Byzantine Consensus

Definition

- Agreement: No two correct processes decide on different values
- Validity:
  - (a) Weak Unanimity: if all processes start from the same value v and all processes are correct, then v is the only possible decision
  - (b) Strong Unanimity: if all correct processes start from the same value v, then v is the only possible decision value
- Termination:…

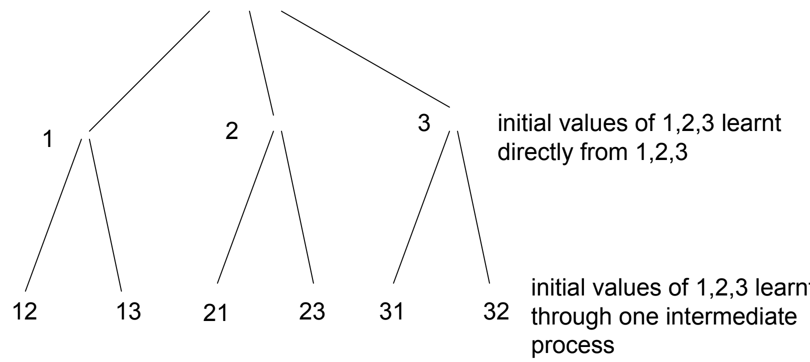
Structure of Consensus algorithms

- Throughout an execution, processes learn about initial values of other processes
- If failures occur, some values are learnt indirectly:
  - i sends 1 to j and fails: j knows that $init_i=1$
  - j sends 1 to k and fails: k knows that j knows that $init_i=1$
  - etc…

EIG Tree

- In general, such a chain can be constructed for every initial value
- We can design an algorithm that maintains these chains explicitly
- Maintain a tree to hold all possible value propagation chains
  - Each path from a leaf to the root represents a propagation chain

EIG Tree



EIG Algorithm

- Round 1:
  - decorate root with $init_i$
  - send $init_i$ to all processes
  - decorate level 1 with received values: value from j decorates label j
- Round r, $2 \le r \le t+1$:
  - relay level r-1 values in the form (label, value)
  - for every (x,val) received from j, decorate level r node $x \cdot j$ with val
- W={values in the tree}, if |W|=1, decide $v \in W$, Otherwise, decide a default value

SilentConsensus

- Round 1:
  - If initi=1, send 1 to all processes;
- Round r+1, $1 \leq r \leq t$:
  - If received 1 in round r && has not yet broadcast a message:

    W := W ∪ {1};

    relay 1 to all processes;
- At the end of round t+1:
  - If |W|>0, decide 1, otherwise decide 0

Proof

- Let $r \geq 1$ be a failure-free round
  - ∃ non-failed process p that has received 1 in one of the rounds 1,…,r-1 ➔ p sends 1 to all processes in the beginning of r the latest
  - No such process exists ➔ no messages are sent
- After a failure-free round either all processes either have 1, or remain silent forever

Tolerating omissions

- Round 1:
  - If $init_i$=1, send 1 to all processes;
- Round r+1, 1 ≤r ≤t:
  - If received (x,1) from j && |x|=r && has not yet broadcast a message:

    W := W ∪ {1}

    relay (x·j, 1) to all processes;
- At the end of round t+1:
  - If |W|>0, decide 1, otherwise decide 0

Authenticated Byzantine

- Round 1:
  - If $init_i$=1, send $[1]_{si}$ to all processes;
- Round r+1, 1 ≤r ≤t:
  - If received $[m]_{sj}$ from j &&
  - m is correctly signed by j &&
  - m is correctly signed by r different processes &&
  - has not yet broadcast a message:

    W := W ∪ {1}

    relay $[m \cdot s_j]_{si}$ to all processes;
- At the end of round t+1:
  - If |W|>0, decide 1, otherwise decide 0

A simpler solution

- Round 1:
  - If $init_i$=1, broadcast $[1]_{si}$ to all processes;
- Round r+1, 1 ≤r ≤t:
  - If received $[1]_{sj}$ from at least r different processes &&
  - has not yet broadcast a message:

    W := W ∪ {1}

    broadcast $[1]_{si}$ and relay all messages that caused it to be broadcast

  At the end of round t+1:
  - If |W|>0, decide 1, otherwise decide 0

Consistent (Echo) Broadcast

- **Correctness**: if correct process p broadcasts a message (p,m,k) in round k, then every correct process accepts (p,m,k) in the same round
- **Unforgeability**: if correct process p does not broadcast (p,m,k), then no correct process ever accepts (p,m,k)
- **Relay**: If a correct process accepts (p,m,k) in round r≥k, then every correct process accepts (p,m,k) by round r+1

Proof of CB algorithm (Relay)

- Message (i,m,k) is accepted by non-faulty process j at round r'➔
- j receives n-t (echo,i,m,k), at least n-2t>t of which are from correct processes
- At r', t+1 correct processes sent (echo,i,m,k) to all correct processes ➔
- Every one of n-t correct processes will echo (i,m,k) at the round r'+1

Implementing CB with n>3t

- Broadcast (i,m,k) at round k: send (init,i,m,k) to all processes
- if process j receives (init,i,m,k) at round k, it sends (echo,i,m,k) to all processes
- if before any round r'≥r+1, j has received (echo,i,m,k) from at least t+1 processes, it sends (echo,i,m,k) to all processes
- if by the end of any round r' ≥r, j has received (echo,i,m,k) from at least n-t processes, j accepts (i,m,k)

Consensus using CB

- Round 1:
  - If $init_i$=1, broadcast (i,1,1) to all processes;
- Round r+1, 1 ≤r ≤t:
  - If accepted 1 from at least r different processes &&
  - has not yet broadcast a message:

    $W := W \cup \{1\}$

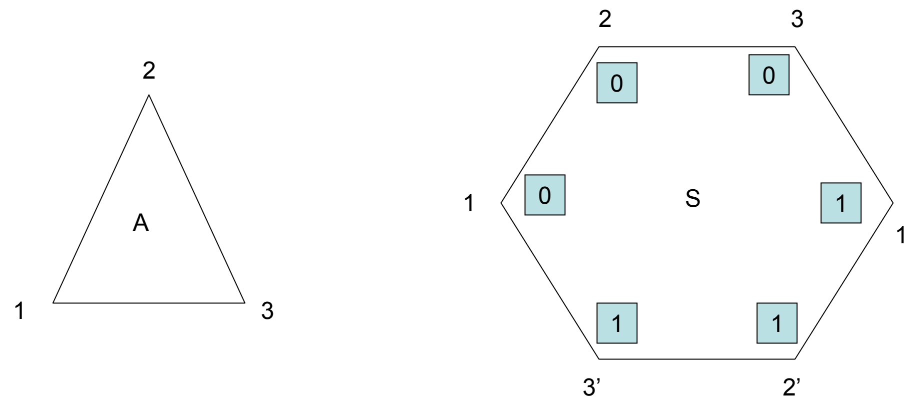
    broadcast (i,1,r+1)

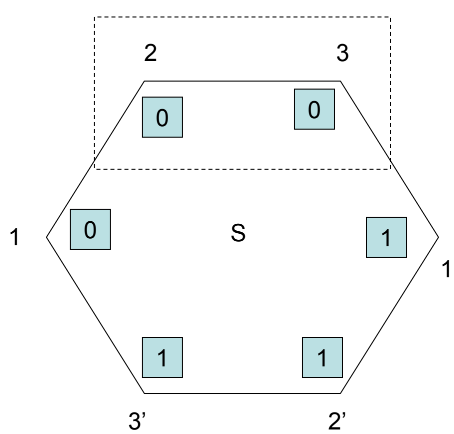
At the end of round t+1:
  - If |W|>0, decide 1, otherwise decide 0

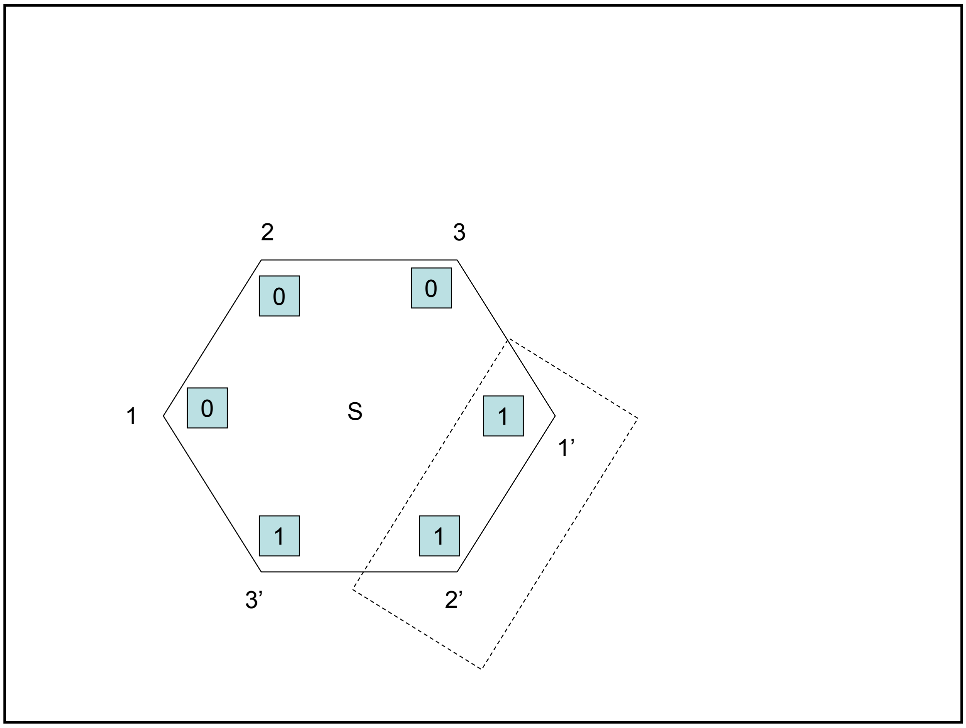
Impossibility with n≤3t

- We show impossibility for strong unanimity
  - Fischer, Lynch and Merritt
  - Found in textbooks
- Impossibility for weak unanimity can be proved using a similar approach:
  - Fischer, Lynch and Merritt
  - Section 6.6, Theorem 6.30, Distributed Algorithms, by N. Lynch

n=3



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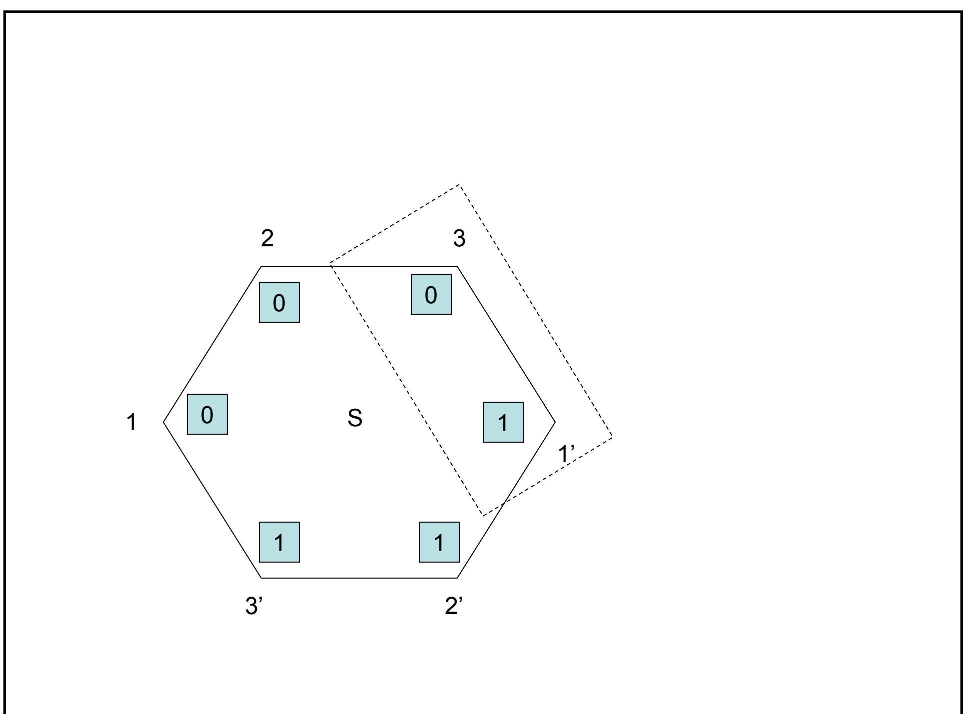
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