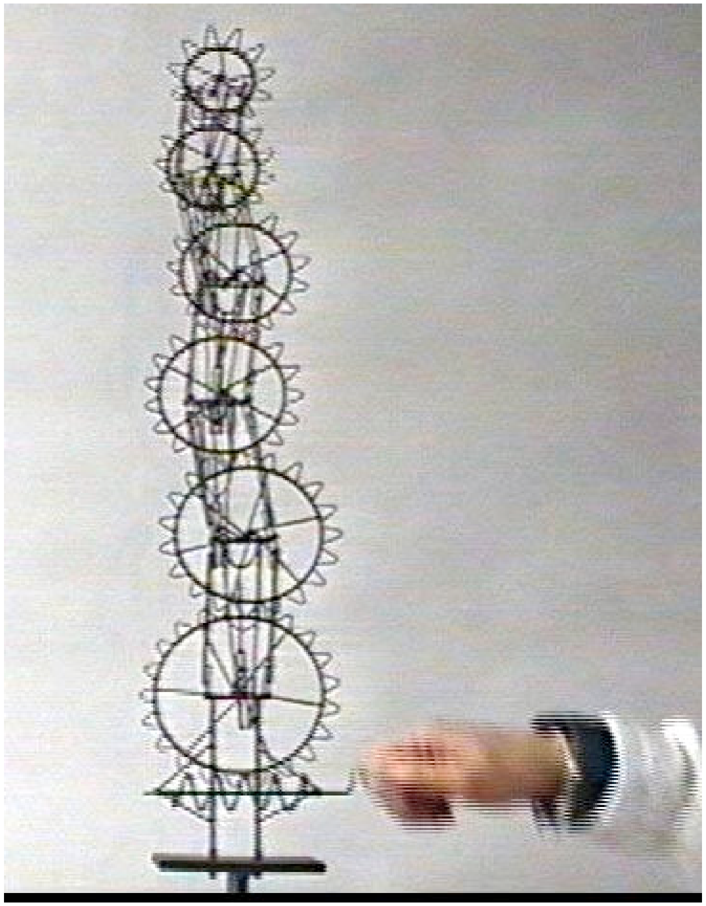

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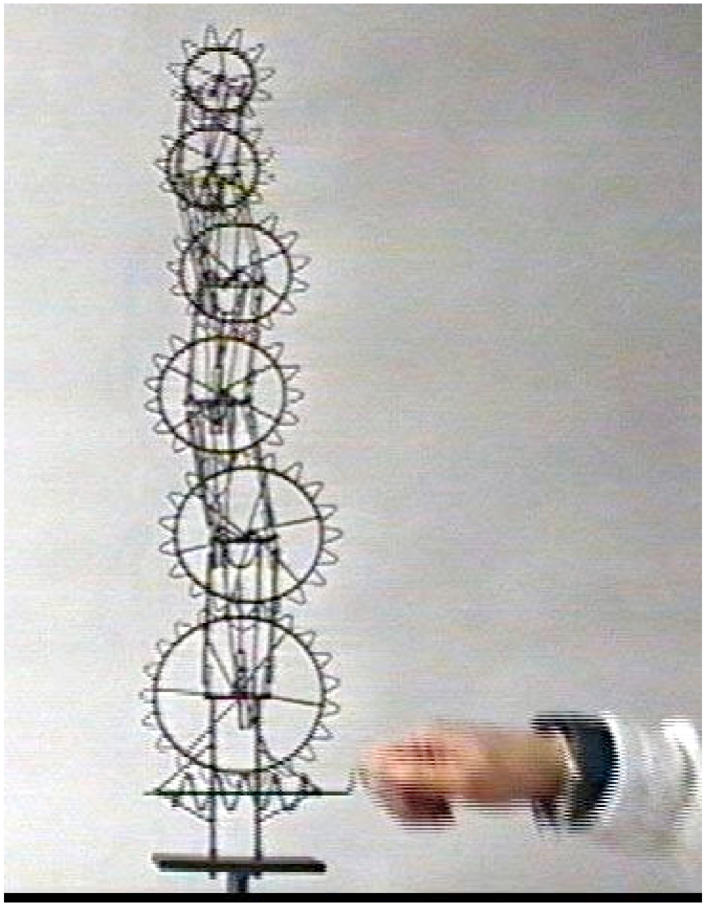
Alloy’s analysis can ‘execute’ a model
- forwards or backwards
- without test cases
- no ad hoc restrictions on logic



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a numbering problem

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given
- document whose paragraphs are tagged with styles
- style sheet that gives numbering rules for styles

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produce

- document with numbered paragraphs
  (like my Marktoberdorf notes)

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\section Overview
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A.1.1 Why?
A.2 Overview
Part B: Conclusions
B.1 Unrelated Work
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style sheet assigns to each style

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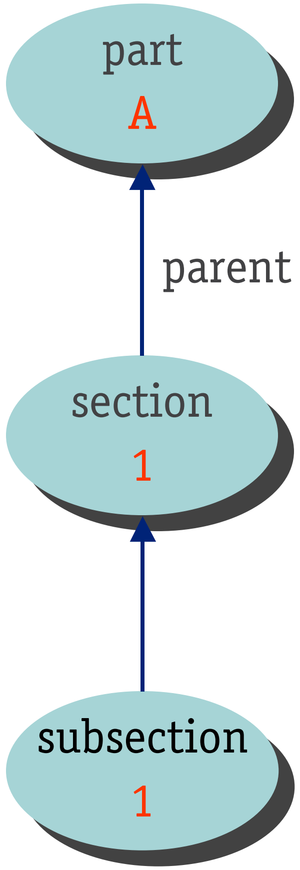

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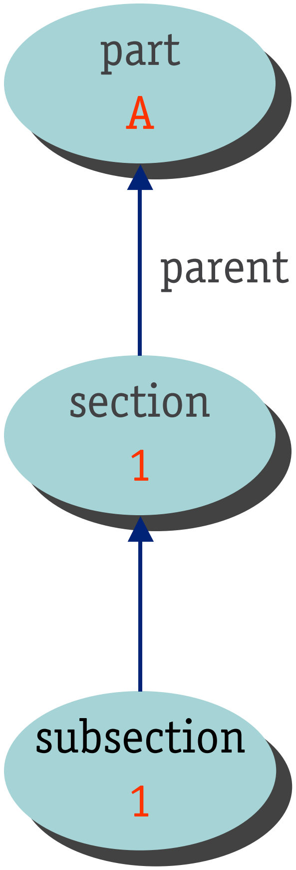

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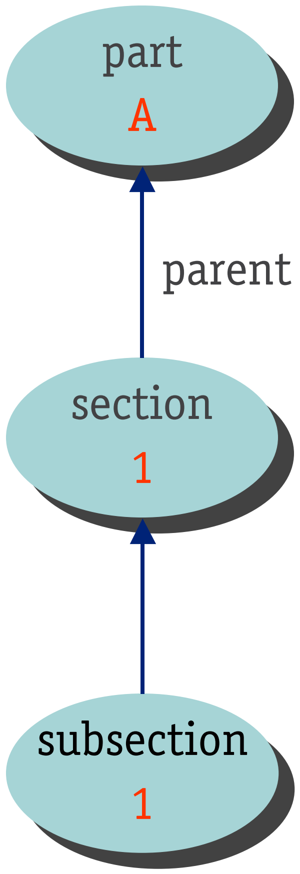

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declare styles & parent relation

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sig Style {parent: option Style}
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fun Show () {some parent}
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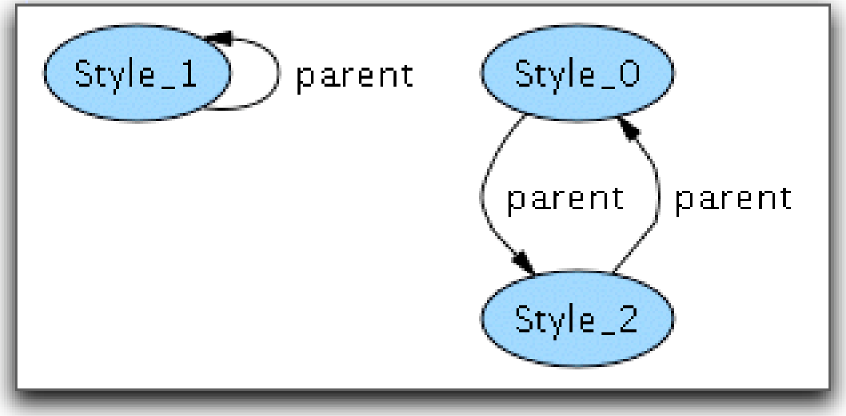

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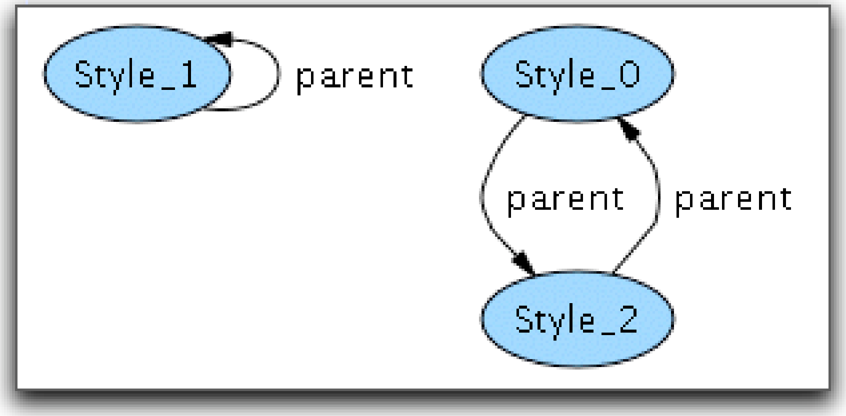

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fact {Acyclic (parent)}
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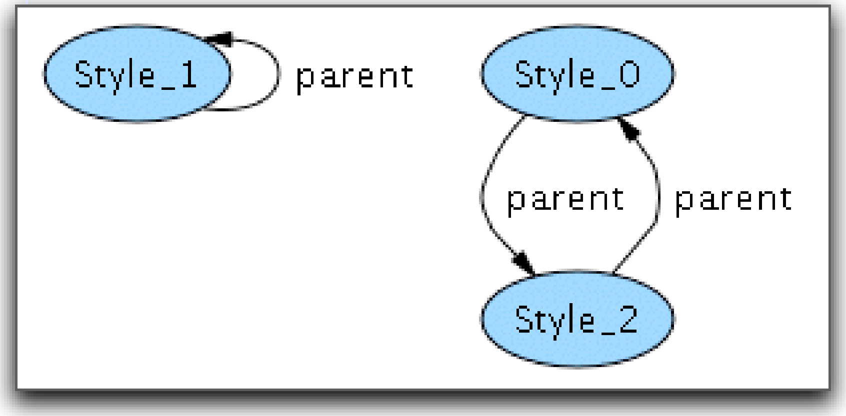

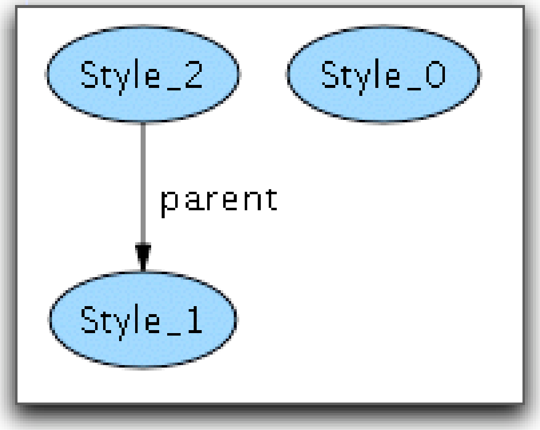

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how to define acyclic

```
fun Acyclic [t] (r: t -> t) {no iden[t] & ^r}
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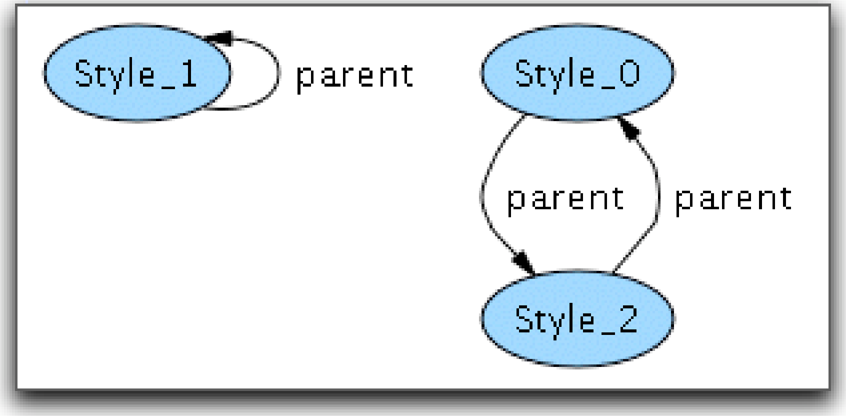

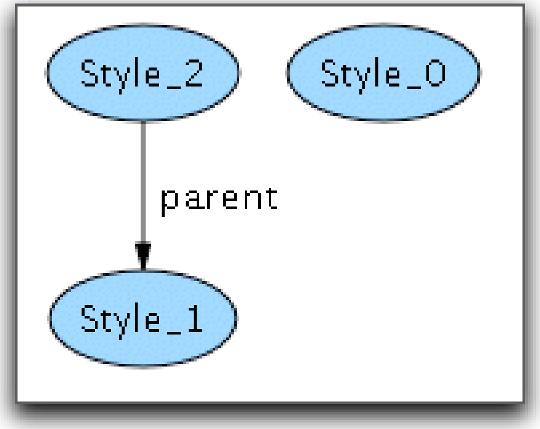

numbers

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introduce numbers

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sig Number {
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add numbers to styles

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sig NumberedStyle extends Style {init: Number}
fact {Style = NumberedStyle}
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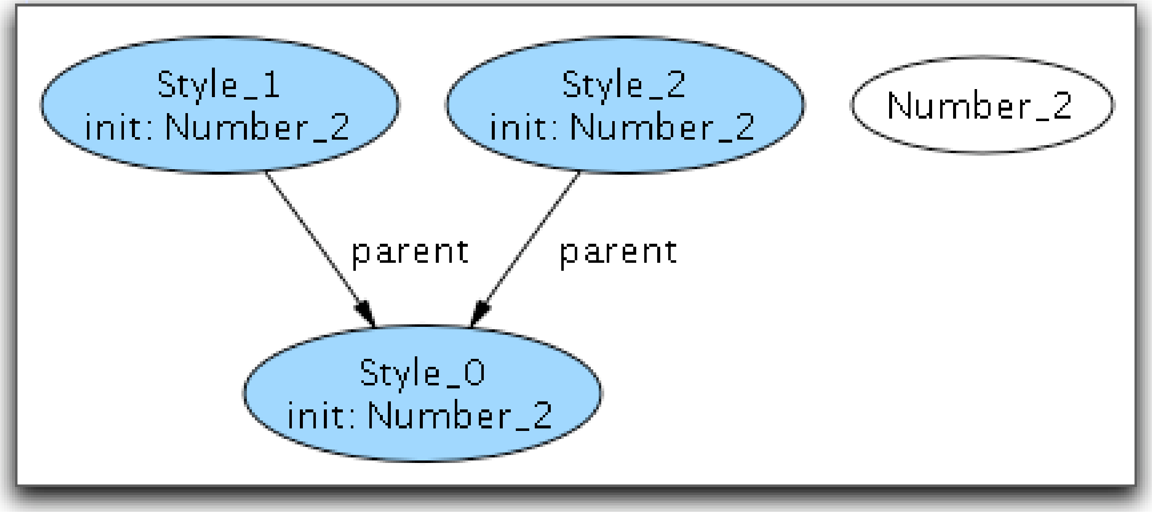

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fun ShowNumbering () {some num}
run ShowNumbering
  for 2 but 1 Numbering
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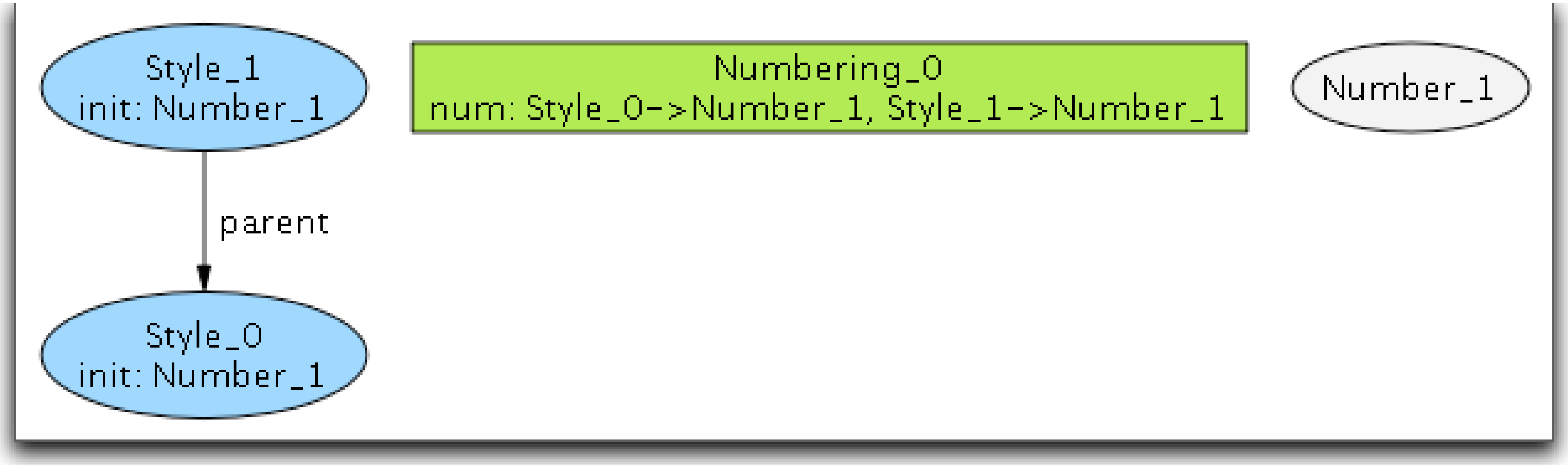

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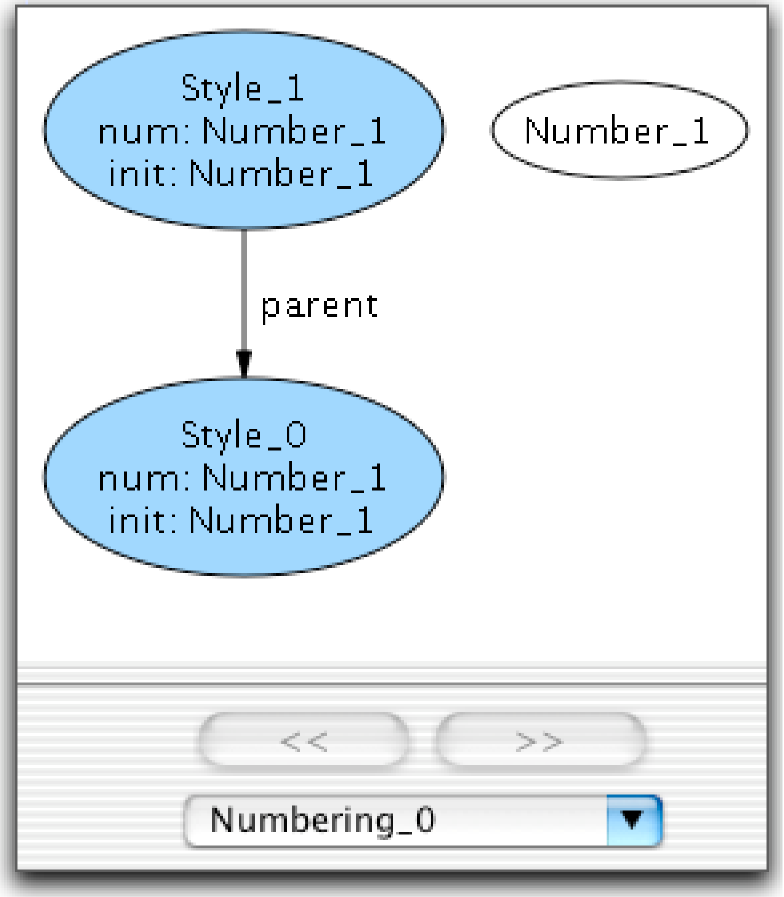

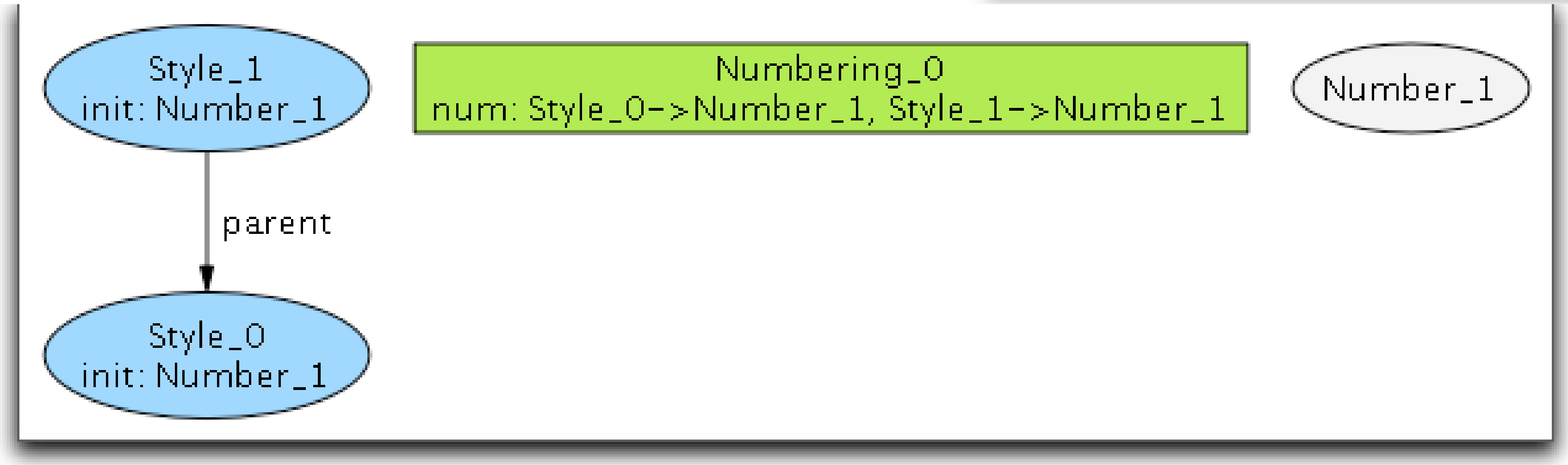

numbering algorithm

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what numbering n' follows n for paragraph of style s?

- ie, just gave numbering n
- encounter paragraph with style s
- must now generate numbering n'

numbering algorithm

what numbering *n’* follows *n* for paragraph of style *s*?

- ie, just gave numbering *n*
- encounter paragraph with style *s*
- must now generate numbering *n’*

an attempt:

```
fun Next (n,n': Numbering, s: Style) {
  n'.num =
    {d: s.^parent, x: Number | x = n.num[d]} +
    s -> if no n.num[s] then s.init else n.num[s].next
  }
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showing next

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```
run Next for 3 but 2 Numbering
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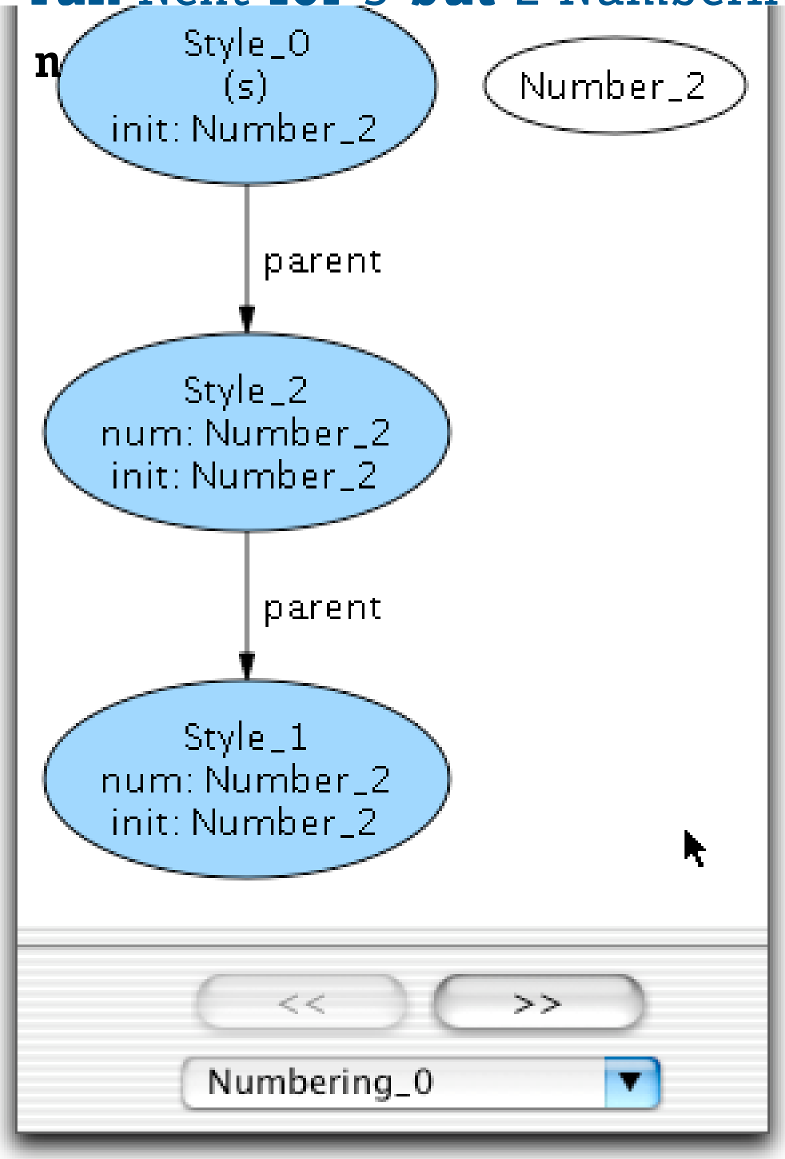

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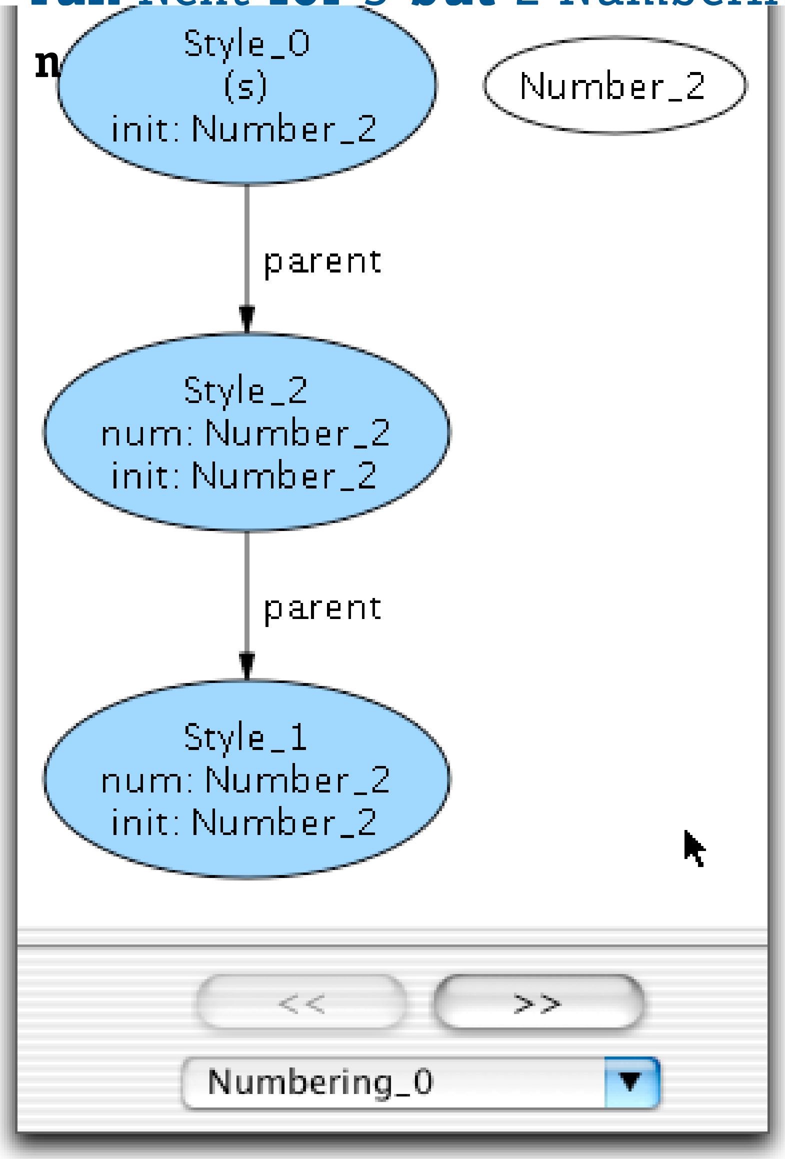

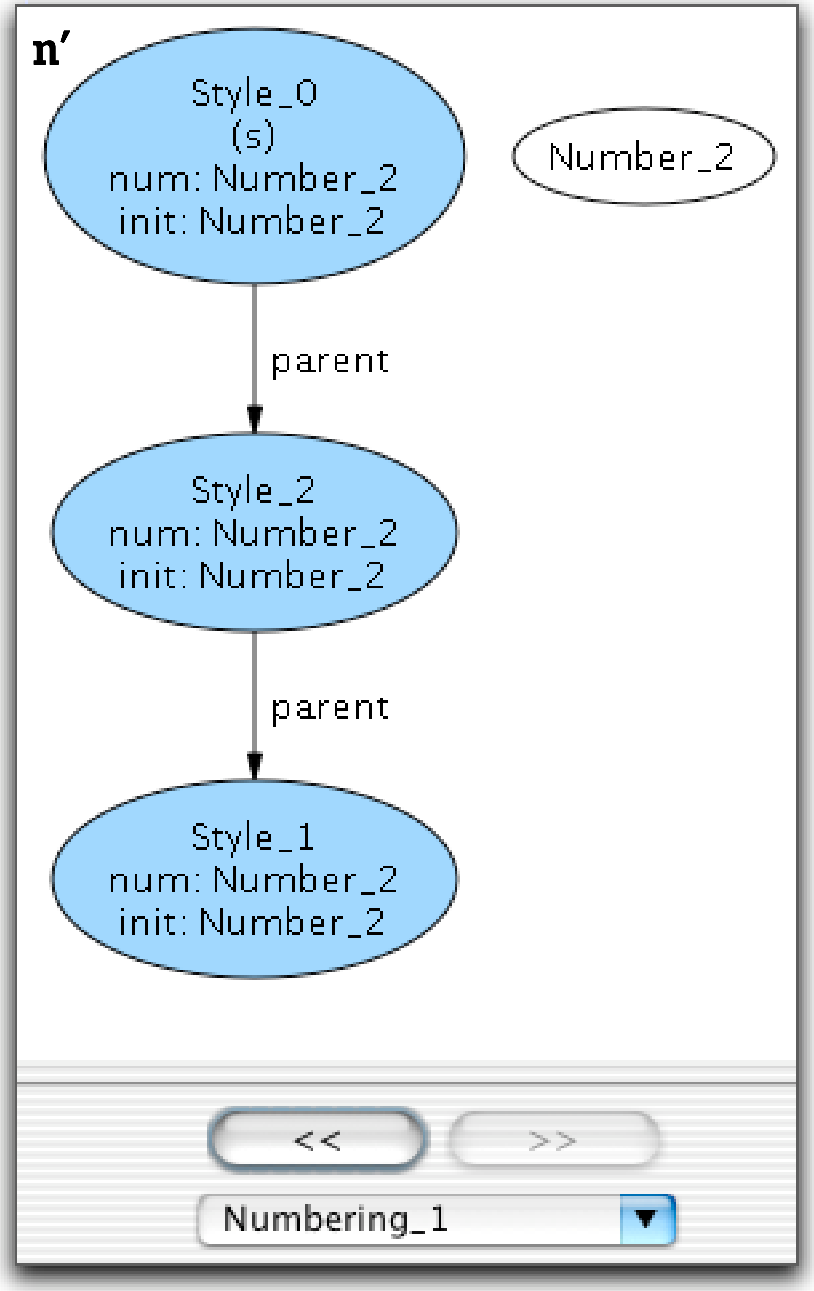

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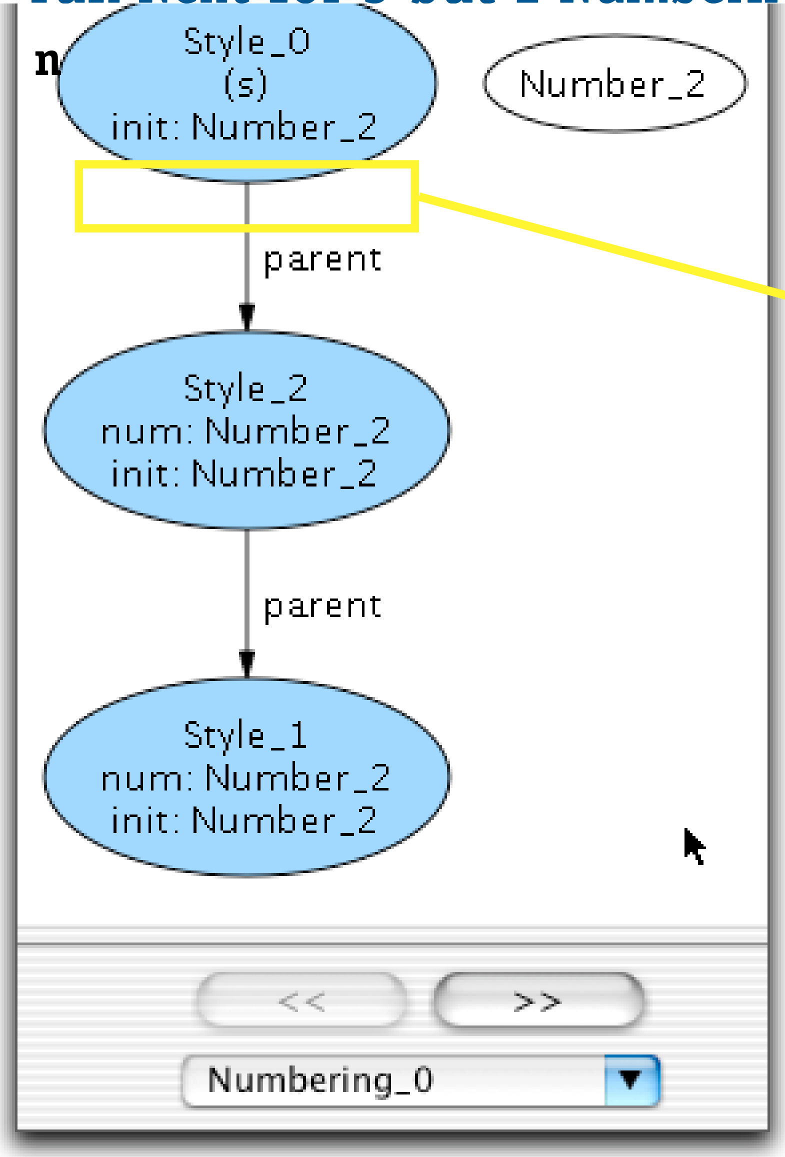

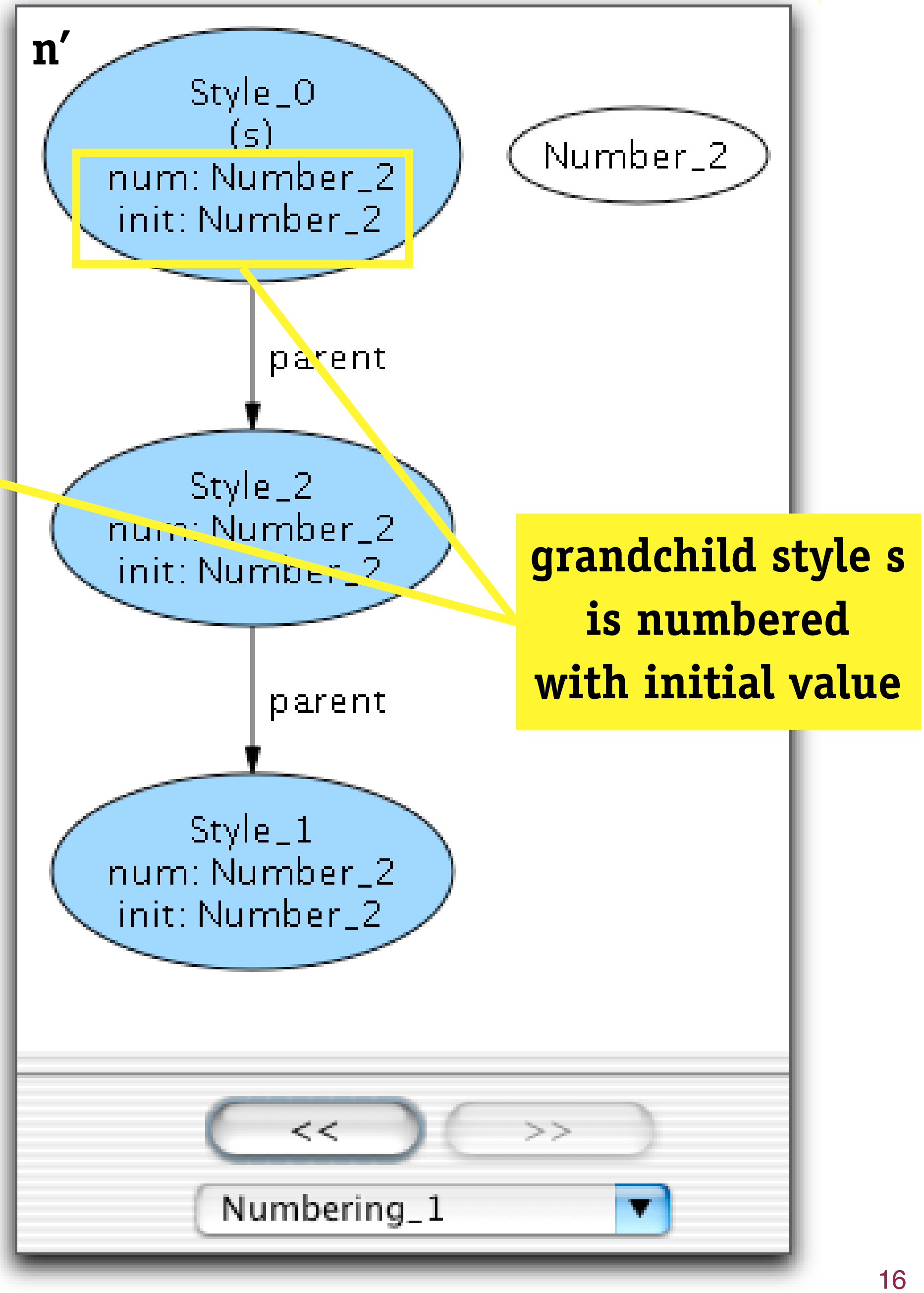

grandchild style s is numbered with initial value

guiding the simulation

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  Next (n,n',s)  && some n.num[s.~parent]}
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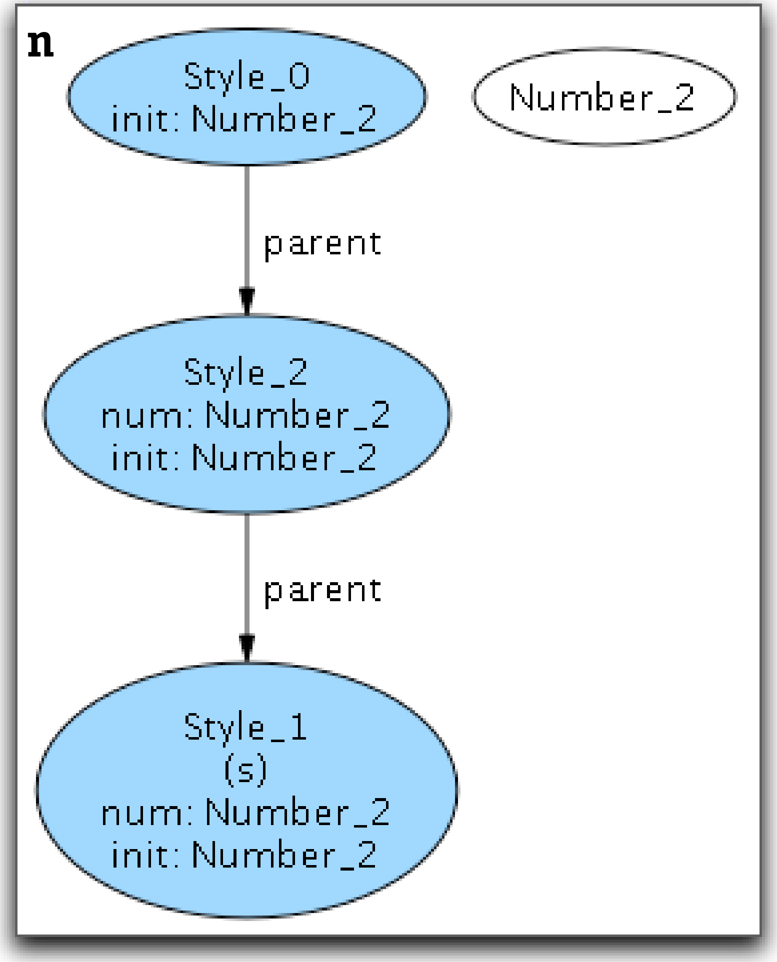

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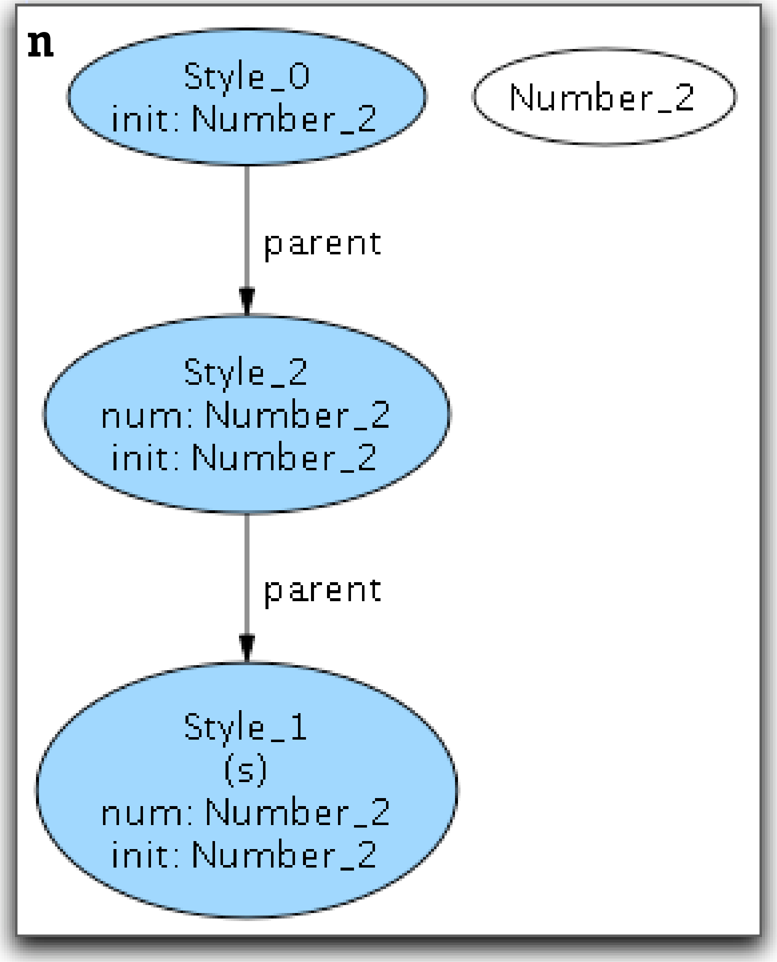

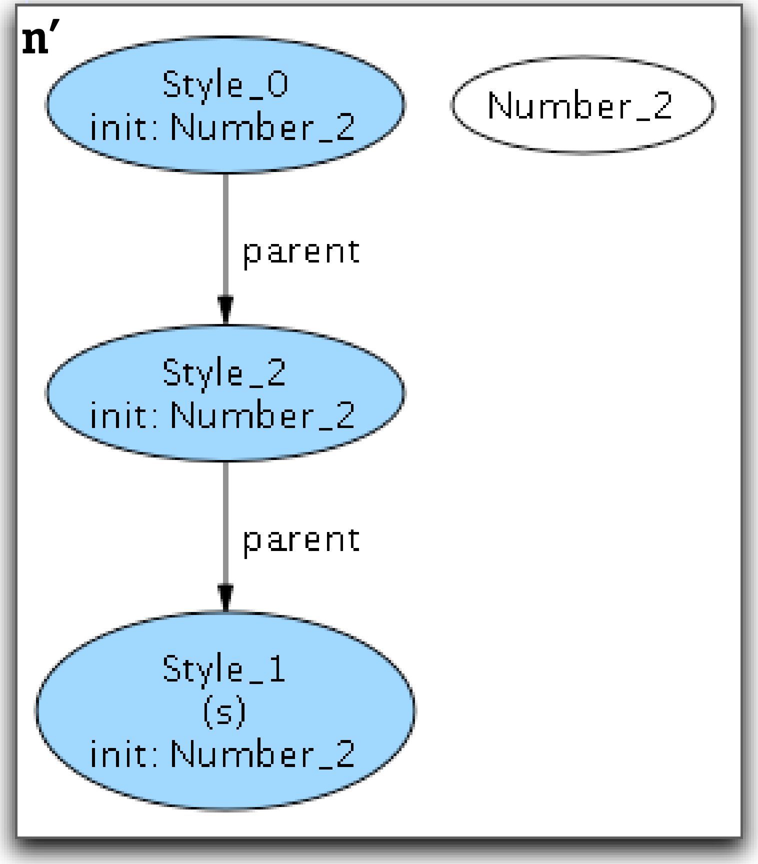

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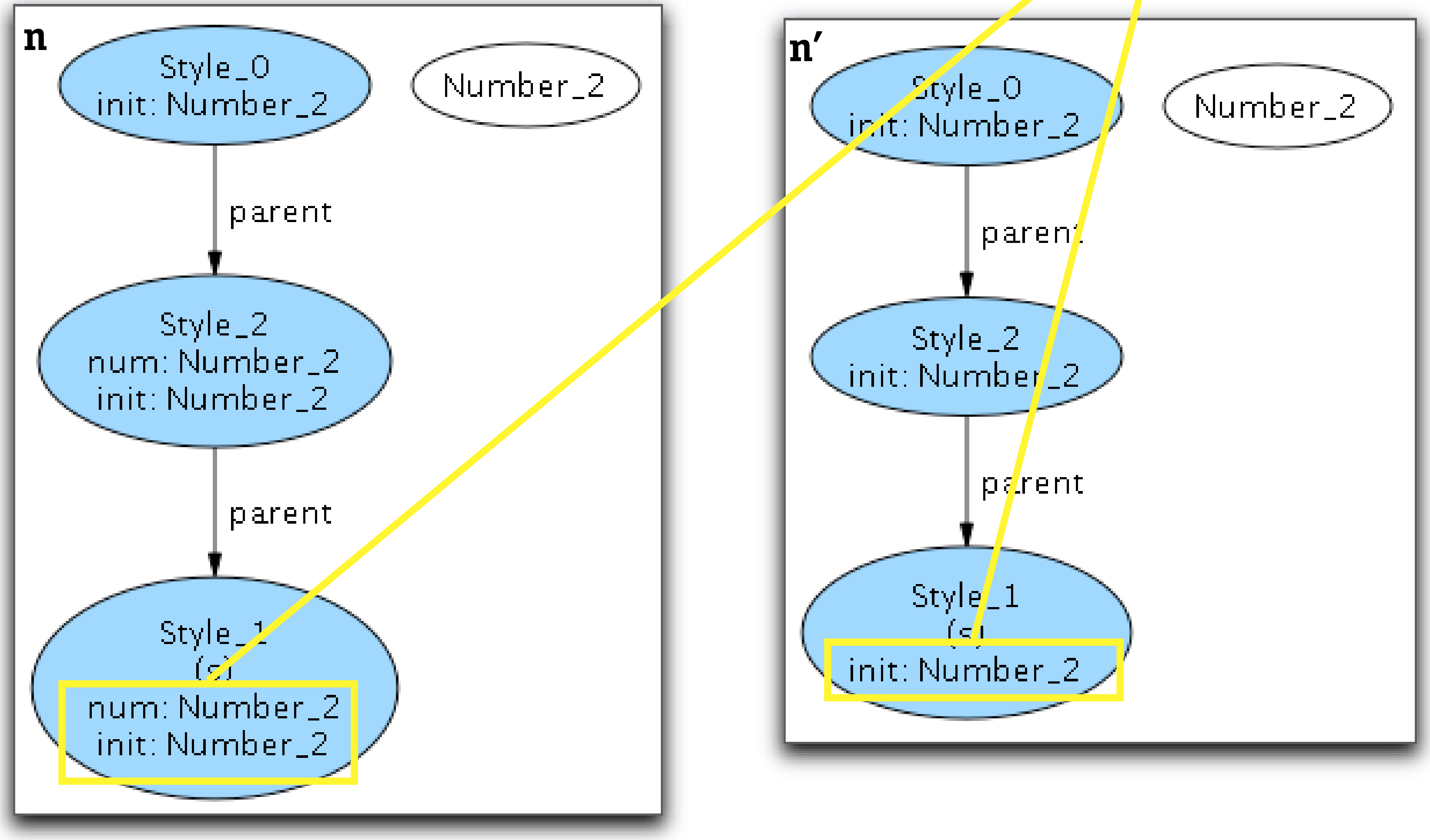

root style s loses its number because no next!

fixing the operation

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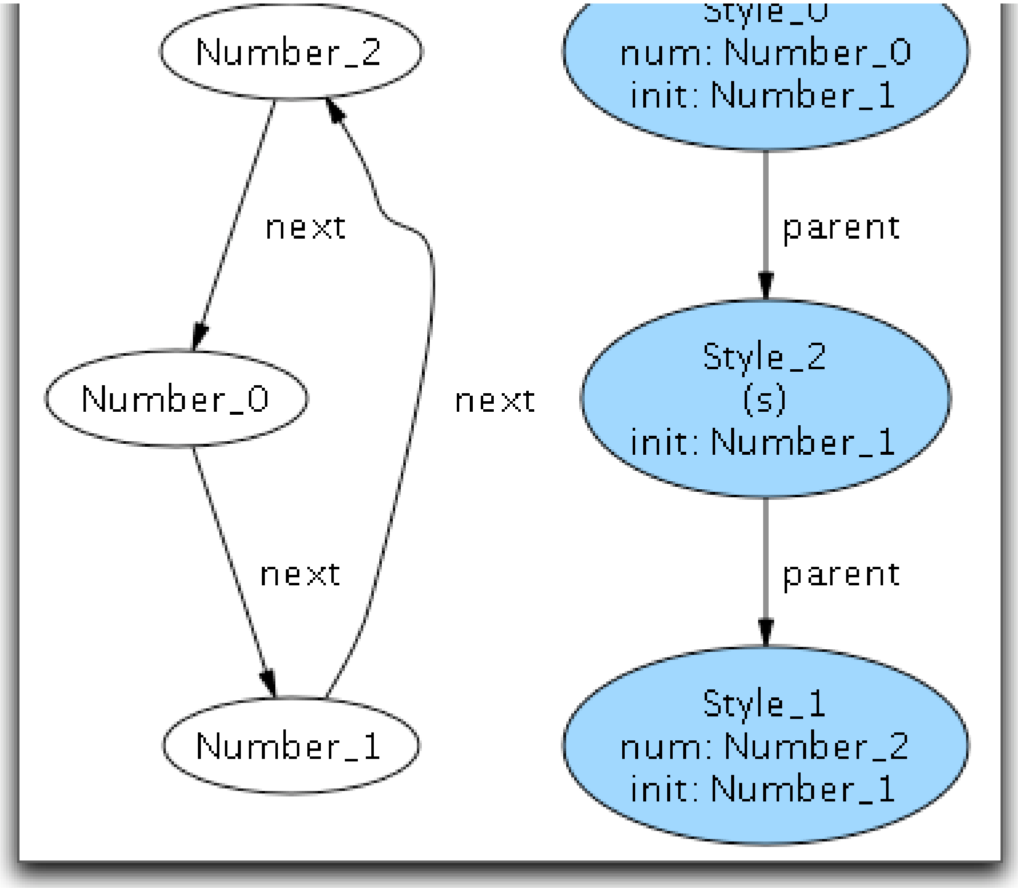

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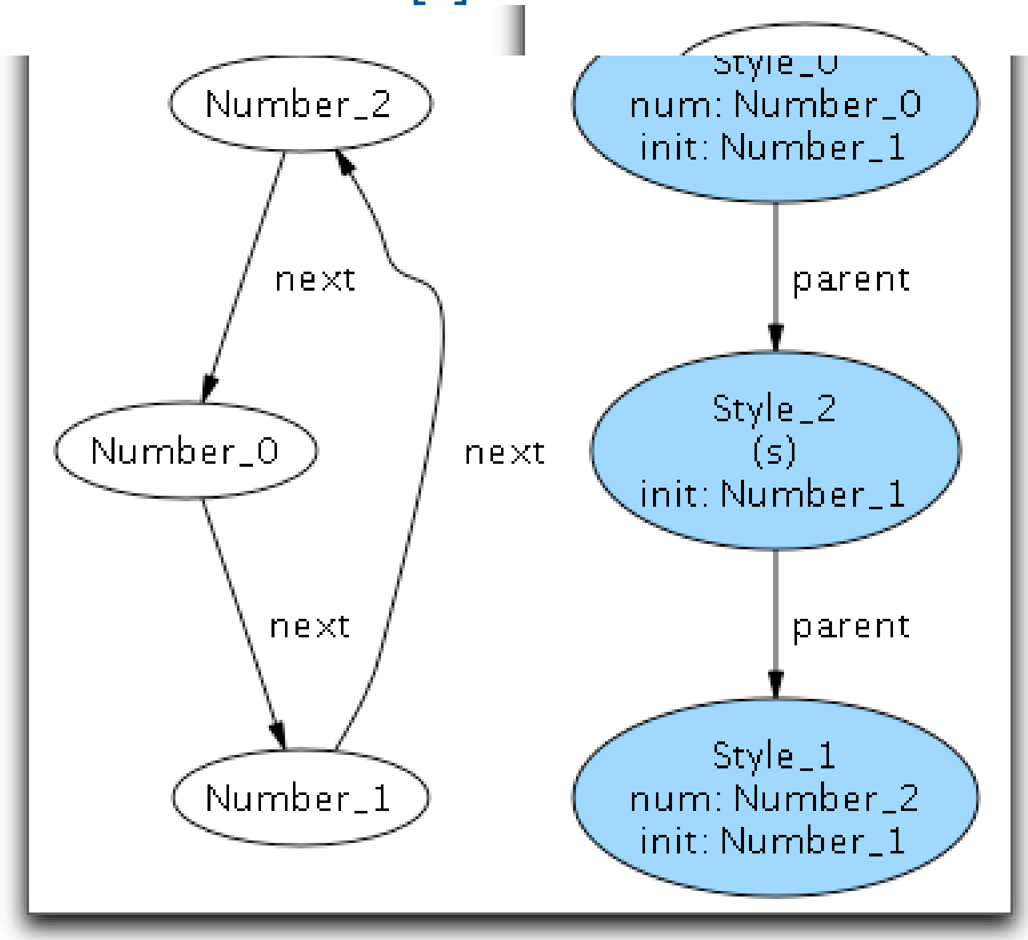
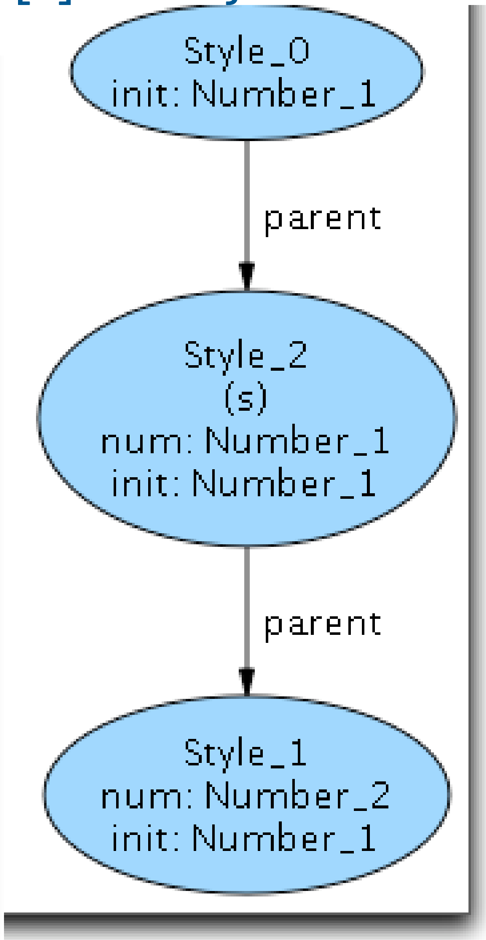
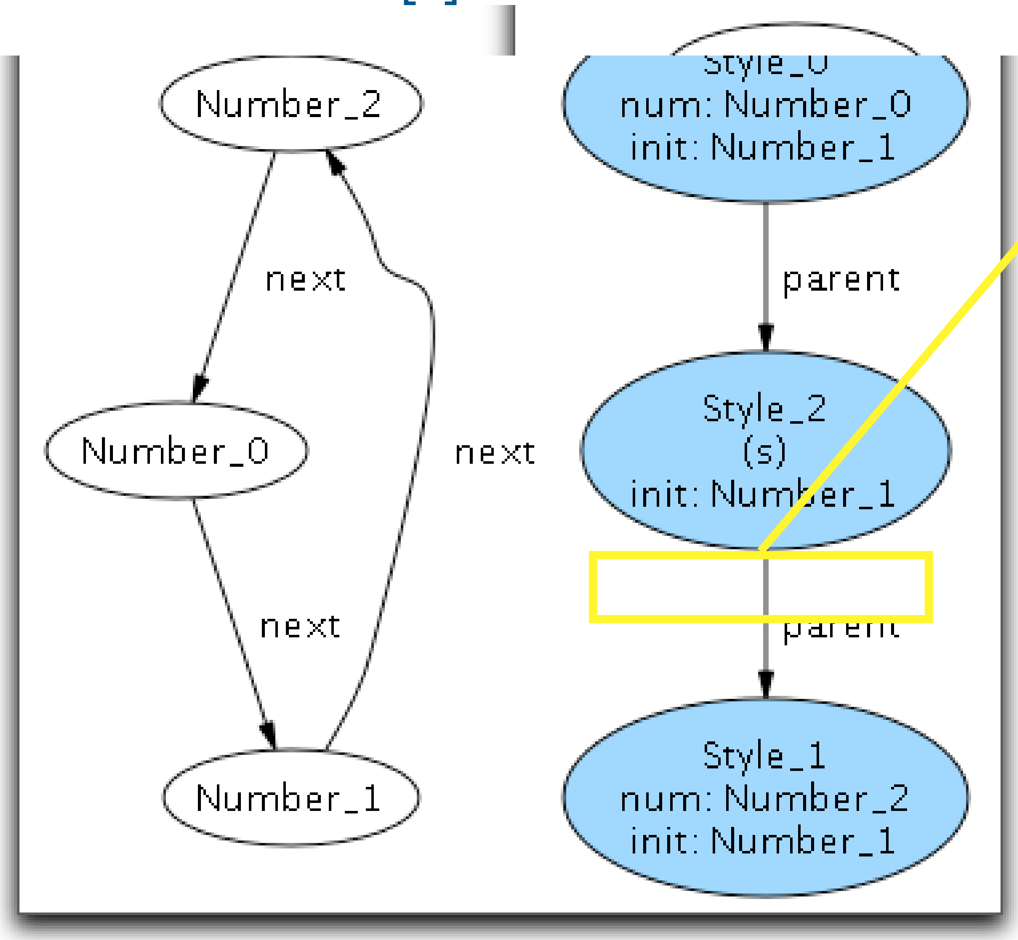


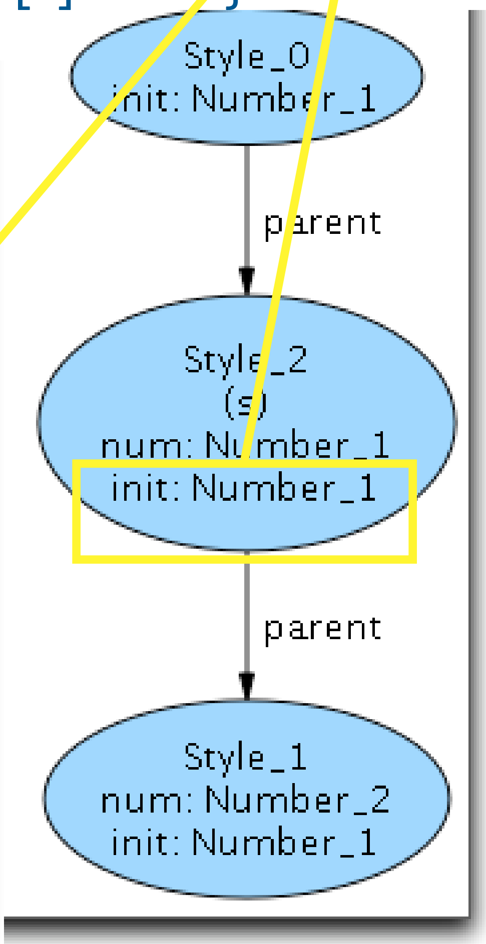


fixing the operation

```
fun Next (n,n': Numbering, s: Style) {
  let i = n.num[s] | some i => some i.next
  n'.num = {d: s.^parent, x: Number | x = n.num[d]} +
    s -> if no n.num[s] then s.init else n.num[s].next }
```

style s gets numbered with initial value





guiding the simulation

guiding the simulation

```
fun ShowNext (n,n': Numbering, s: Style) {
  Next (n,n',s)  && some n.num[s.~parent] && some n.num[s]}
run ShowNext for 3 but 2 Numbering
```

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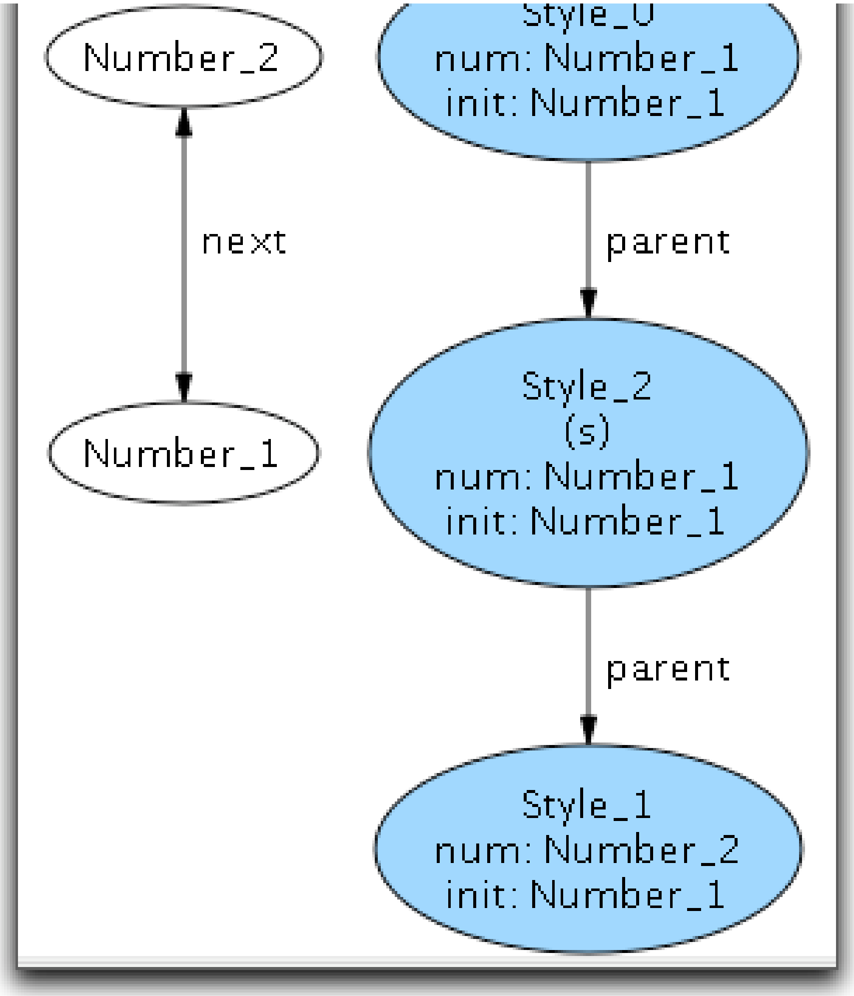

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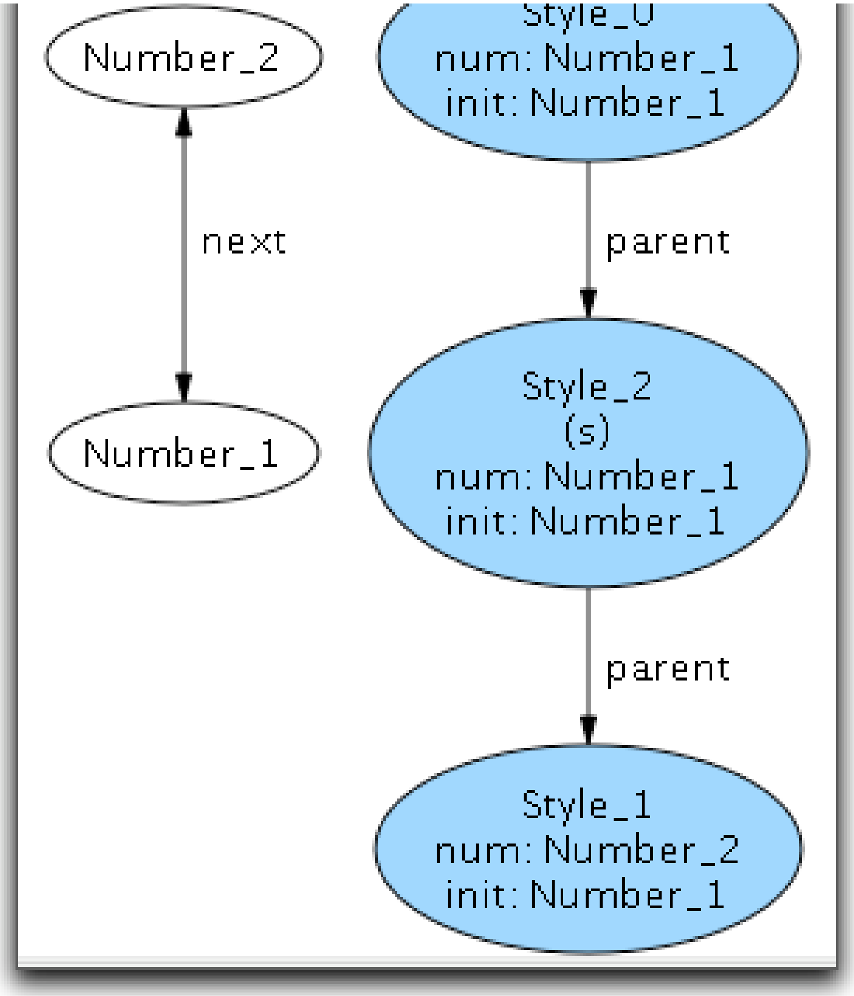

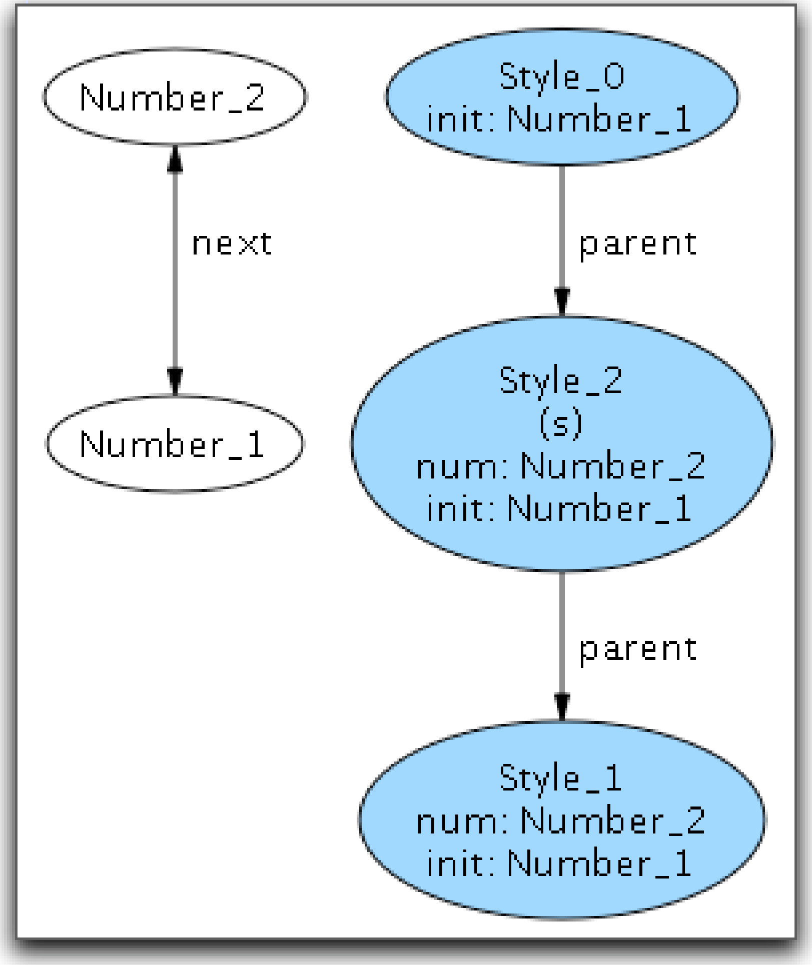

checking a property

checking a property

if style is not a parent, step is reversible

```
assert Reversible {
  all n0, n1, n: Numbering, s: Style - Style.parent |
    Next(n0,n,s) && Next(n1,n,s) => n0.num = n1.num}

check Reversible
```

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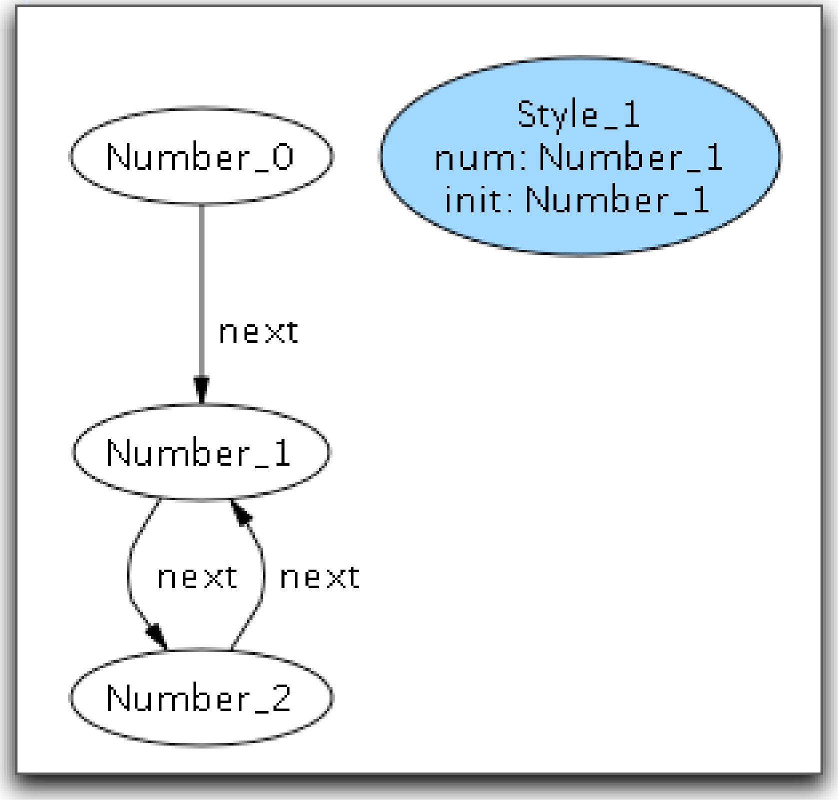

trying again…

trying again…

make numbering injective

```
fact {Injective (next)}
```

trying again…

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```
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```

does this fix the problem?

counterexample

counterexample

after numbering n

counterexample

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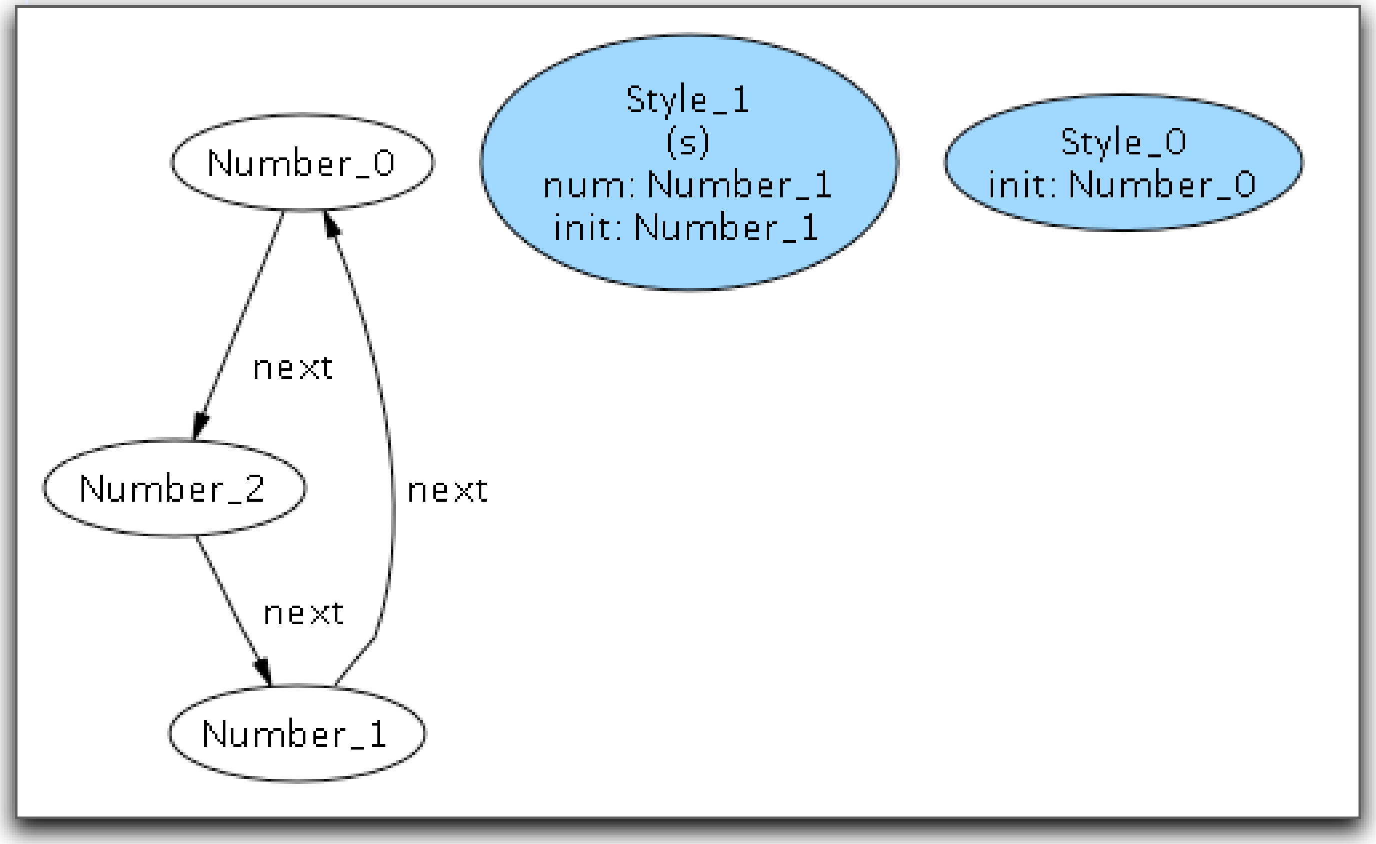
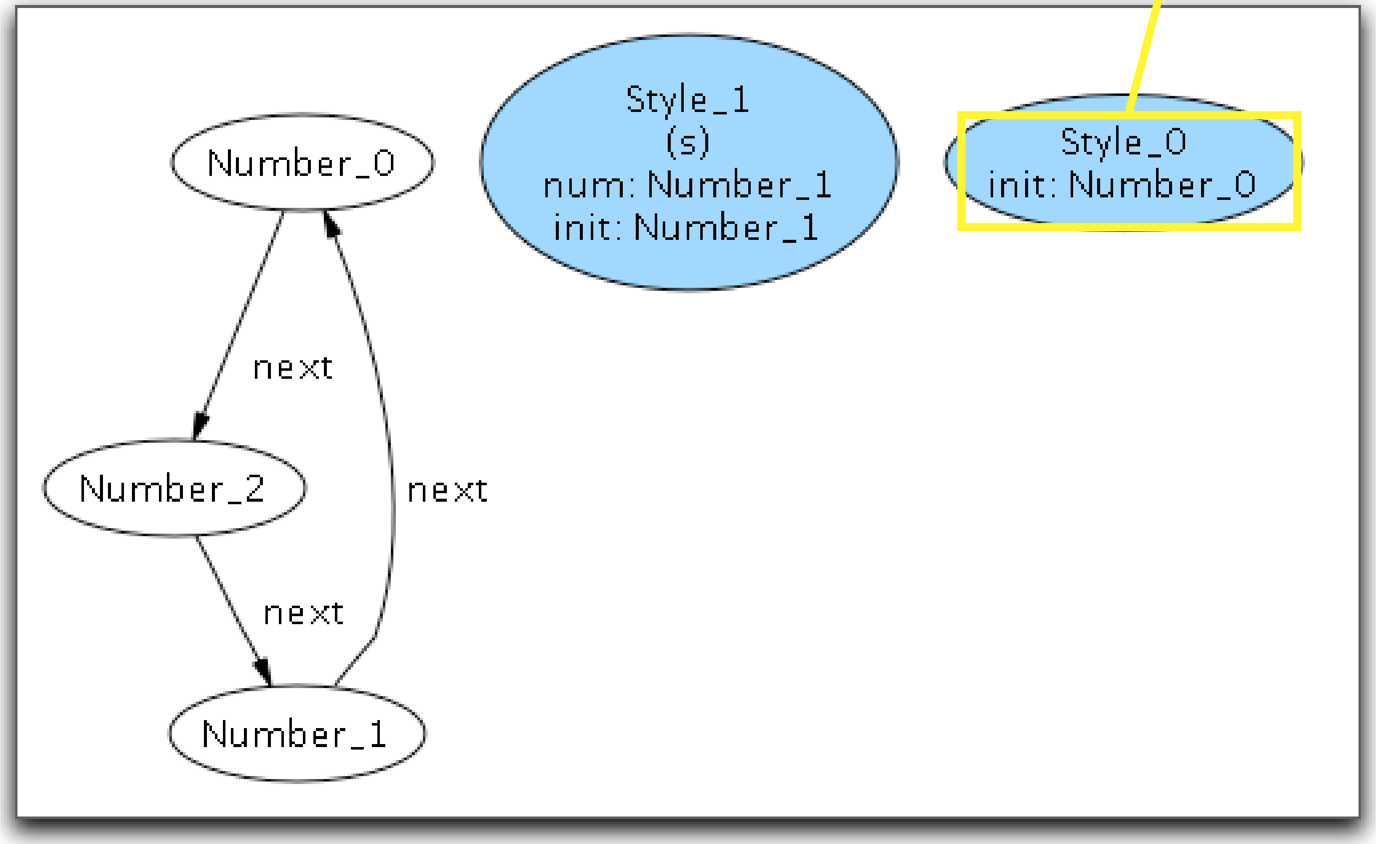


counterexample

after numbering *n*



adjacent style has no number afterwards

counterexample, ctd

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before
numberings
n0 and *n1*

counterexample, ctd

before
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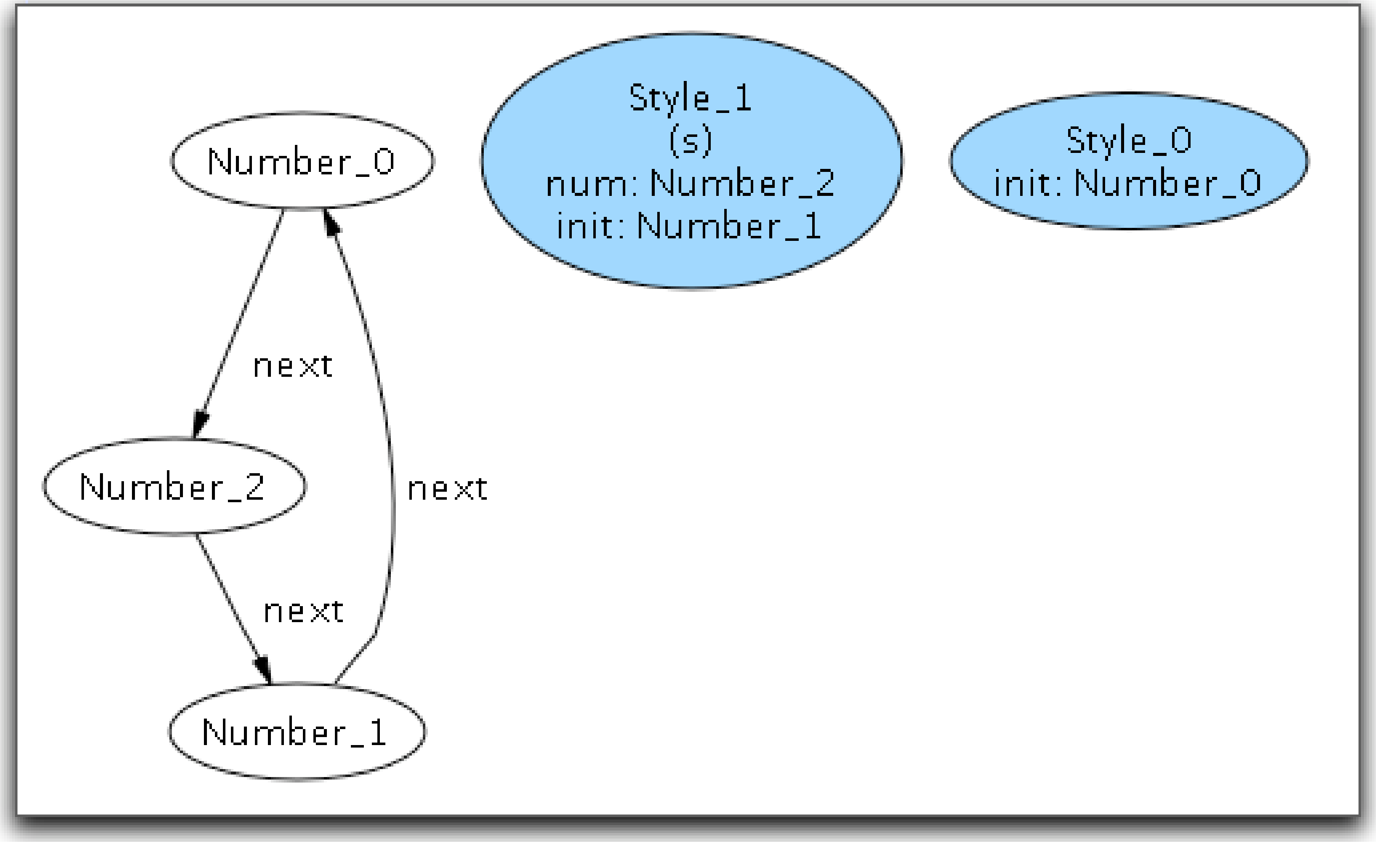
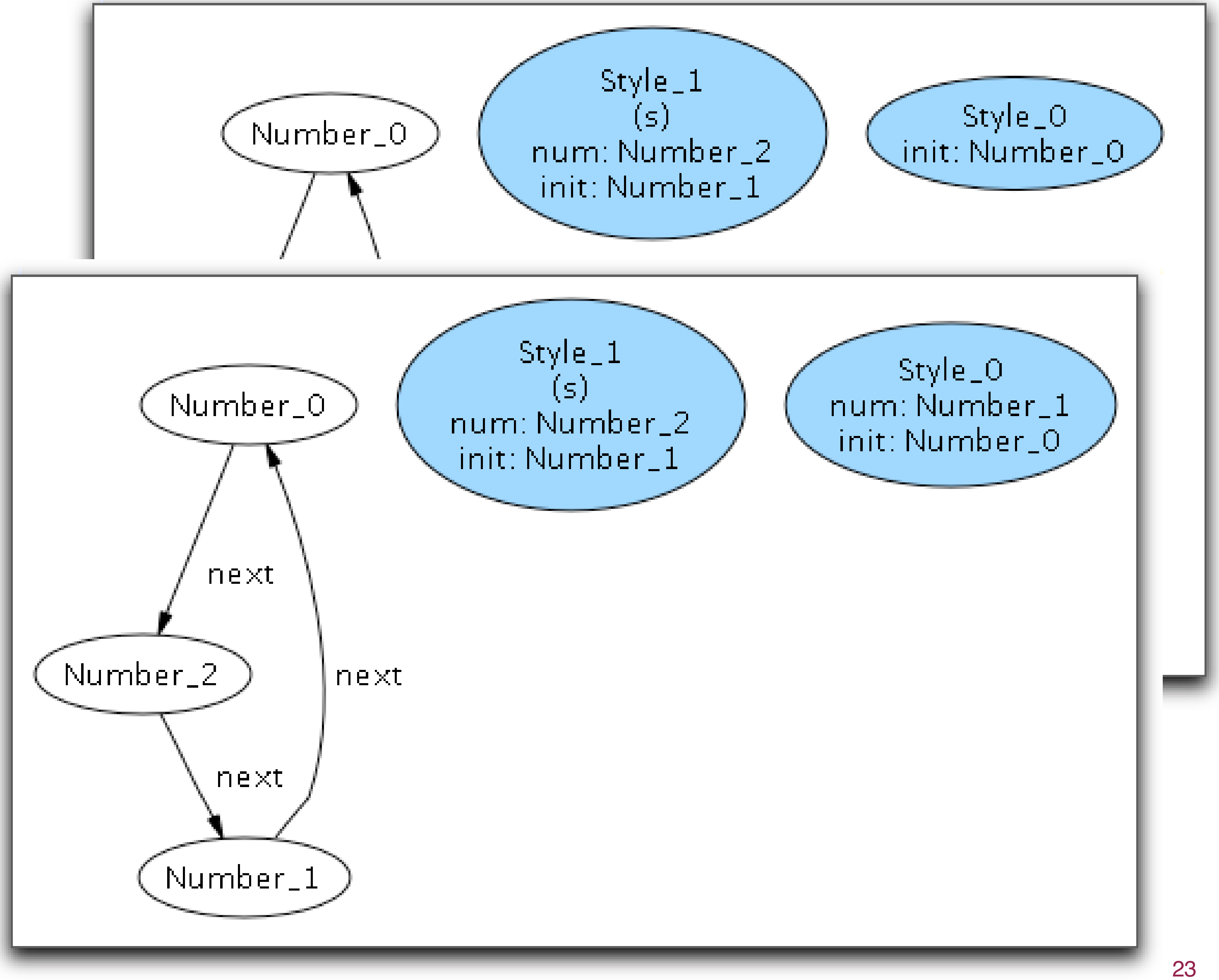


counterexample, ctd

before numberings *n0* and *n1*



counterexample, ctd

before
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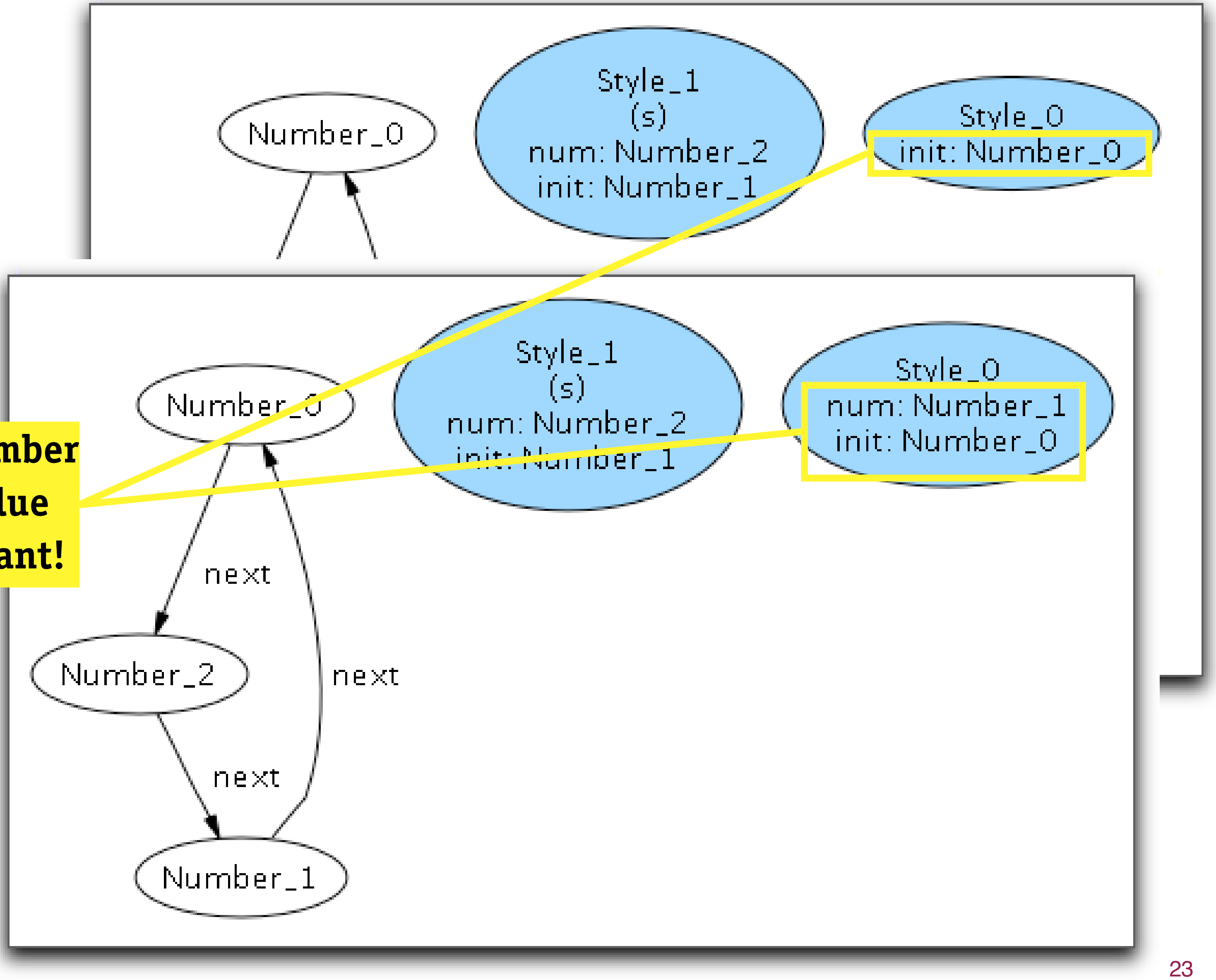
n0 and *n1*



adjacent style's number eliminated, so value before was irrelevant!

masking

masking

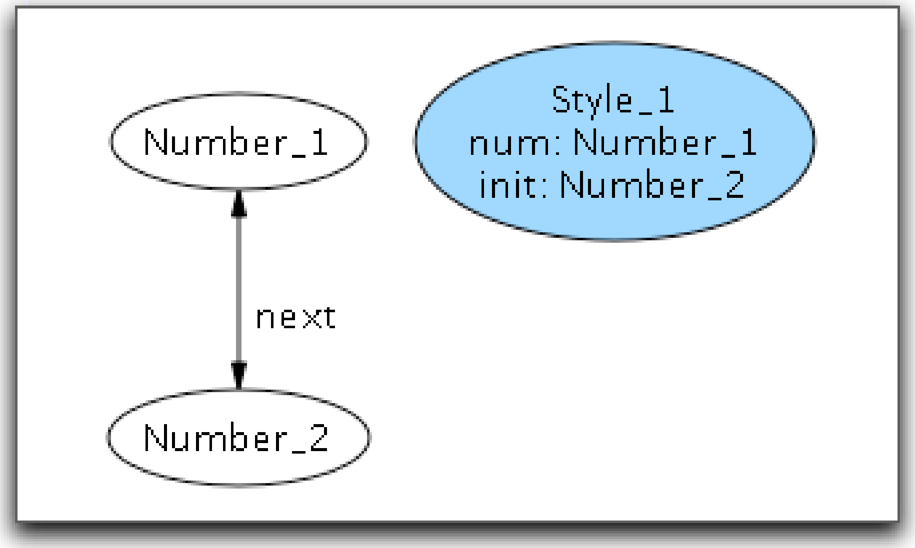
check again, assuming styles form a line

```
assert ReversibleWhenLine {
  Injective(parent)
  && (some root: Style | Style in root.*~parent) =>
  all n0, n1, n: Numbering, s: Style - Style.parent |
    Next(n0,n,s) && Next(n1,n,s) => n0.num = n1.num}

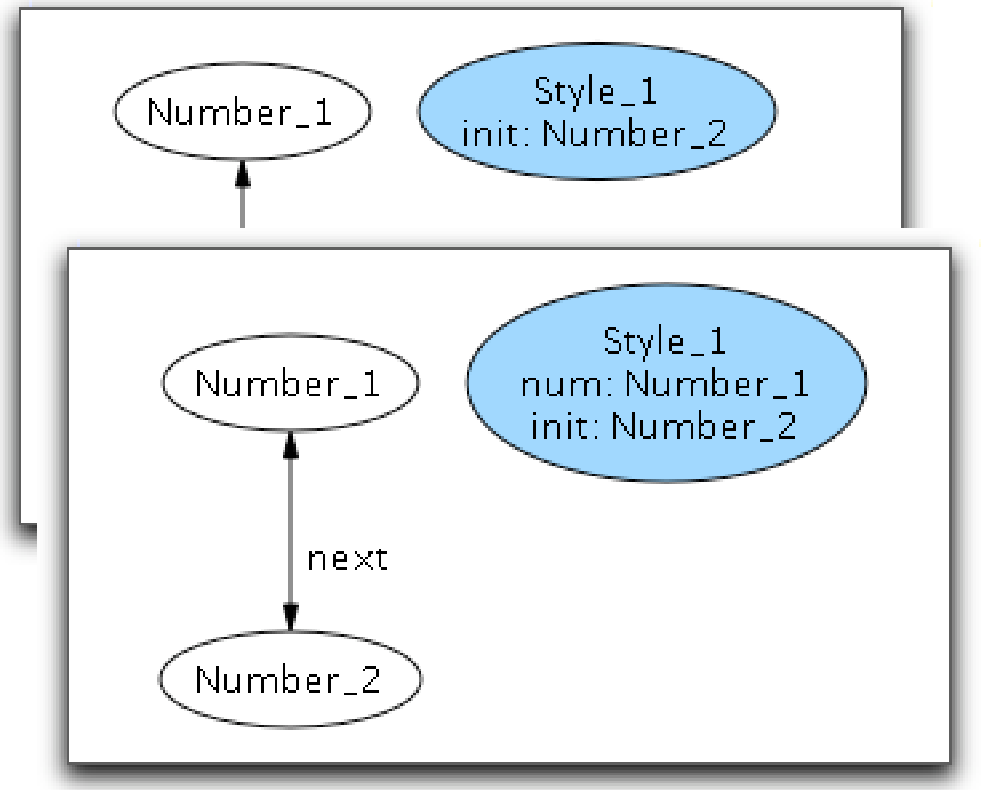
check ReversibleWhenLine
```

counterexample, again

counterexample, again



counterexample, again



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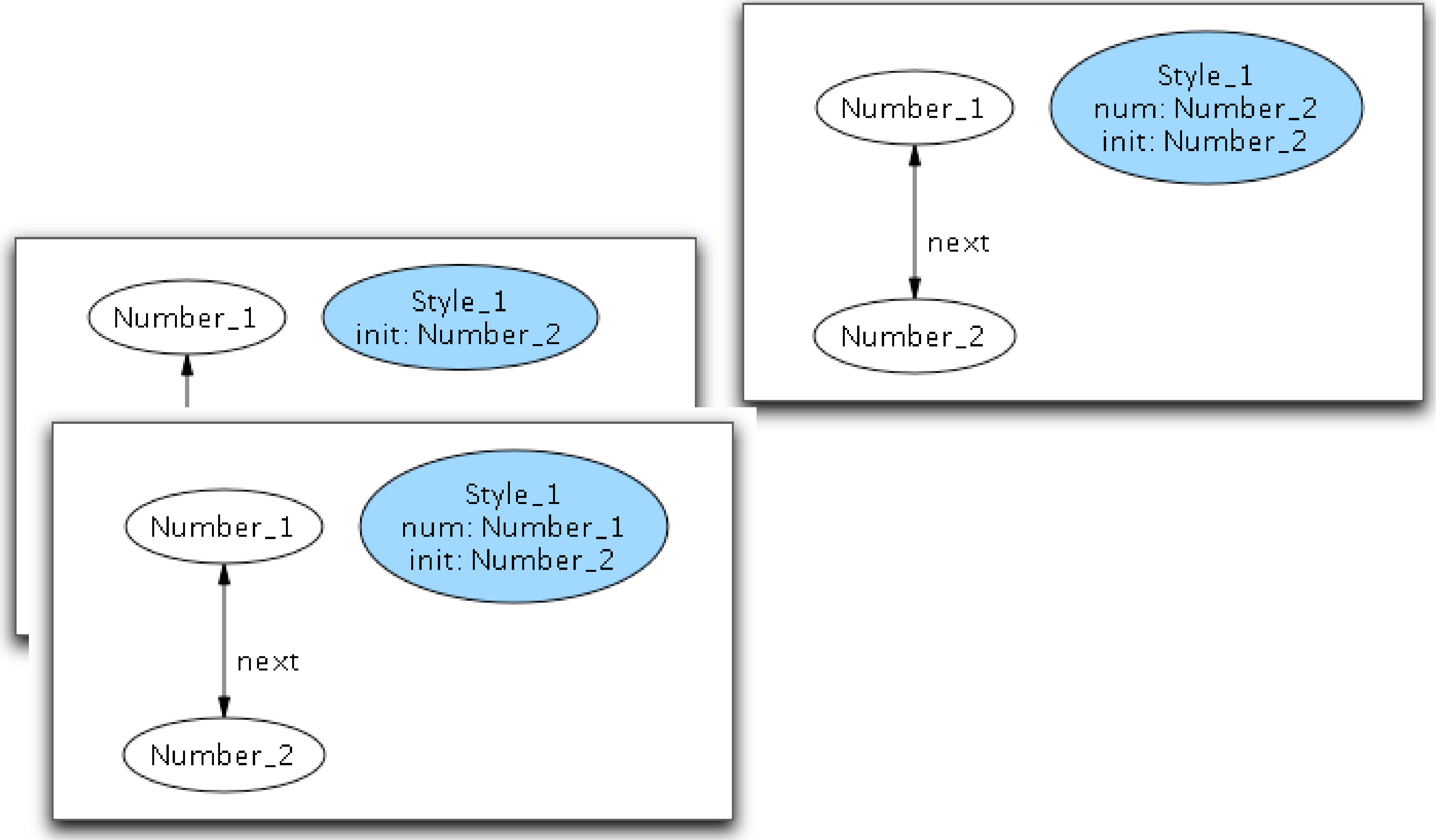

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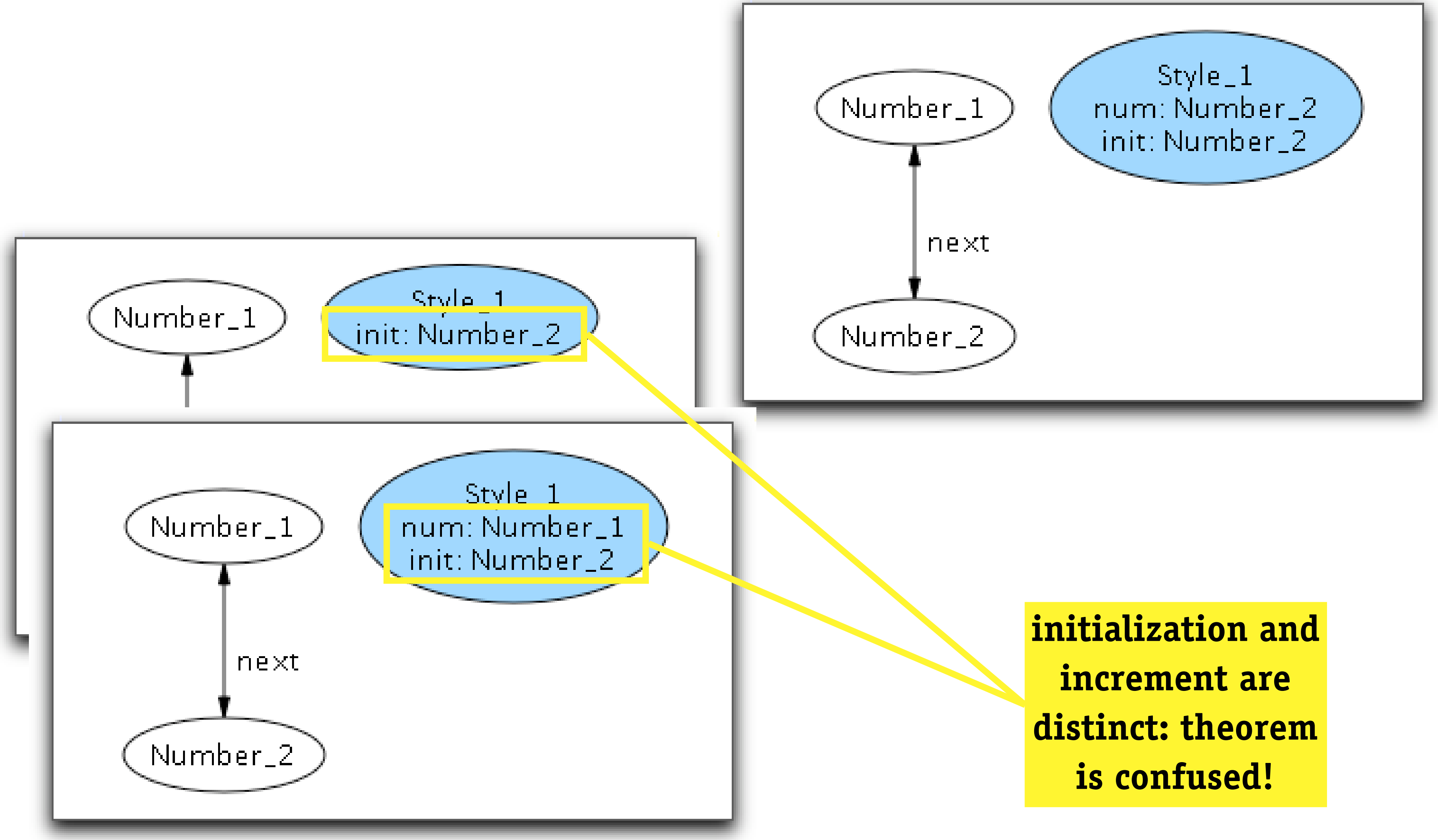

initialization and increment are distinct: theorem is confused!

checking a refactoring

checking a refactoring

are these equivalent?

```
fun Next1 (n,n′: Numbering, s: Style) {
  n′.num =
    {d: s.^parent, x: Number | x = n.num[d]} +
    s -> if no n.num[s] then s.init else n.num[s].next
  }

fun Next2 (n,n′: Numbering, s: Style) {
  all d: s.^parent | n′.num[d] = n.num[d]
  n′.num[s] = if no n.num[s] then s.init else n.num[s].next
  }
```

checking a refactoring

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  }
```

ask the tool:

```
assert Same {
  all n,n′: Numbering, s: Style | Next1(n,n′,s) iff Next2(n,n′,s)}
```

what happened

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- constrain just enough to get key properties
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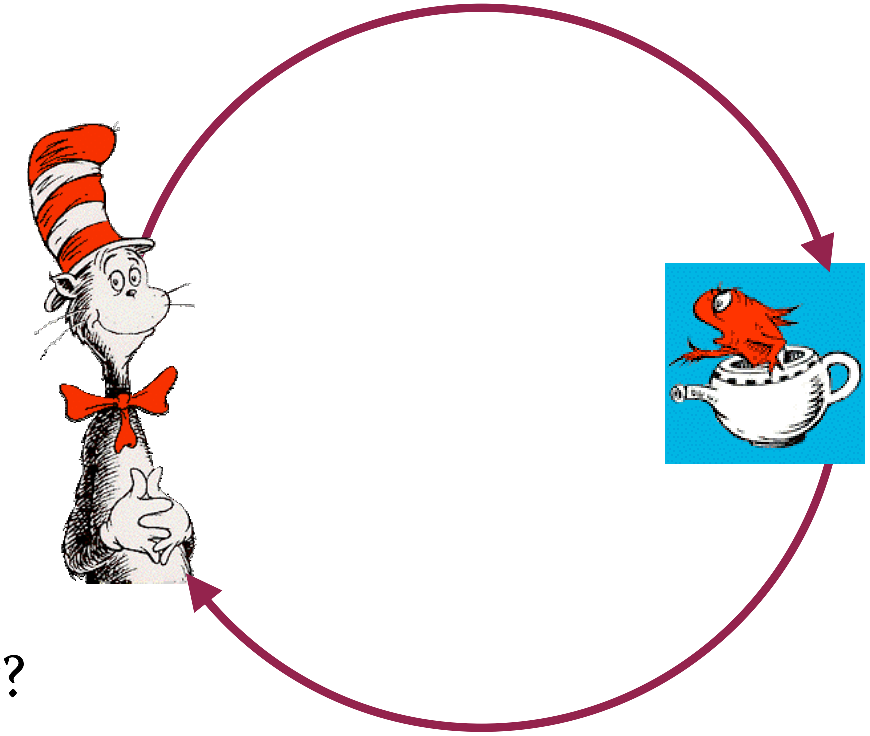
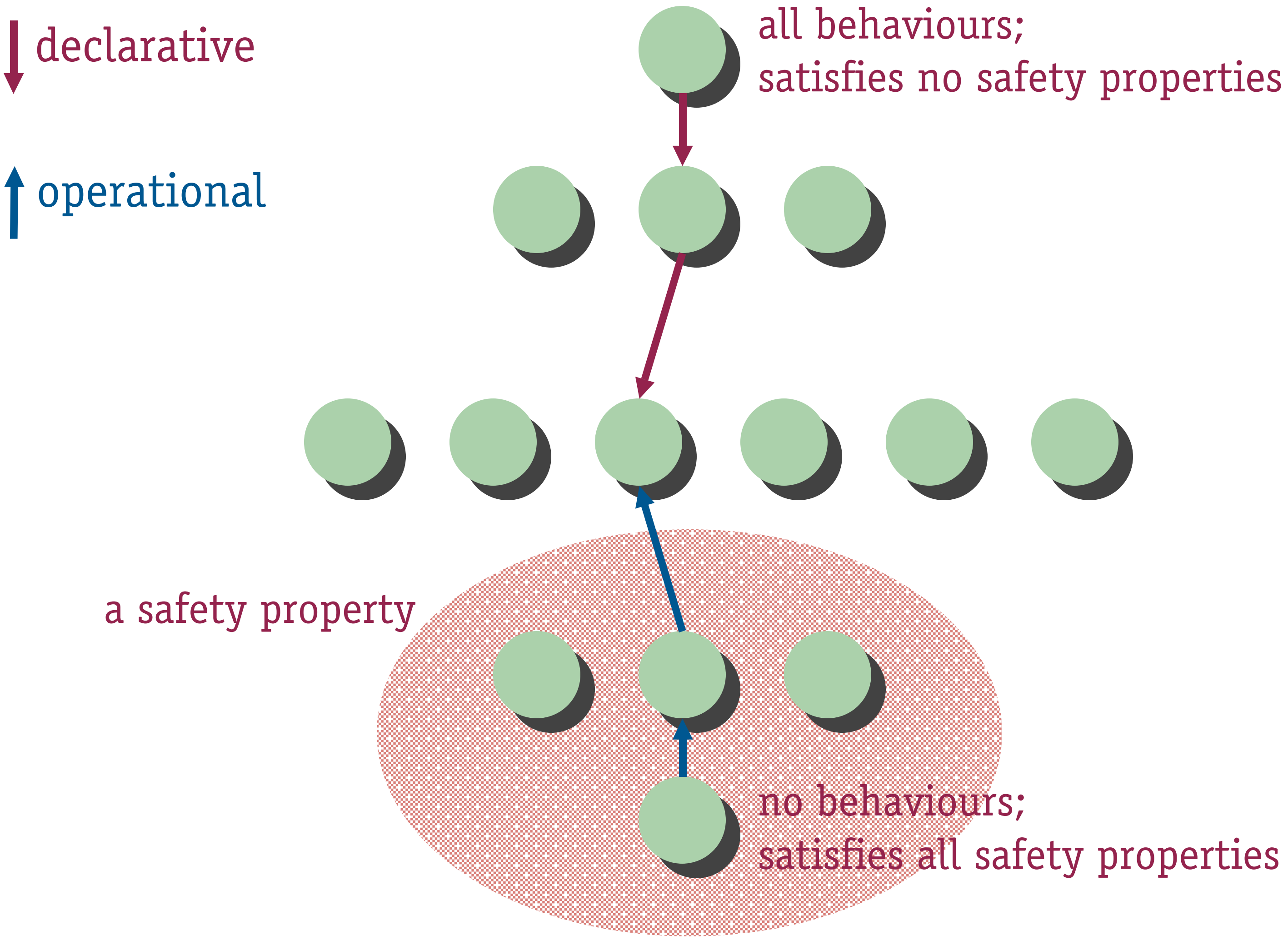


declarative vs. operational development



declarative

operational

all behaviours;
satisfies no safety properties

a safety property

no behaviours;
satisfies all safety properties

what’s been done?

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analyzing implemented systems

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- Chord peer-to-peer lookup (Wee)
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typically

- 200 lines of Alloy, 30-200 hours work

example: intentional naming

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query scheme

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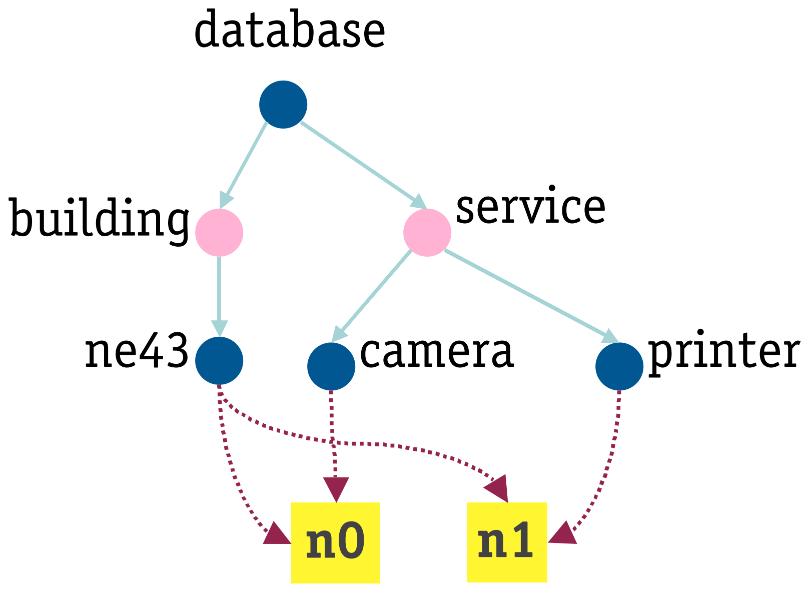

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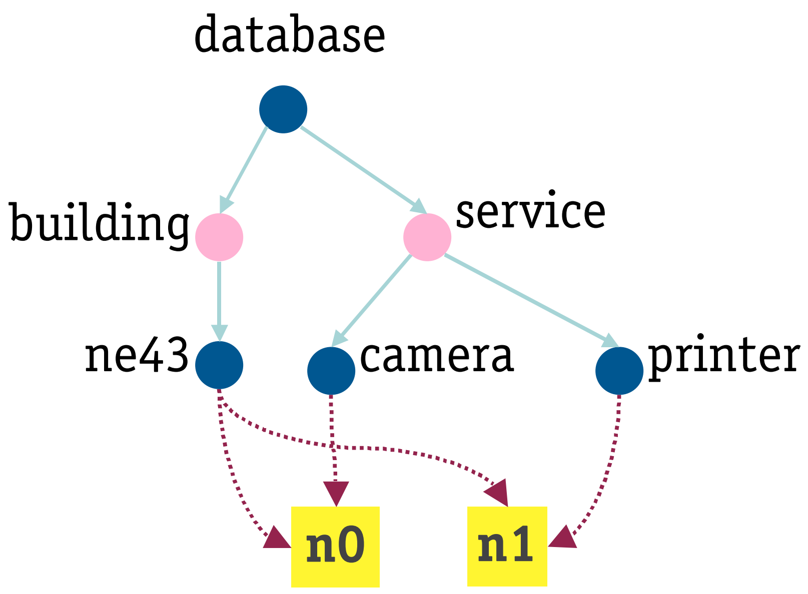

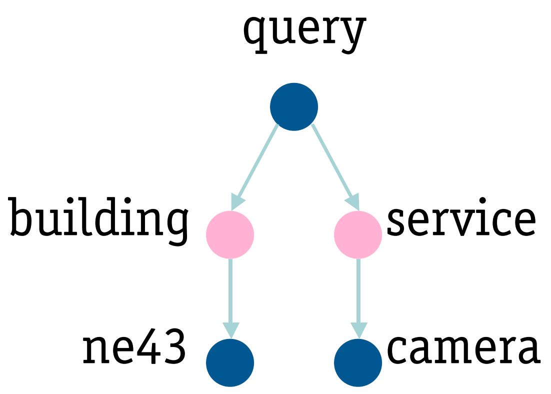

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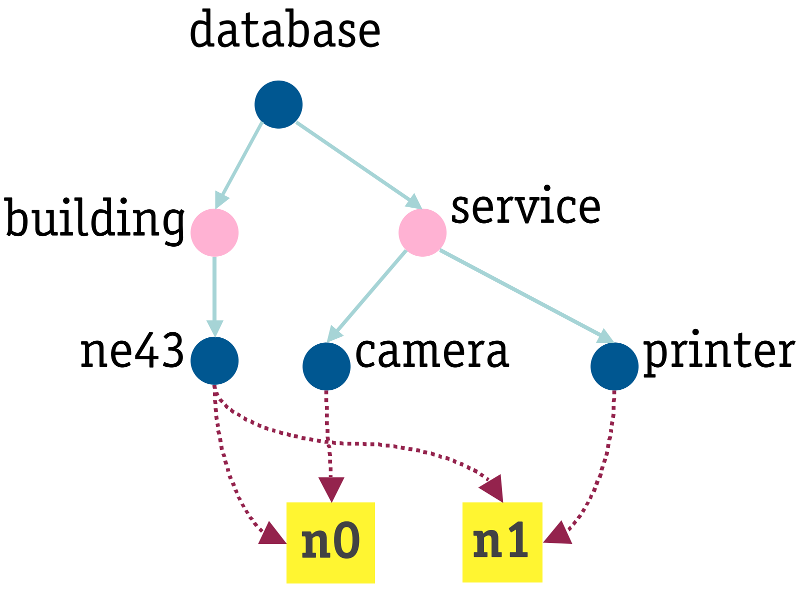

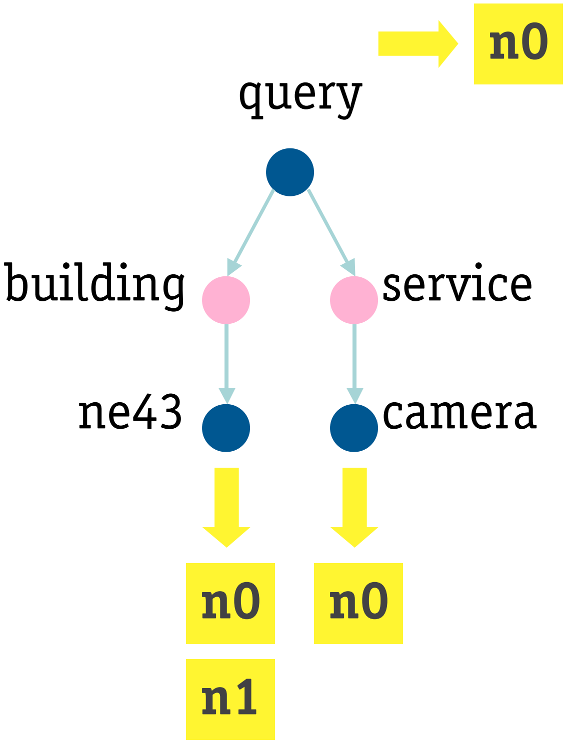

results

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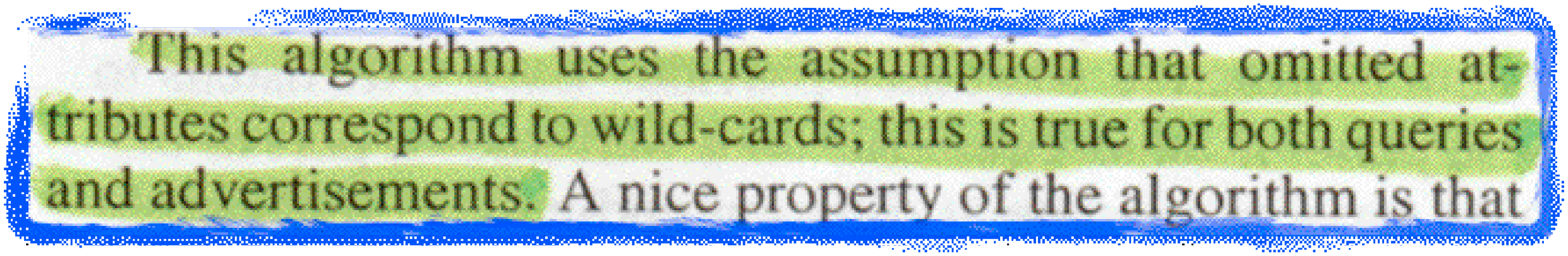
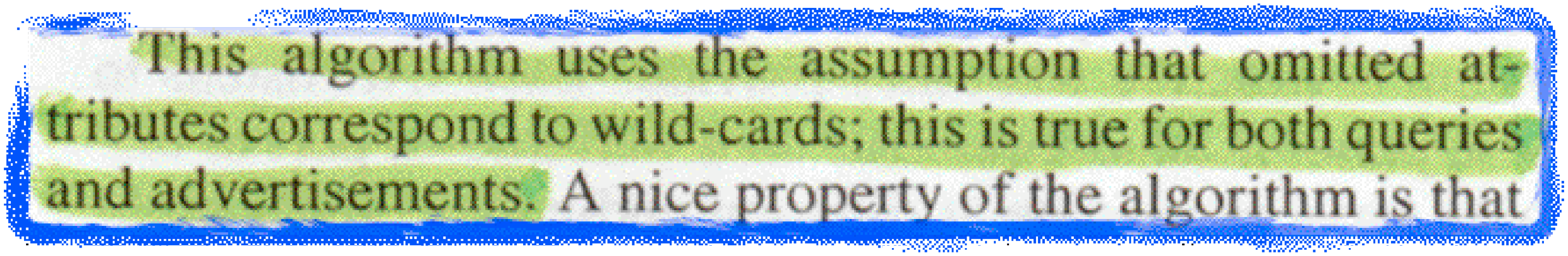


This algorithm uses the assumption that omitted attributes correspond to wild-cards; this is true for both queries and advertisements. A nice property of the algorithm is that

results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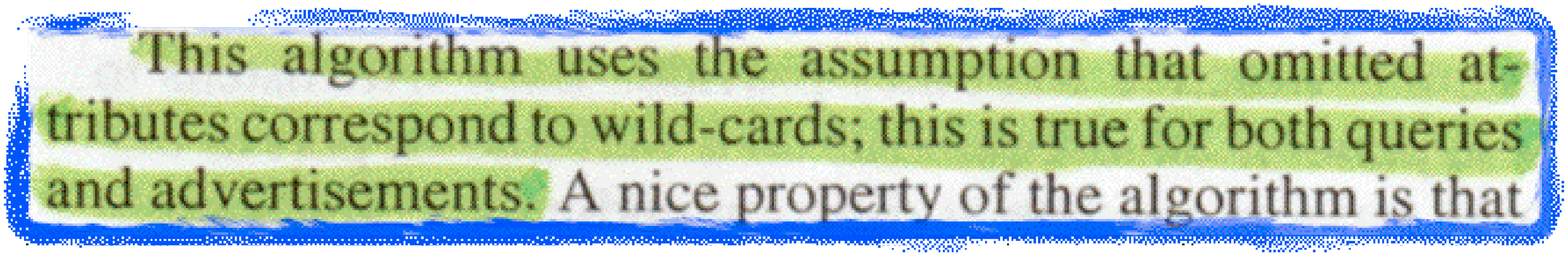

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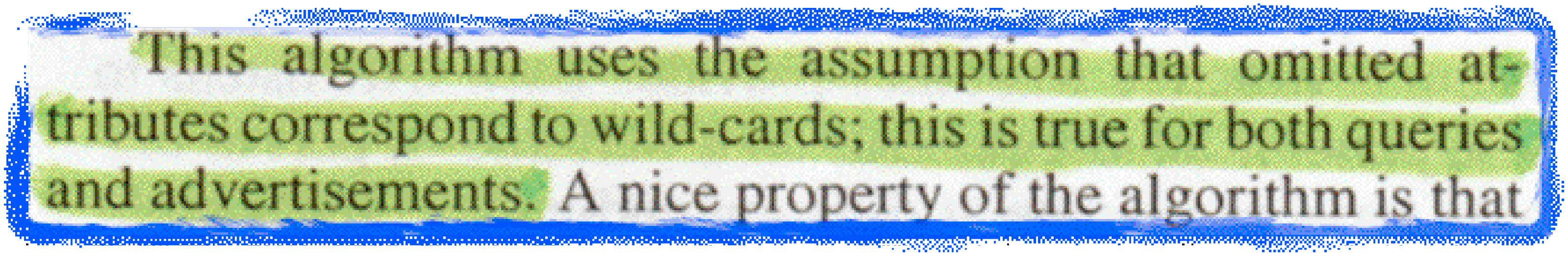

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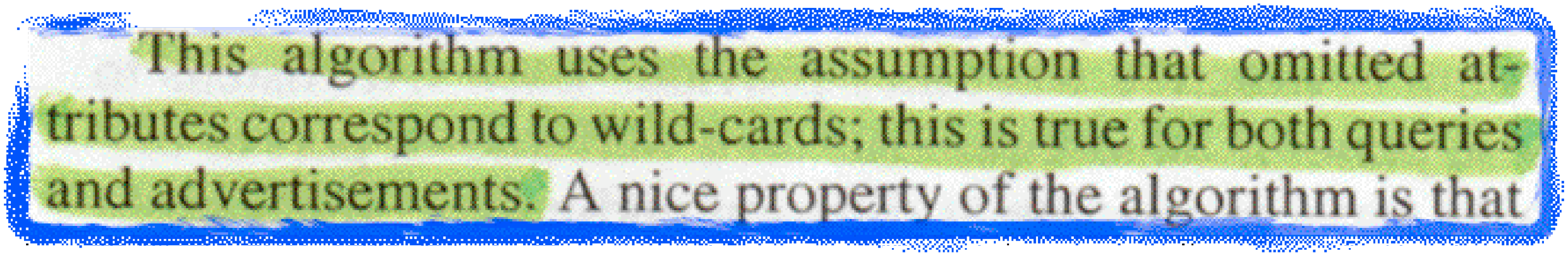

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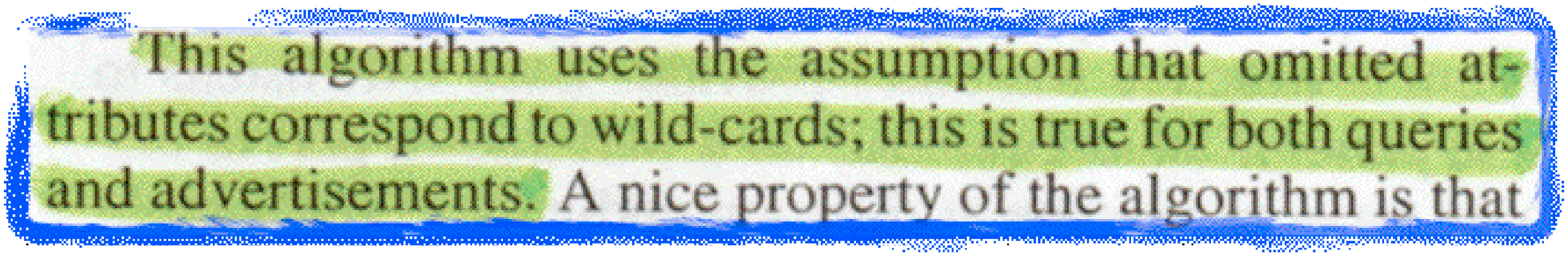

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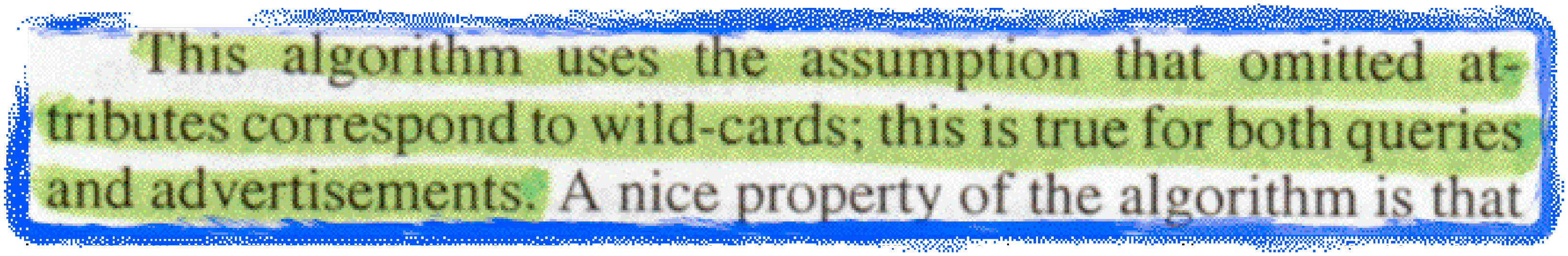

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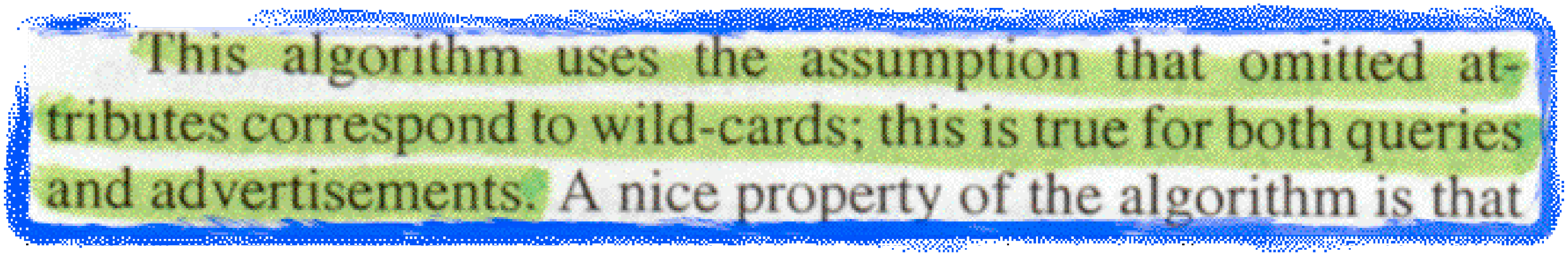

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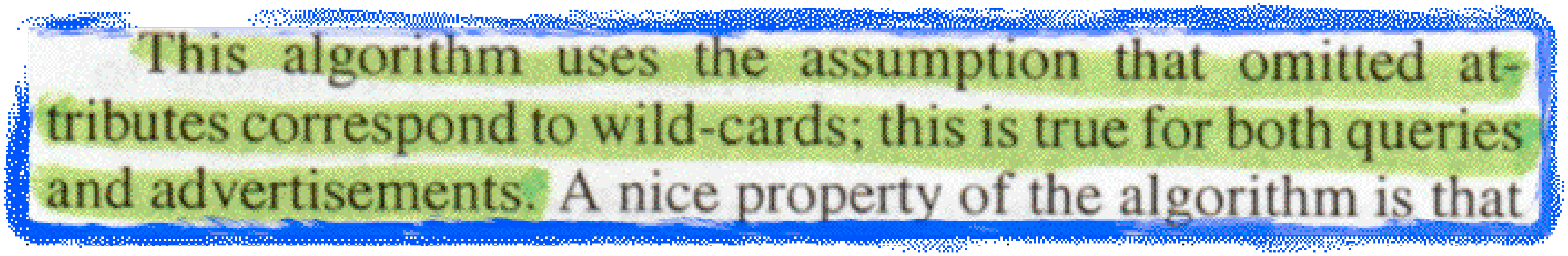

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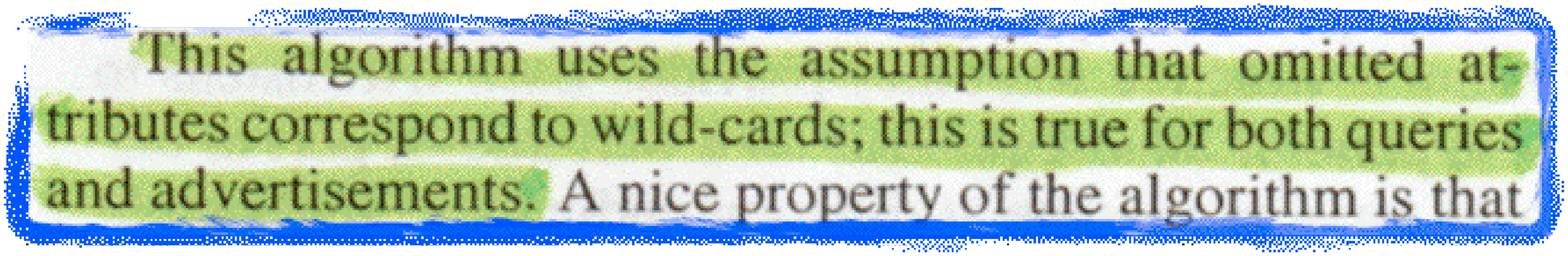

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- found all bugs in trees of 4 nodes or less -- approx 10 secs
- 2000 lines of tests hadn’t found bugs in a year

challenge: get numbering right

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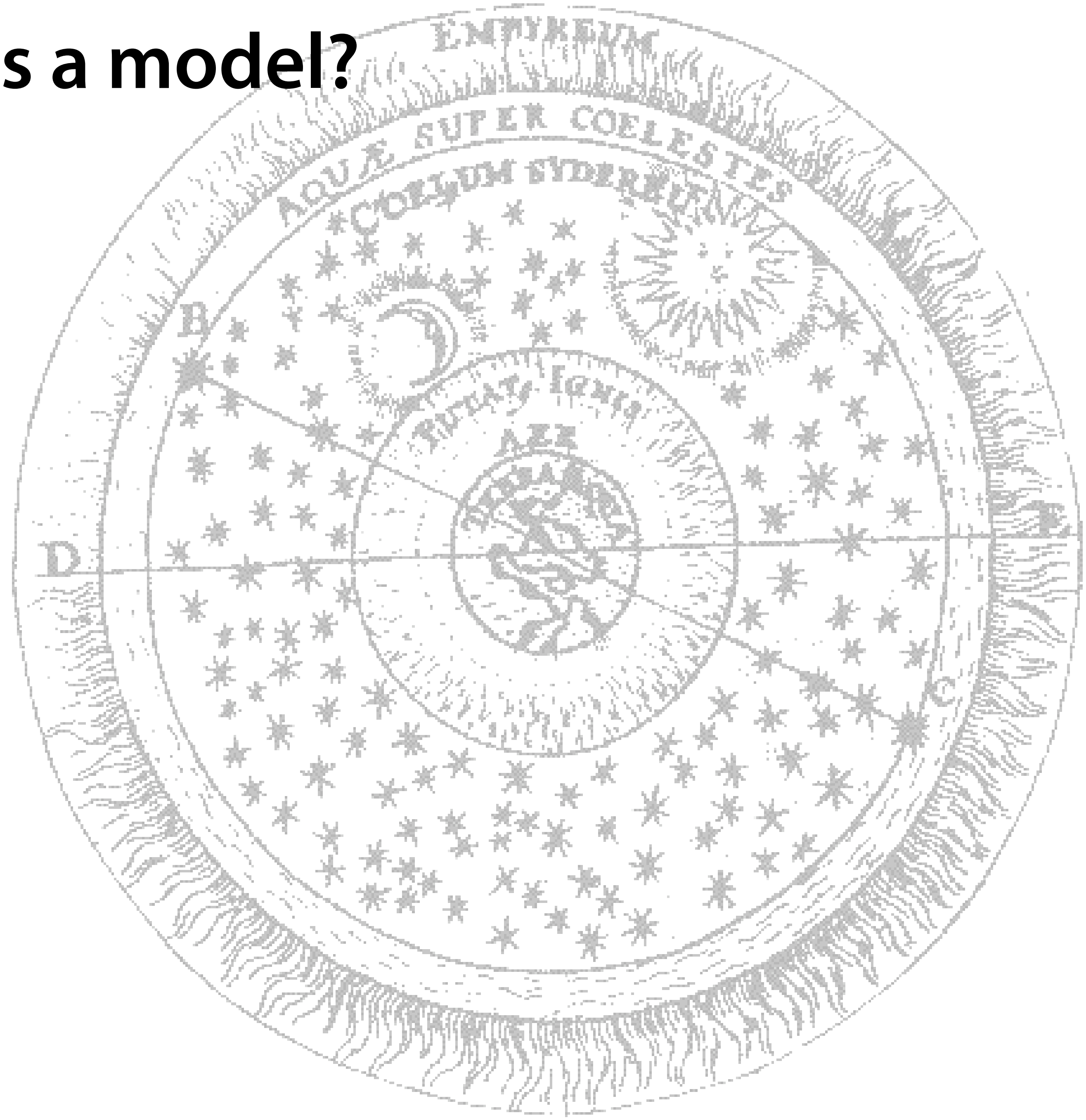
fix the numbering mechanism to handle

- multiple children
  - section and figure have parent chapter
- multiple parents
  - section has parent chapter and appendix

what is a model?



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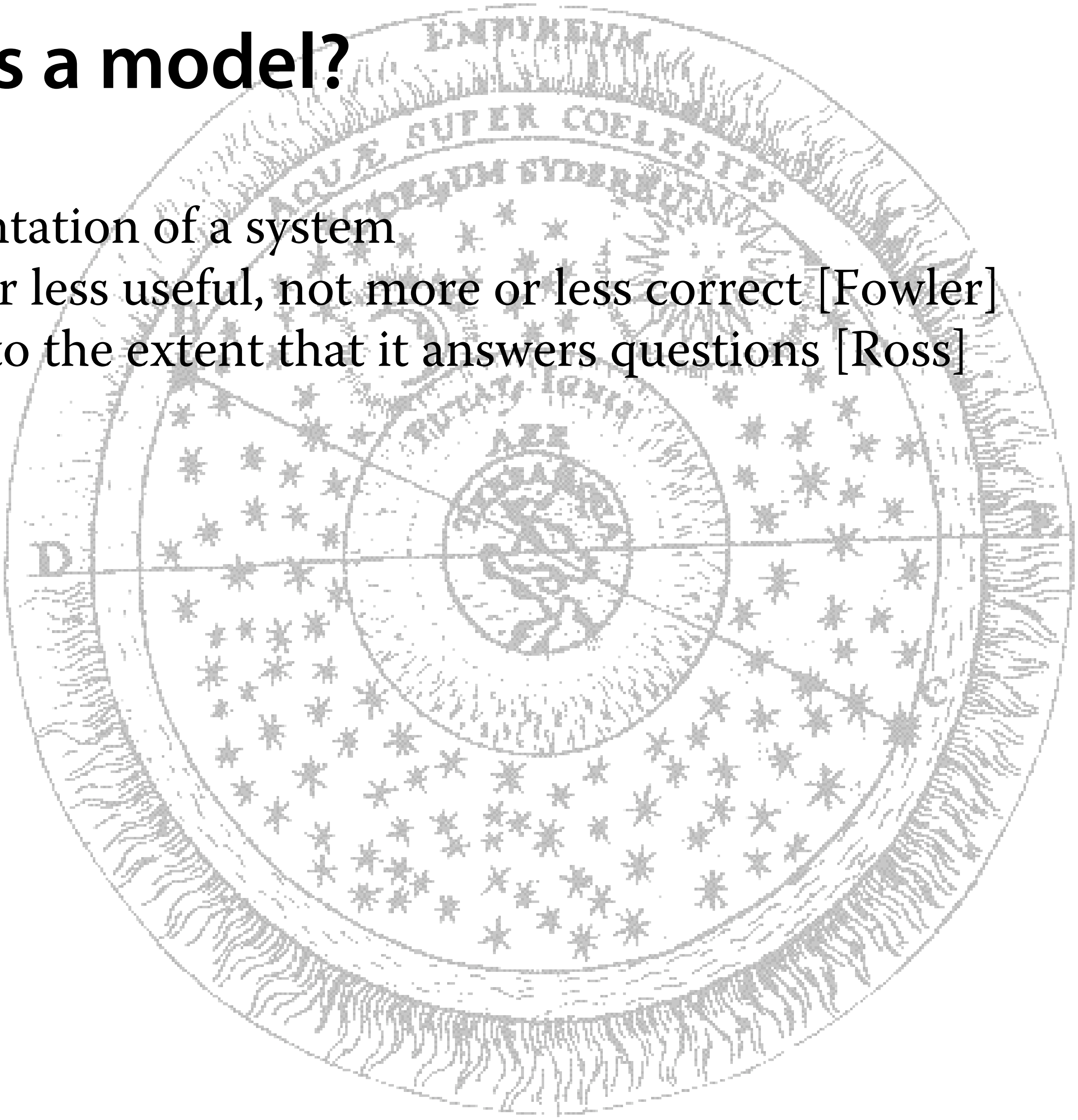

a representation of a system

- › more or less useful, not more or less correct [Fowler]
- › useful to the extent that it answers questions [Ross]

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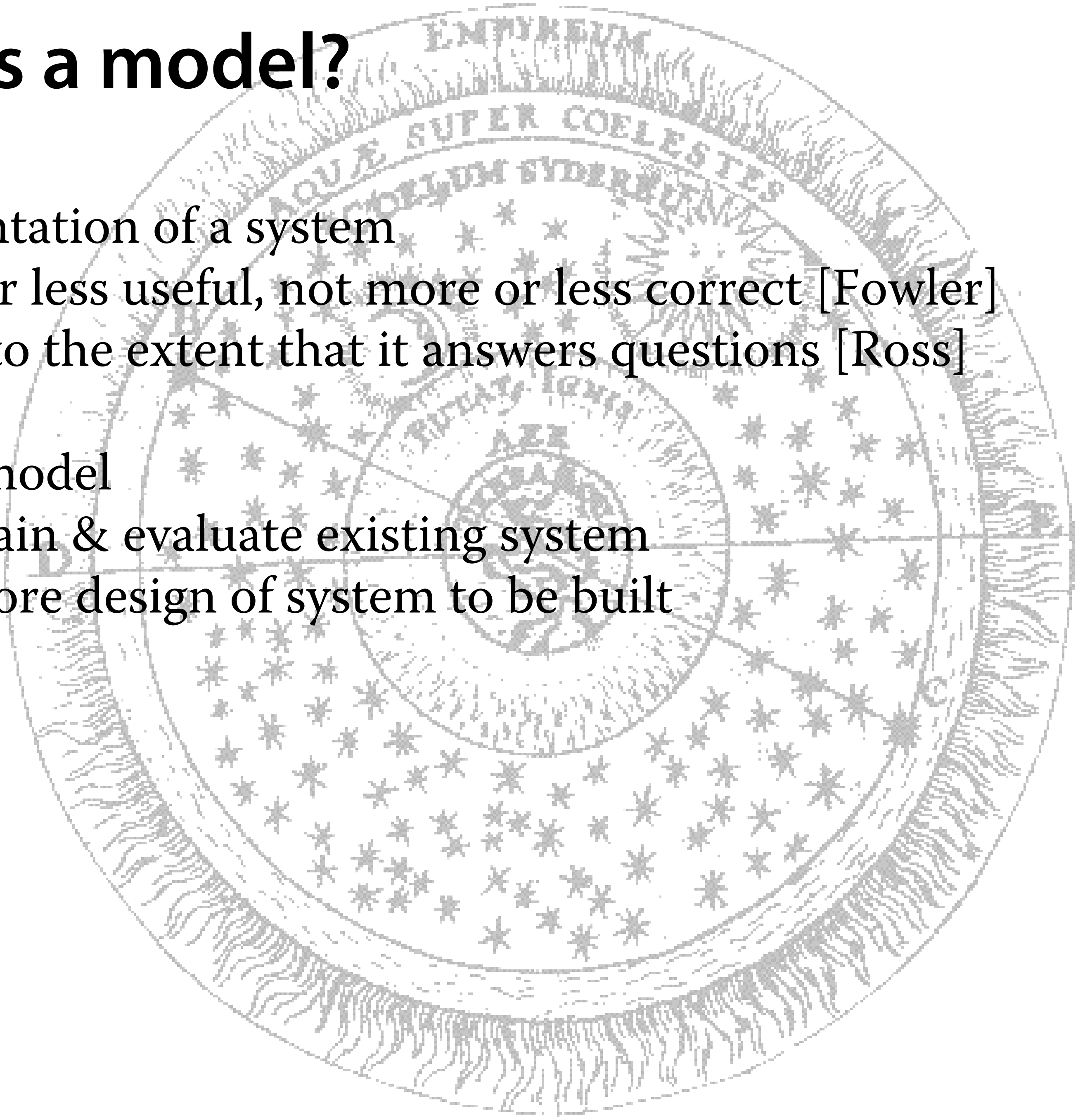

a representation of a system

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role of a model

- to explain & evaluate existing system
- to explore design of system to be built

why model?

why model?

‘plan to throw one away’ [Brooks]
- 100 line model or 100k lines of code?
- nasty surprises happen sooner

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designs with clear conceptual models
- › easier to use and implement
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separation of concerns
- conceptual flaws get mired in code
- not a good use of testing

lightweight formal methods

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elements

- small & simple notations
- partial models & analyses
- full automation

lightweight formal methods

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- small & simple notations
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focus on risky aspects
- hard to get right, or to check
- structure-determining
- high cost of failure