

Objectives:

What is the weakest failure detector
to emulate a shared memory?

Model:

- Message passing system with reliable channels
- Crash failures

Why important?:

Can find the weakest failure detector

to implement consensus with a majority of
faulty process

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faulty process (up to $n-1$)

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\geq

$\Omega(\diamond S)$

Can tolerate $n/2$ failures

Shared Memory vs. Message Passing

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$$\begin{array}{ccccccc} & & \text{Can tolerate } n-1 \text{ failures} & & & & \\ & & \geq & ? & \geq & & \\ S & & & & & & \Omega(\diamond S) \\ \text{Can tolerate } n-1 \text{ failures} & & & & & & \text{Can tolerate } n/2 \text{ failures} \end{array}$$

weakest failure detector:

If we can find the weakest failure detector to emulate registers, say \underline{F} , then the failure detector class $\underline{F} \times \underline{\Omega}$ is the weakest for consensus.

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Implementing consensus

Consensus can be implemented using registers and Ω , in every environment (tolerate up to $n-1$ failures)

weakest failure detector:

If we can find the weakest failure detector to emulate registers, say \underline{F} , then the failure detector class $\underline{F} \times \underline{\Omega}$ is the weakest for consensus.

Weakest?

For any failure detector class D that implements consensus, 1). $D \geq \underline{\Omega}$ (proved in the class)

Using consensus as a building block, we can implement register by D . Thus 2). $D \geq \underline{F}$.

Quorum failure detector, Σ :

Outputs a list of *trusted* processes

Properties satisfied with Σ :

1. *Intersection:*

Any two outputs at any time, for any process includes at least one same process.

2. *Completeness:*

Eventually no faulty process is ever trusted by any correct process.

Emulate SWSR register using Σ :

Atomic actions in the model:

1. Receive from other processes
2. Query the failure detector
3. State change and send to other processes

Local vars for each process

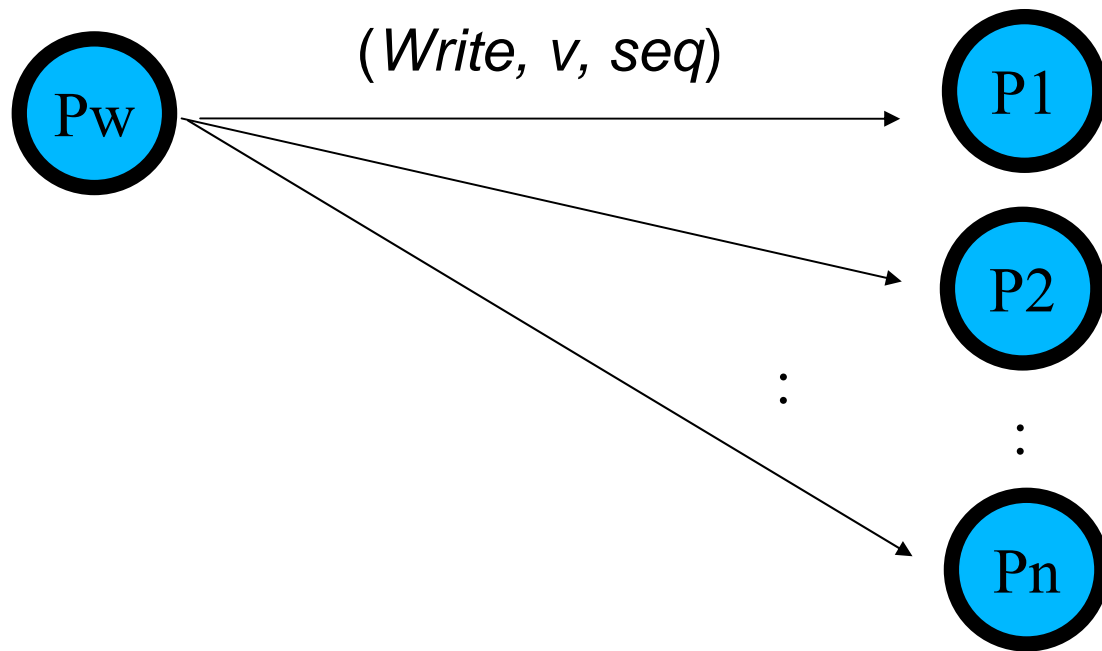
current : current value of emulated register

last_write : keep track of the time stamp for
the current value (initially set to -1)

Shared Memory vs. Message Passing

Emulate SWSR register using Σ :

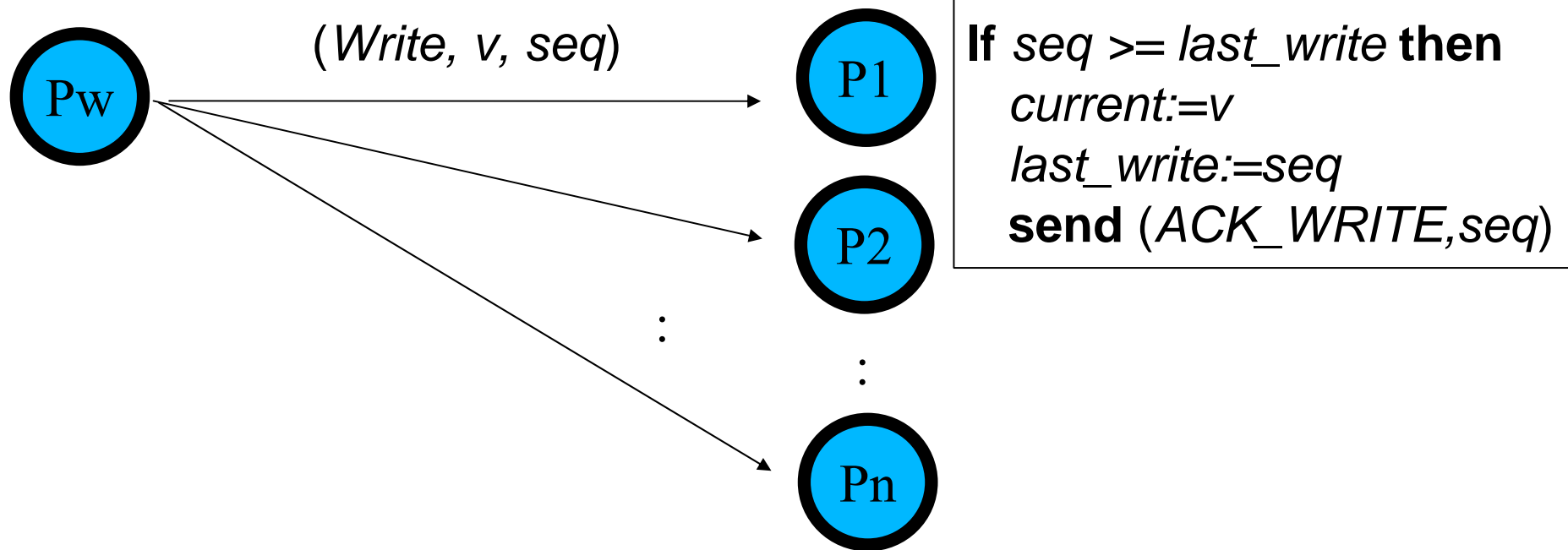
Initially $seq = 0$



Shared Memory vs. Message Passing

Emulate SWSR register using Σ :

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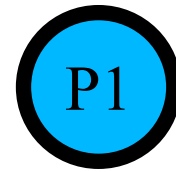
Shared Memory vs. Message Passing

Emulate SWSR register using Σ :

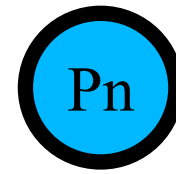
Initially $seq = 0$



(Write, v, seq)



:



If $seq > last_write$ then
current:=v
last_write:=seq
send (ACK_WRITE,seq)

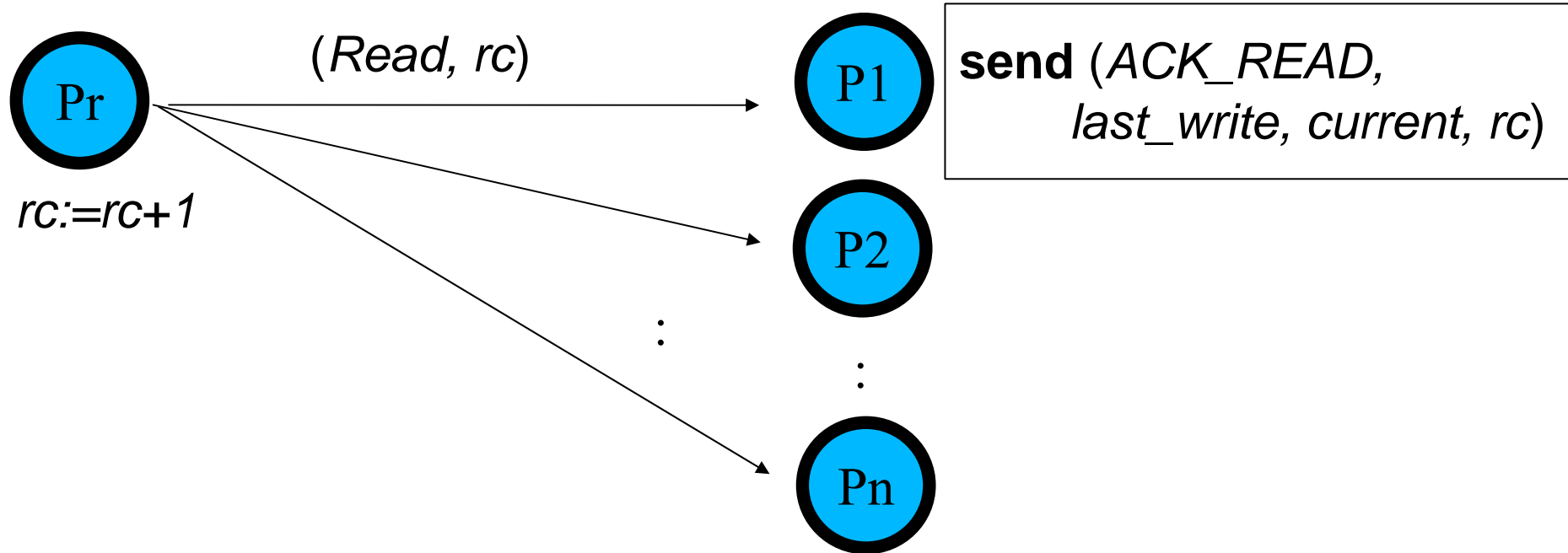
Wait until it receives ACK
from all trusted processes,
and $seq++$

Trusted processes would change before
write terminates

Shared Memory vs. Message Passing

Emulate SWSR register using Σ :

Initially $rc = 0$ (read counter)



Shared Memory vs. Message Passing

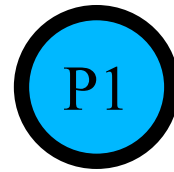
Emulate SWSR register using Σ :

Initially $rc = 0$ (read counter)



$rc := rc + 1$

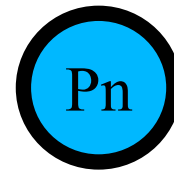
(Read, rc)



send (*ACK_READ*,
last_write, *current*, *rc*)



:



Wait until it receives ACK
from all trusted processes,
 $m/w = (\text{max of second field of ACK})$

if $m/w > \text{last_write}$ **then**

$\text{current} := (\text{third field of ACK})$

$\text{last_write} := m/w$

return *current*

Correctness:

Assertion 1.

if P_w has not finished its k -th writing, then

for all processes, $last_write \leq k$

Termination for write

from *completeness* of Σ

Eventually, Σ outputs only correct processes.

From assertion 1, all correct processes
acknowledge.

Termination for read is similar.

Correctness:

Assertion 2.

If any process sends $(ACK_READ, s, v, *)$,
then v is the value of the s -th write operation.

Validity

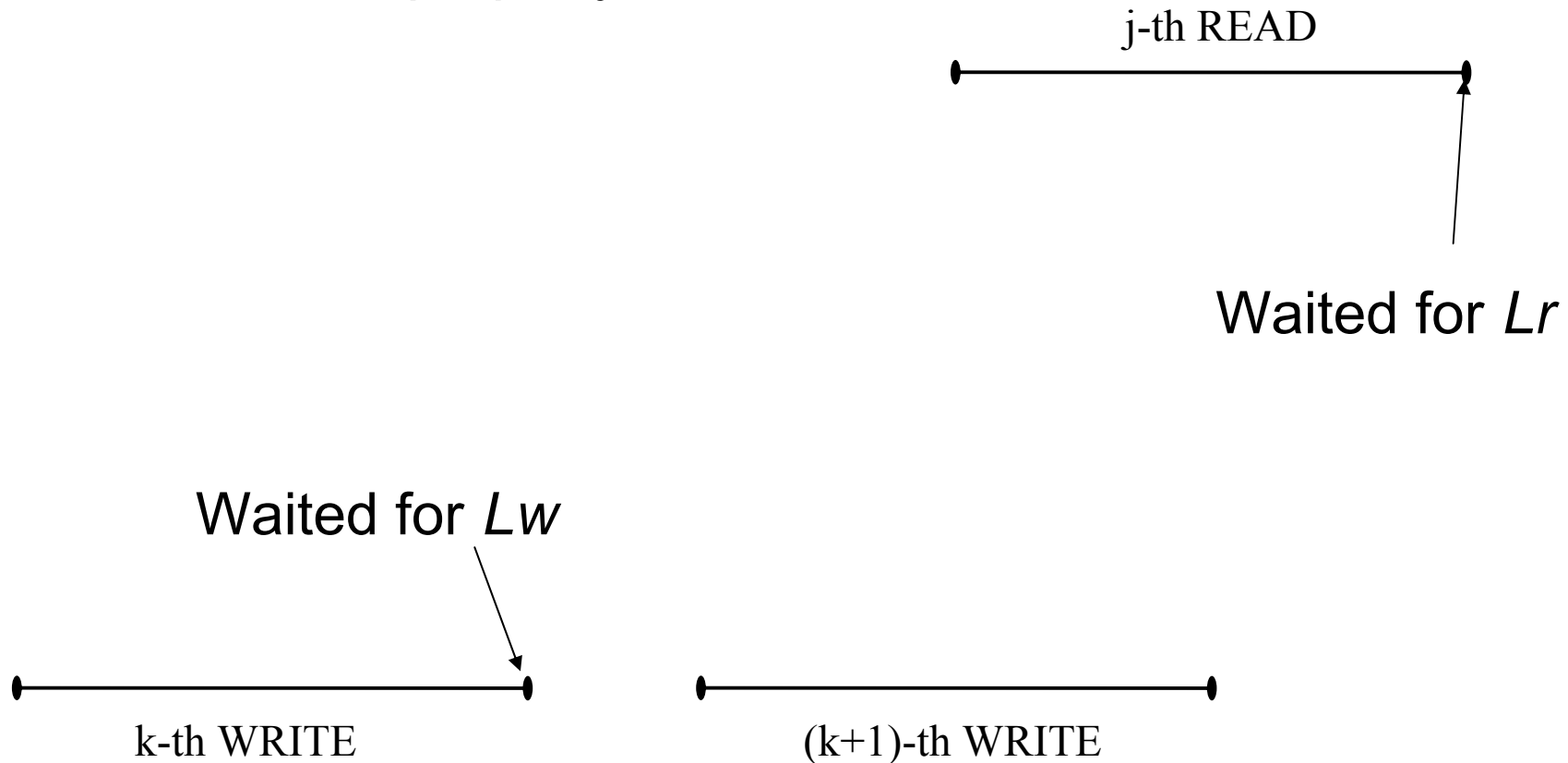
Have to show: every read operation returns either
the value written by the last write that precedes it,
or a value written concurrently with this read.

(If there is no overlapping read/write, read should
return the last value written.)

Shared Memory vs. Message Passing

Validity

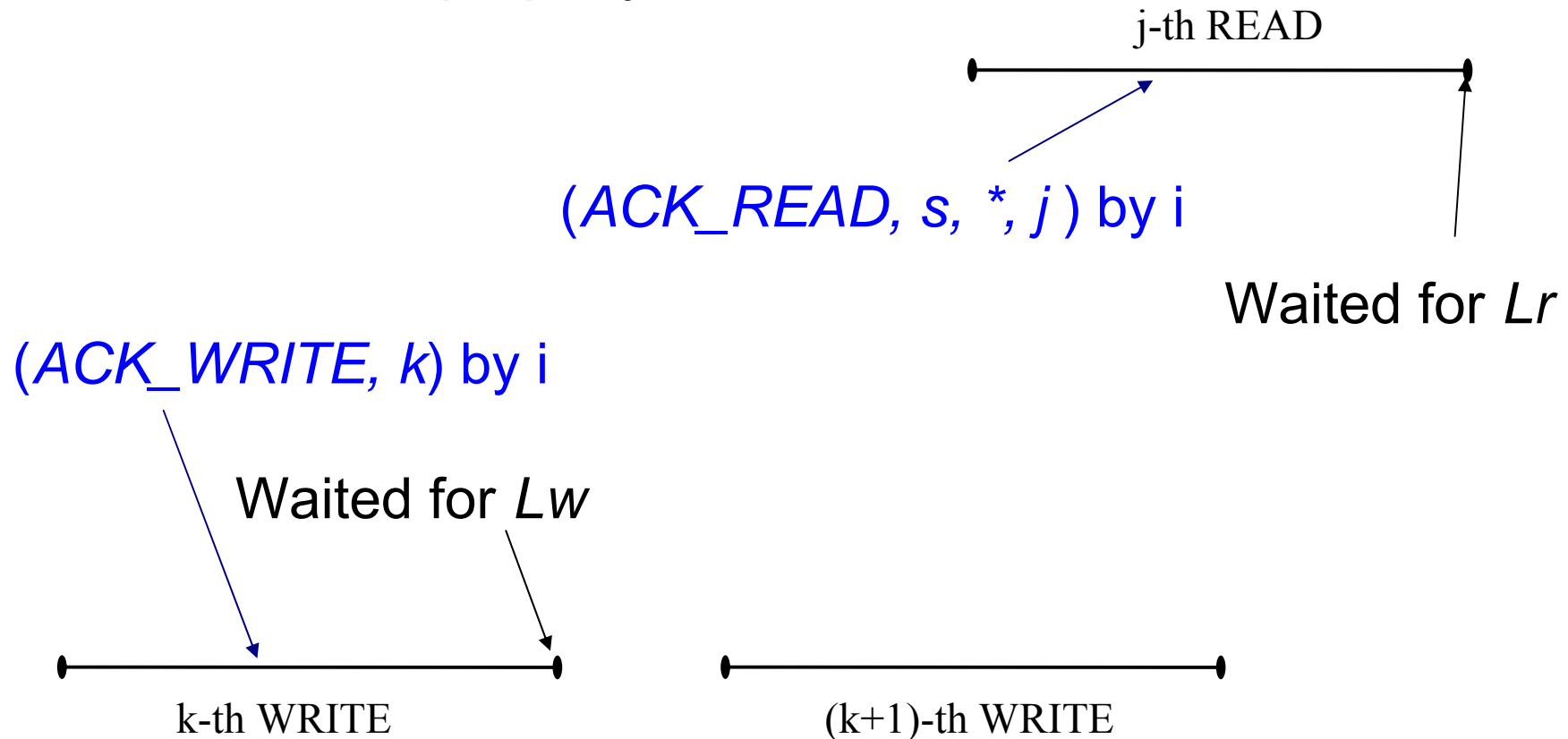
P_i is in $L_w \cap L_r$ because of the intersection property of Σ



Shared Memory vs. Message Passing

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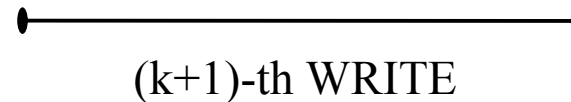
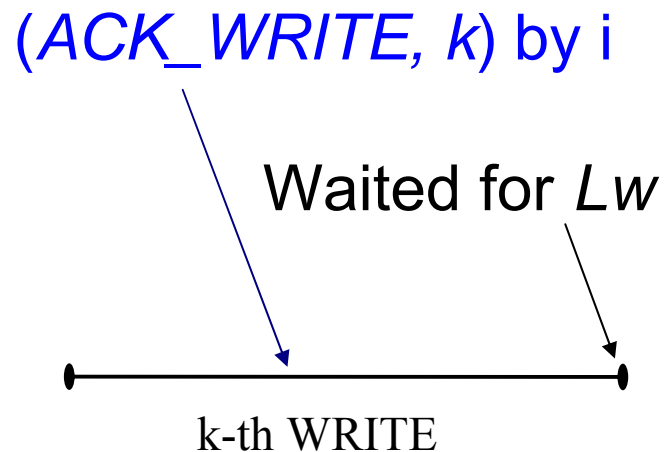
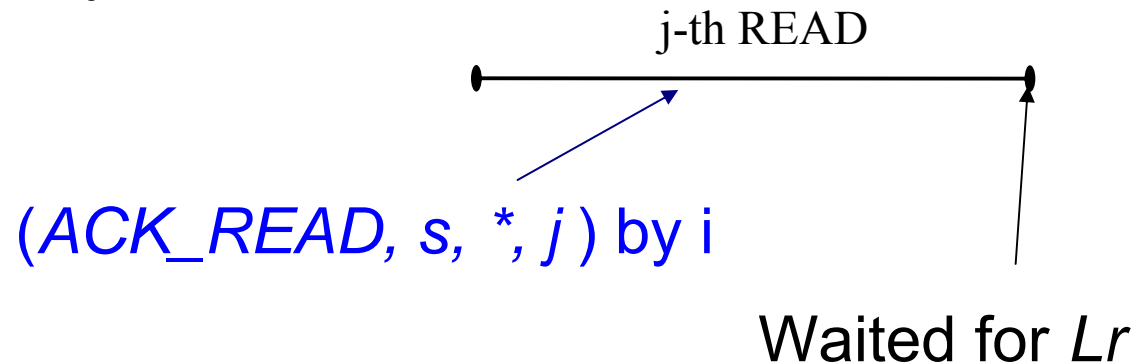


Shared Memory vs. Message Passing

Validity

P_i is in $L_w \cap L_r$ because of the intersection property of Σ

$$s \geq k$$

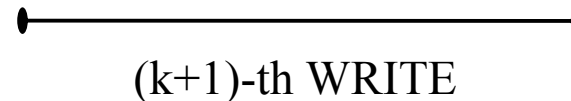
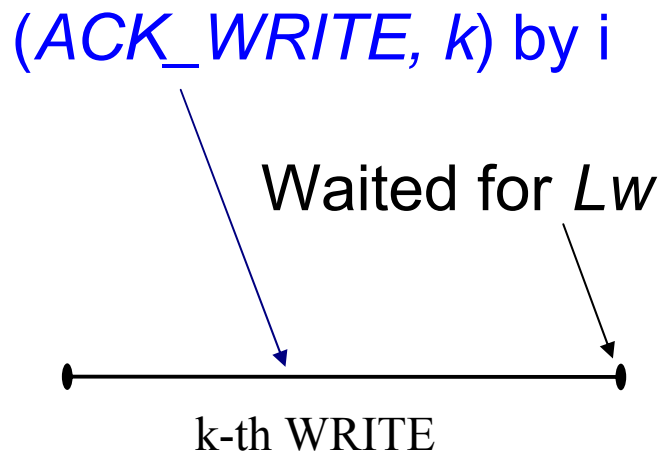
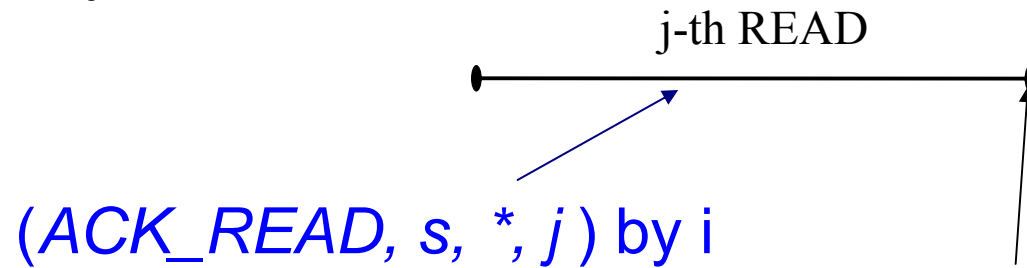


Shared Memory vs. Message Passing

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Waited for L_r

$$mlw \geq s \geq k$$

Shared Memory vs. Message Passing

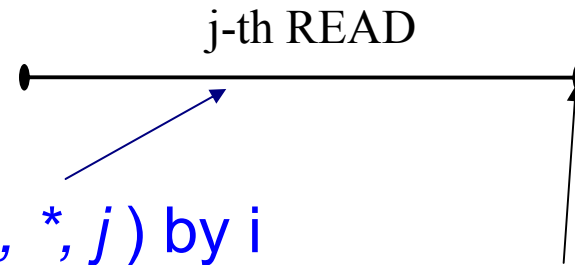
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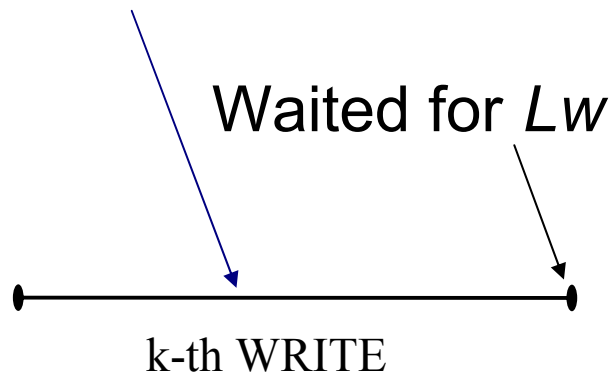
$$mlw \geq k$$

$(ACK_READ, s, *, j)$ by i



Waited for L_r
 mlw

(ACK_WRITE, k) by i



Waited for L_w

Shared Memory vs. Message Passing

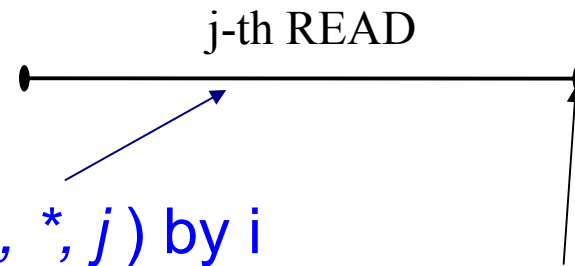
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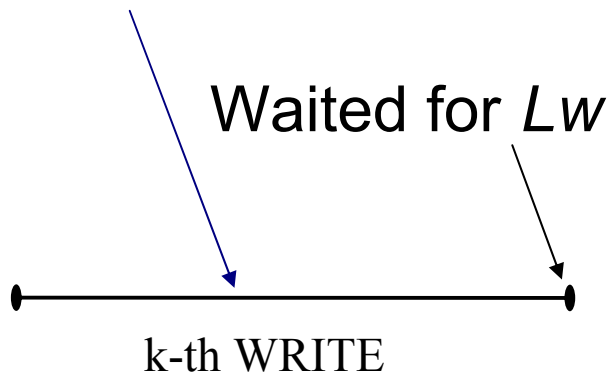
$$mlw \geq k$$

$(ACK_READ, s, *, j)$ by i



Waited for L_r
 mlw

(ACK_WRITE, k) by i



If the $k+1^{\text{th}}$ write has not started, then all processes have a last_write $\leq k$

\Rightarrow for any $(ACK_READ, x, *, j)$,
 $x \leq k$

Shared Memory vs. Message Passing

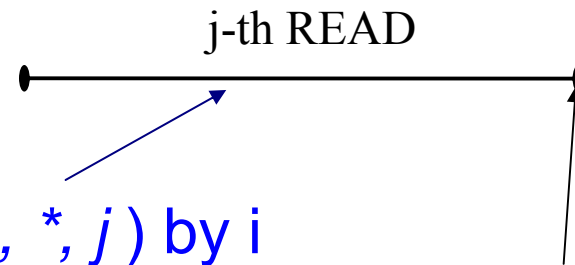
Validity

P_i is in $Lw \cap Lr$ because of the intersection property of Σ

$$s \geq k$$

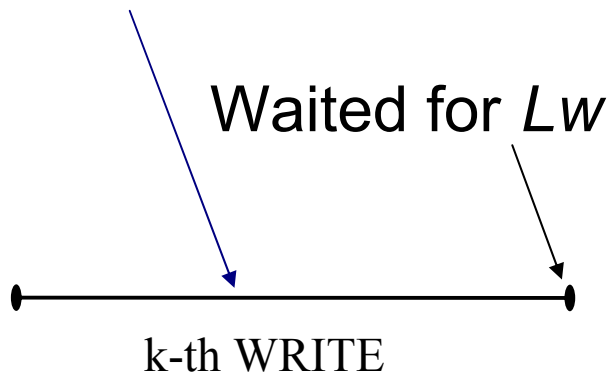
$$mlw \geq k$$

$(ACK_READ, s, *, j)$ by i



Waited for Lr
 mlw

(ACK_WRITE, k) by i



Waited for Lw

If the $k+1^{\text{th}}$ write has not started, then all processes have a `last_write` $\leq k$

\Rightarrow for any $(ACK_READ, x, *, j)$,
 $x \leq k$

\Rightarrow $mlw=k$, which implies
 j -th read returns the value
written by the k -th write

Shared Memory vs. Message Passing

Correctness:

Ordering:

Have to show : if a read operation r precedes a read operation r' , then r' cannot return a value written before the value returned by r .

Proof sketch:

last_write for a reader makes sure consistency.

Shared Memory vs. Message Passing

Have to show that Σ is the weakest.

=> have to emulate Σ using a failure detector
that implements register.

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Proof: not for this time...

Summary: we showed Σ is weakest failure detector to implement register.

$$\begin{array}{ccccc} & \text{Can tolerate } n-1 \text{ failures} & & & \\ S & \geq & \Omega \times \Sigma & \geq & \Omega(\diamond S) \\ \text{Can tolerate } n-1 \text{ failures} & & & & \text{Can tolerate } n/2 \text{ failures} \end{array}$$