The Weakest Failure Detector for Consensus

References and Thanks

- [CHT] Chandra, Hadzilacos and Toueg, The Weakest Failure Detector for Solving Consensus, JACM 43(4), 1996
- Gärtner, Guerraoui and Kouznetsov, The CHT Play
  - A highly recommended informal and very accessible overview of the CHT proof
- Some of the following slides were provided by Rachid Guerraoui

What we need to prove

- To prove that a failure detector class C is the weakest for a problem P, one needs to show that
  - If: P can be solved with $D \in C$, and
  - Only if: For all C' that can be used to solve P, $C' \geq C$
- $\Diamond S = \Diamond W$ is sufficient for Consensus with $n>2t$
- Need to prove that for all D that can be used to implement Consensus, $D \geq \Diamond W$ with $n>2t$

The Outline

- Define a failure detector Ω (*leader oracle*):
  - Output: process id
  - Eventually all correct processes permanently output the same process id p ,and p is correct
- Lemma 1: For any failure environment: $\Omega \geq \Diamond W$
  - Proof:?
- Lemma 2: For any failure environment:

  If D solves Consensus, then $D \geq \Omega$
  - Proof: ? ☺

A question: Is it possible that $\Omega > \Diamond W$?

The Outline

- Theorem: For any failure environment:

  If D can be used to solve Consensus, then D $\geq \Diamond W$

Proof:

- If D solves Consensus, then D $\geq \Omega$ (Lemma 2). $\Omega \geq \Diamond W$ (Lemma 1). Transitivity: If D solves Consensus, then D $\geq \Diamond W$

D solves Consensus $\Rightarrow$ D $\geq \Omega$

- Let A be a consensus algorithm using D
- Construct an algorithm T that emulates Ω on top of D

Overview of the emulation

1. The exchange
2. The simulation
3. The tagging
4. The stabilization
5. The extraction

(1) The Exchange

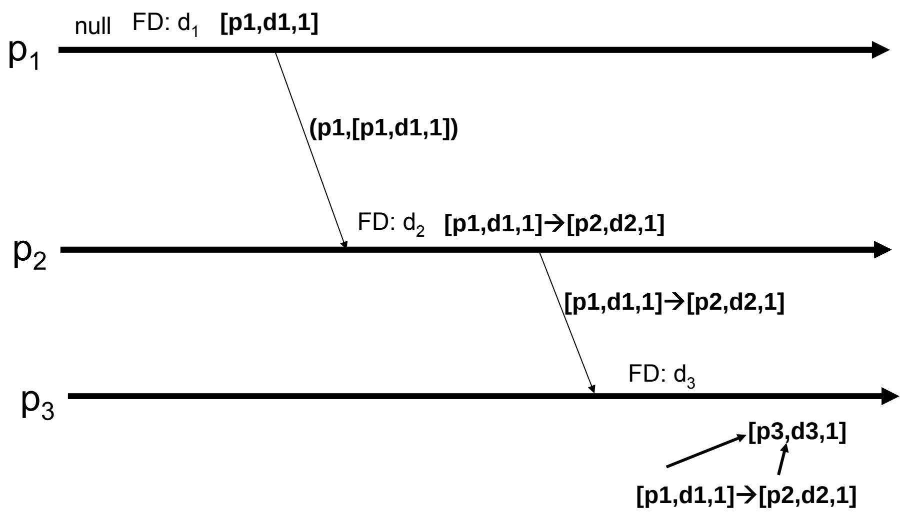
- Every process periodically queries its failure detector module (D) and sends all outputs it has seen to all
- A process builds a growing DAG using the outputs provided by other processes
- A vertex of the DAG is a triple:
  - (process, f. d. value, f. d. query#)
- An arrow $(p_1,d_1,k_1) \rightarrow (p_2,d_2,k_2)$ means that p_1 saw d_1 before p_2 saw d_2

(1) The Exchange Algorithm

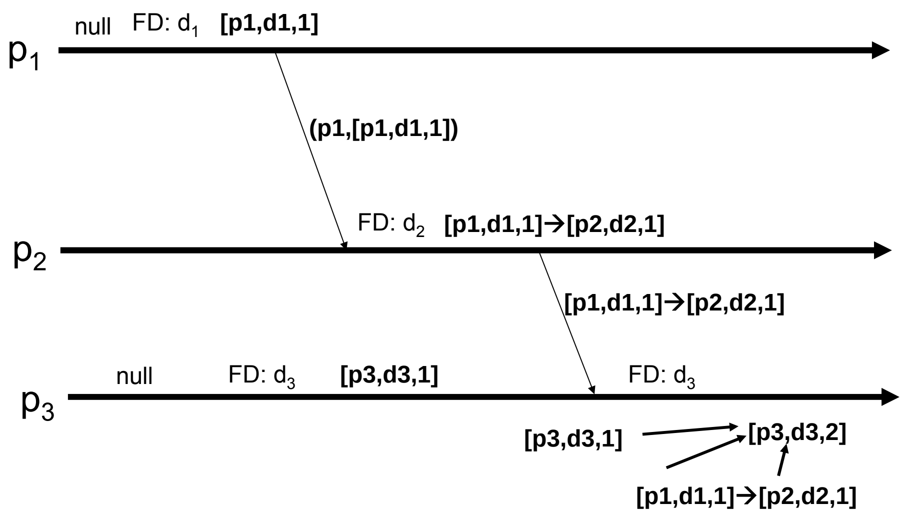
DAG := empty graph;
k := 0;
Forever do:

- k := k+1;
- p receives (q,DAG_q)   // maybe null
- d := output of p's failure detector
- DAG := DAG $\cup$ DAG_q;
  Add [p,d,k] to DAG and edges from all vertices of DAG to to [p,d,k];
  Send (p,DAG) to all processes

(1) The Exchange Algorithm

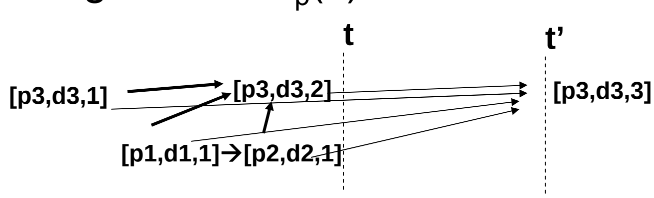


(1) The Exchange Algorithm



Properties of Local DAGs

- For any correct process p and time t
  - (1) $DAG_p(t)$ is transitively closed
    - An easy induction
  - (2) There is a time $t' \geq t$, d and k such that $\forall v \in \text{Vertices}(DAG_p(t))$, $v \rightarrow (p,d,k)$ is an edge of $DAG_p(t')$



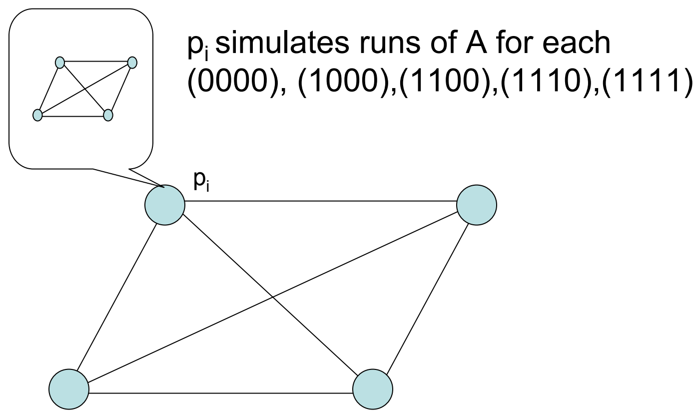
Properties of Local DAGs

- The DAG of each **correct** process is ever increasing finite approximation of the **same** infinite limit graph
  - The common portion of correct process DAGs grows without limit

(2) The Simulation

- Every process p_i uses its DAG to simulate runs of A in the system, i.e., every process locally plays the role of all other processes
- Whenever p_i updates its DAG, p_i triggers runs of A for:
  - All paths in the DAG
  - All input vectors I_0, I_2,.. I_n, where I_i makes processes p_1-p_i propose 1 and the rest propose 0

(2) The Simulation



(2) The Simulation

Forever do:

- p receives (q,DAG_q)   // maybe null
- d := output of p's failure detector
- DAG := DAG $\cup$ DAG_q;
  Add [p,d] to DAG and edges from all vertices of DAG to to [p,d];
  Simulate(A,DAG);
  Send (p,DAG) to all processes

The Simulation Algorithm

For each $I=I_j$, $0 \leq j \leq n$ do

$Y_I := \emptyset$;

For each path g in DAG_p:

R_g := a run of A from I with the sequence of failure detector events induced by g;

$Y_I := Y_I \cup R_g$;

Simulation output is a collection of trees Y_{Ij}

(2) The Simulation

Forever do:

- p receives (q, DAG_q) // maybe null
- d := output of p's failure detector
- $DAG := DAG \cup DAG_q$;

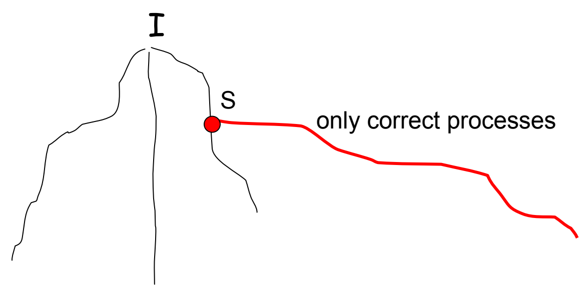
  Add [p,d] to DAG and edges from all vertices of DAG to to [p,d];

  $\{Y_{I0},\ldots,Y_{In}\}$:= Simulate(A,DAG);

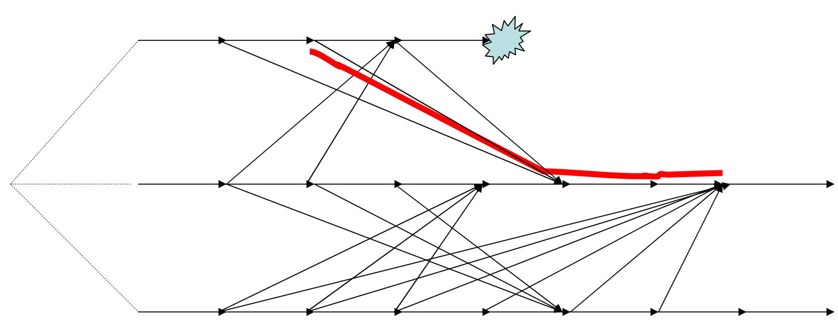
  Send (p,DAG) to all processes

Properties of the simulation at correct processes

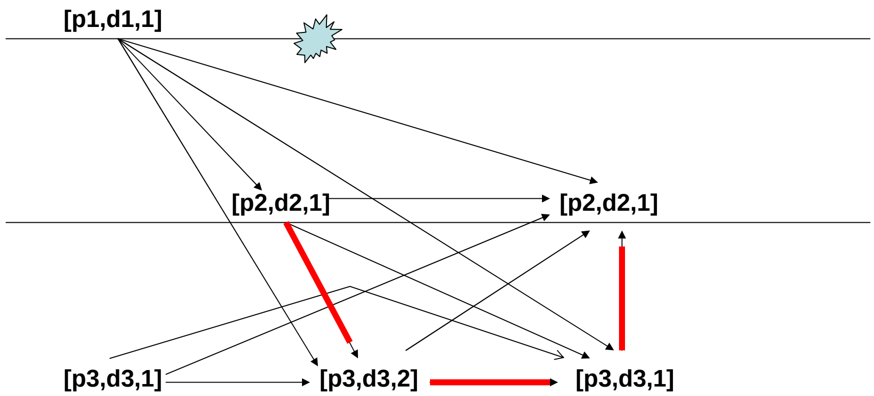
Property 1: For any vertex S of Y_I, there exists a finite trace E containing only the steps of correct process such that S·E is in Y_I and all correct process decide in S·E



Intuition Behind Property 1



An Example



(3) Tagging

Forever do:

- p receives (q, DAG_q) // maybe null
- d := output of p's failure detector
- $DAG := DAG \cup DAG_q$;
  Add [p,d] to DAG and edges from all vertices of DAG to to [p,d];
  $\{Y_{I0},\ldots,Y_{In}\}$:= Simulate(A,DAG);
  TAG($\{\mathbf{Y_{I0},\ldots,Y_{In}}\}$);
  Send (p,DAG) to all processes

The Tagging Algorithm

- For every vector YIj: Tag Ij as
  - **0-valent** if only 0 are decided in Y_{Ij}
  - **1-valent** if only 1 are decided in Y_{Ij}
  - **Bivalent** if both 0 and 1 are decided in Y_{Ij}

(3) Tagging

Forever do:

- p receives (q,DAG_q)   // maybe null
- d := output of p's failure detector
- DAG := DAG $\cup$ DAG_q;
  Add [p,d] to DAG and edges from all vertices of DAG to to [p,d];
  $\{Y_{I0},\ldots,Y_{In}\}$:= Simulate(A,DAG);
  Tagged_forest := TAG($\{Y_{I0},\ldots,Y_{In}\}$);
  Send (p,DAG) to all processes

Tagging Properties

- By validity of consensus, I_0 is always tagged as 0-valent and I_n as 1-valent
- Other 0 or 1-valent input vector can only get tagged bivalent
- A bivalent input vector stays bivalent forever

Critical Index

- There is some index k in the sequence of vectors such that I_{k-1} is 0-valent and I_k is not: k is called the critical index
- If I_k is 1-valent, then p_i trusts p_k
- (we do not consider here the more complicated case when I_k is bivalent)

(4) The Stabilization

- Eventually, the critical index at a given process does not change anymore: this is because the index can only decrease and cannot go lower than 1

- All DAGs converge to the same infinite DAG and the same critical index k is eventually computed at all processes

(5) The Extraction

Forever do:

- p receives (q,DAG_q)   // maybe null
- d := output of p's failure detector
- DAG := DAG $\cup$ DAG_q;
  Add [p,d] to DAG and edges from all vertices of DAG to to [p,d];
  {Y_{l0},…,Y_{ln}} := Simulate(A,DAG);
  Tagged_forest := TAG({Y_{l0},…,Y_{ln}});
  p := Extract_Leader(Tagged_forest);
  Output p;
  Send (p,DAG) to all processes

The Extraction Algorithm

If k is critical then

If I_{k-1} is 0-valent and I_k is 1-valent then

return p_k;

else // I_{k-1} is 0-valent and I_k is bivalent then

Look for decision gadgets;

choose a process based on a deterministically chosen decision gadget;

Correctness of Extraction

Claim: Eventually, (1) all correct processes permanently return the same process p_k and (2) p_k correct

Proof:

(1) At each correct process, the critical index eventually stabilizes at k.

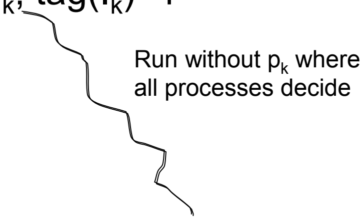
It is eventually the same at all processes.

All correct processes return p_k

Correctness of Extraction

(2) Assume p_k crashes

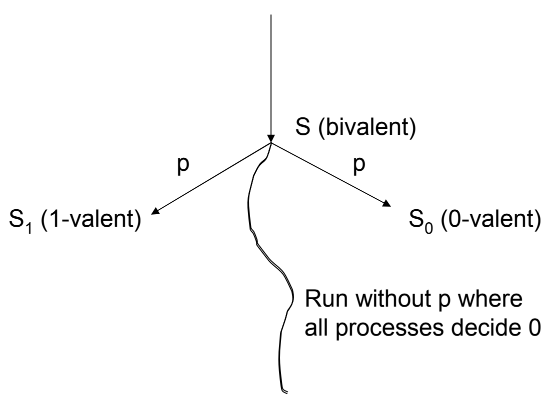
I_{k-1}, $tag(I_{k-1})=0$ I_k, $tag(I_k)=1$



A contradiction

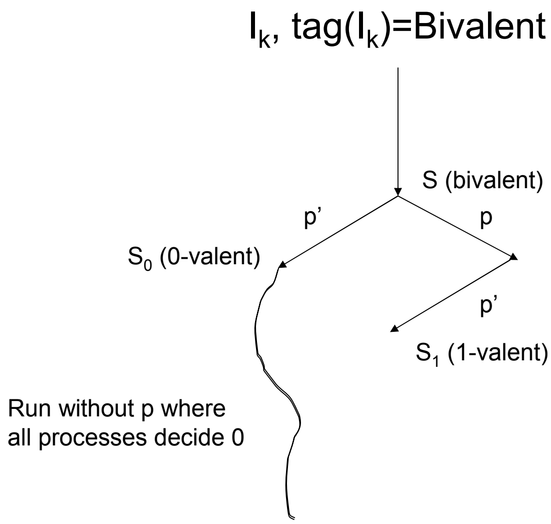
Decision Gadgets: Fork

I_k, $tag(I_k)$=Bivalent



p is correct

Decision Gadgets: Hook



p is correct

What people think ☺

- Actual replies I've got when enquiring about an instructional material on CHT
  - My advice is: don't do it!
  - I tried to understand it for a while and gave up
  - It's a terrible proof
  - It's mind-boggling
  - No way I'm going to try and teach it in a class