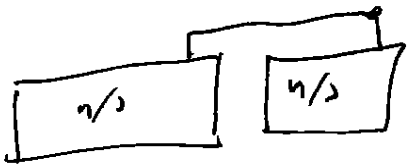
6.896
4-14-2004
Lecture 17.1

Layout:

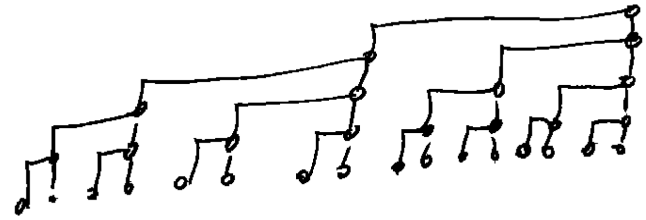
Complete Binary Tree

colinear layout:

Divide + Conquer



e.g.



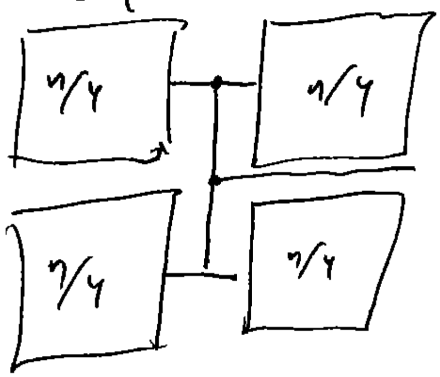
Analysis: width = $\Theta(n)$
height = $\Theta(\lg n)$
Area = $\Theta(n \lg n)$

In fact, can show if all leaves are on a line, ~~wire area~~ total wire length = $\Omega(n \lg n)$

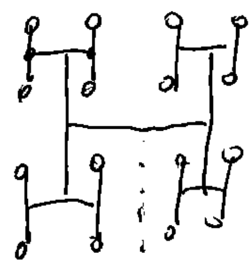
~~H-tree layout~~

H-tree layout

Divide + Conquer



e.g.



Always space for wire to get out.

side length = 2n

Analysis $W(n) = 2W(n/4) + \Theta(1)$
$= \Theta(n)$

Longest wire? $O(\sqrt{N})$ in this layout

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17.2
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Reduce longest wire?

Can get longest wire to $\Theta(\sqrt{n}/\lg n)$

Thm: cannot do ~~better~~:

proof: diameter of net is $\lg n$,

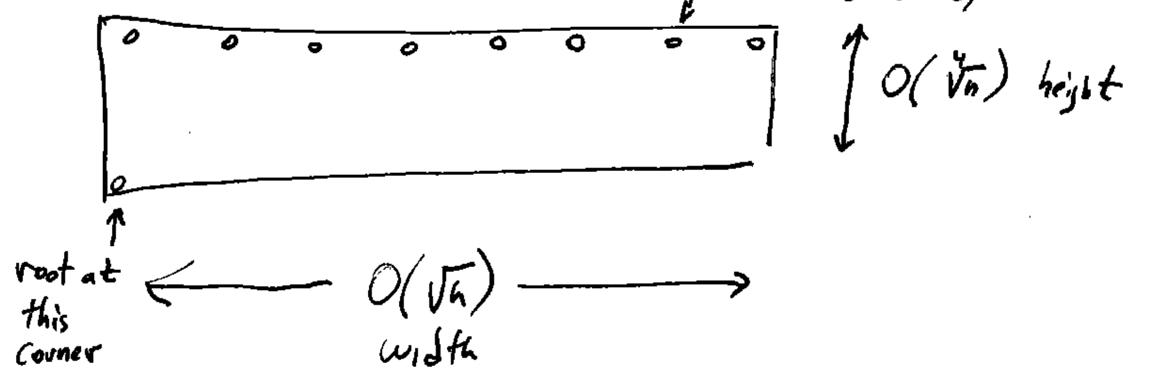
diameter of chip is $\Omega(\sqrt{n})$ (or you can't fit leaves)

$\Rightarrow$ some wire is at least $\Omega(\sqrt{n}/\lg n)$

Thm: can achieve $O(\sqrt{n}/\lg n)$

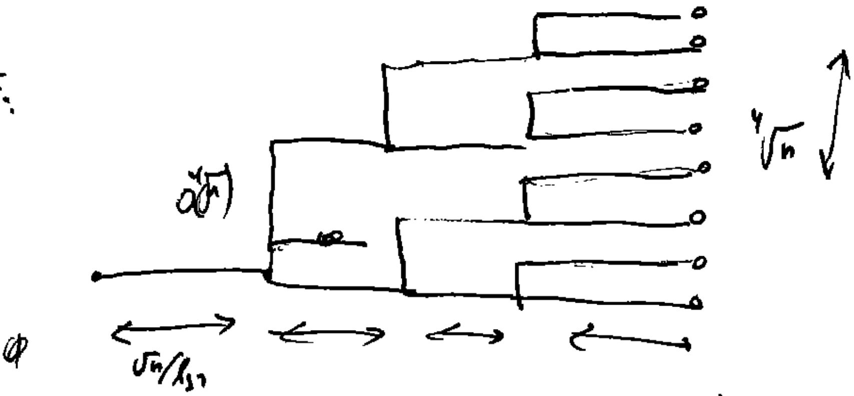
Lemma: Can layout a tree with $\sqrt[4]{n}$ leaves in this box

~~the box with height, width~~,



with max. wire length $O(\sqrt{n}/\lg n)$

proof:



this layout has max wire length $O(\sqrt{n}/\lg n)$, ~~but~~ and fits in the box. But the leaves are on the wrong edge

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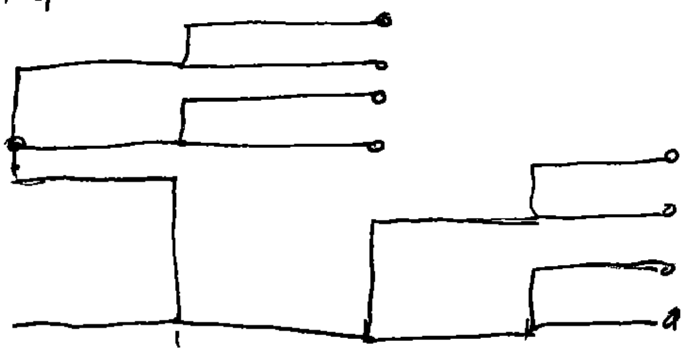
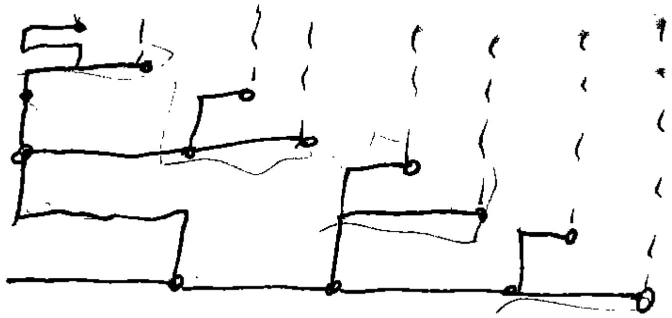
Fix it up



Same height, but half the leaves moved over and some were more left.

Do it again recursively all the way down

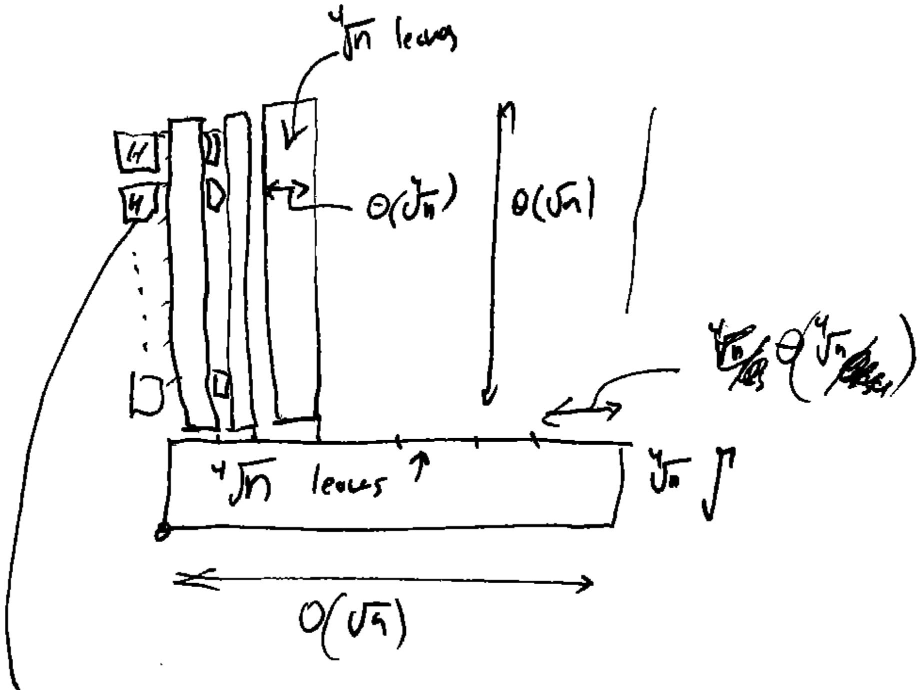


Now each leaf is in the correct column.

Simply add vertical lines to ~~easy~~ get to output

$\square$

Now for proof of thm:



a little H tree containing only $\sqrt{n}$ leaves

has area $\sqrt{n}$

side length $\sqrt[4]{n}$

max wire length $\sqrt[4]{n}$.

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Some basic layout ideas

IDEA:

Multiple Layers ~~Don't~~ Don't Matter Much.

Thm: Given a layout that uses ~~10~~ K layers, we can reimplement the layout to use only 2 layers.

In the first layer wires go only east-west.

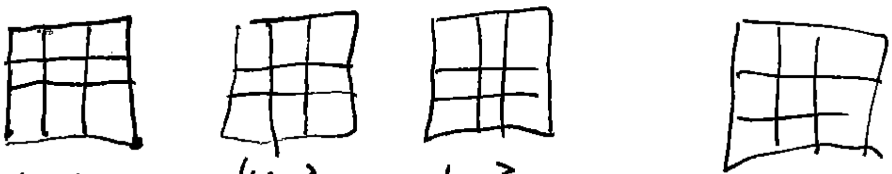
In the second layer wires go only north south.

The side length grows by $O(k)$ in our new layout

The area grows by $O(k^2)$

proof by picture

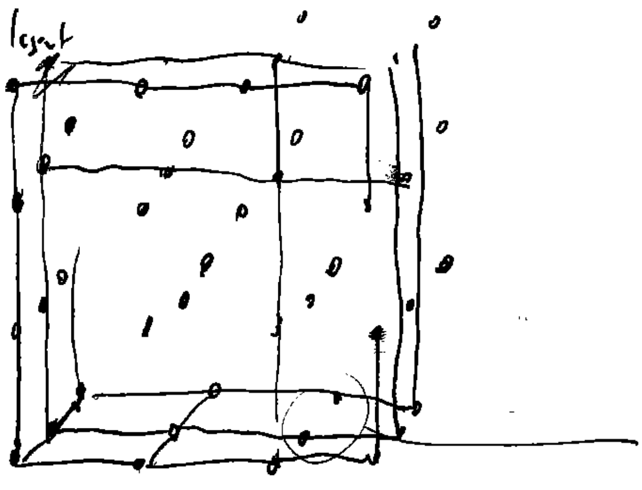
~~Example~~.

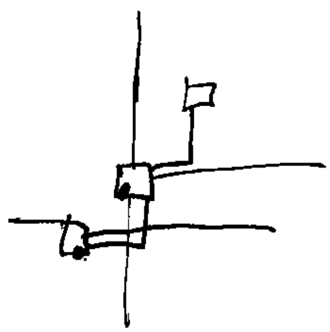


layer 1 layer 2 layer 3 ... layer k

also assume all ~~connecting parts~~ corresponding parts connected

new layout





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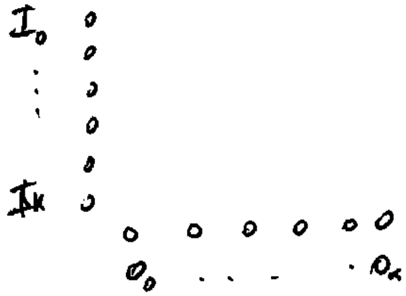
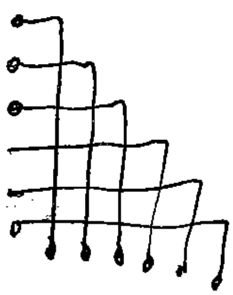
Idea: Any circuit can be made nearly square. (4W)

Idea: Turning a corner is expensive.



Connect input I_i to output O_i

Here is one easy way



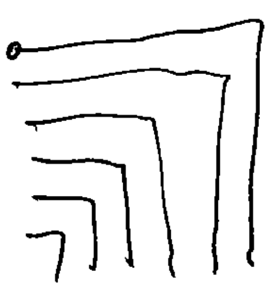
Analysis: ~~Area~~

Bounding Box area $= \Theta(k^2)$

total Wire length $= \Theta(k^2)$ (just following shortest path

is $\Theta(k)$ per wire.

Similarly we can reverse them in one $\Theta(k^2)$



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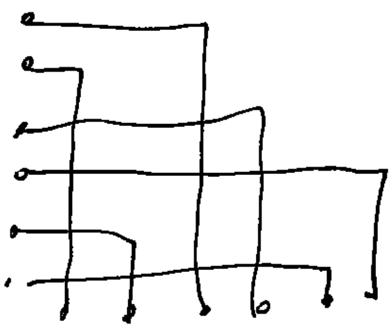
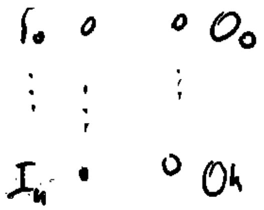
In fact we can perform any permutation in area $O(k^2)$



Idea: Reversing is expensive

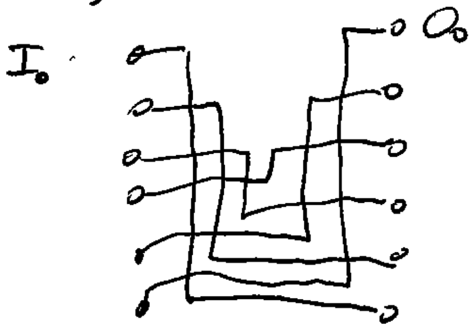


Connect I_0 to ~~I~~ O_0

cheap:

$O(k)$ area

~~But~~ reversing: is $O(k^2)$ area



leave an extra channel in the grid

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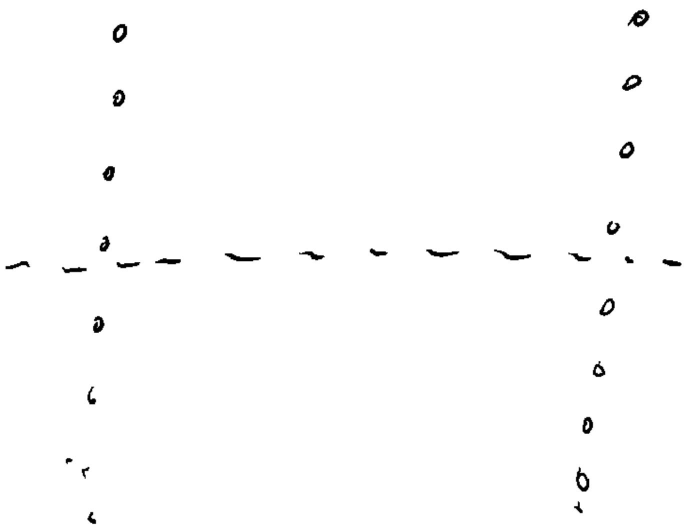
Thm: Reversing is area $\Omega(k^2)$ bounding box

Proof:



height is $\Omega(k)$

width? Consider cut across middle

Q: How many wires cross?

A: $\Theta(k)$ wires

$\Rightarrow$ that cut must be $\Omega(k)$ across.

$\therefore$ area is $\Omega(k^2)$ $\square$

Can show ~~idea~~ it is $\Omega(k^2)$ wire length.

[illegible] 17.9

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4.14.04

~~Thm: Area of n-input Benes network is $\Theta(n^2)$~~

Thm: ~~Layout of butterfly is $\Theta(n \lg n)^2$~~

Layout of butterfly of n inputs ($n \lg n$ nodes) is area $\Theta(n^2)$

proof: $\Omega(n^2)$ lower bound

bisection width argument.

Assume you layed out butterfly

There is some cut that is vertical, maybe with one jog in it that cuts it in half.

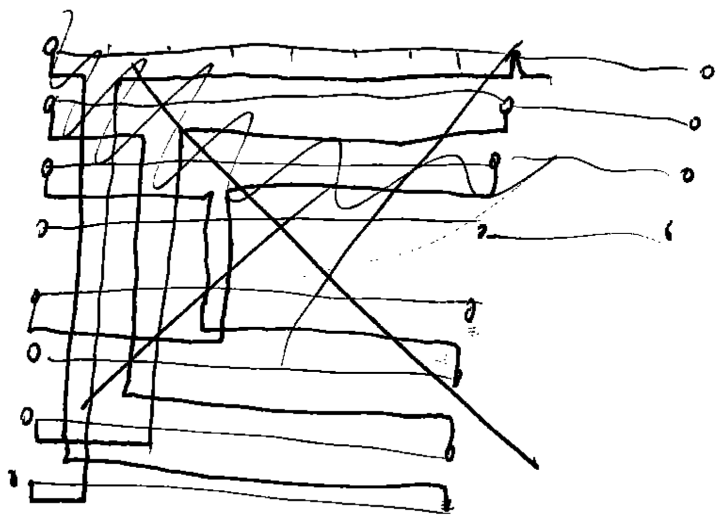
The bisection width of butterfly is $\Theta(n)$, so the height is $\Omega(n)$

Similarly the width.

$\Rightarrow \Omega(n^2)$

~~Thm~~

~~proof $O(n^2)$~~



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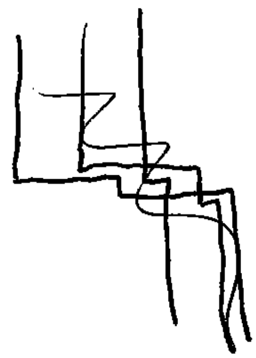
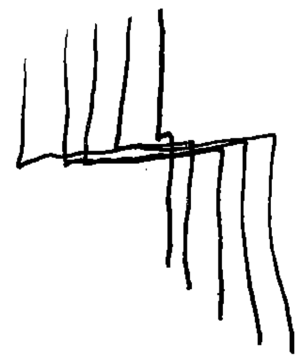
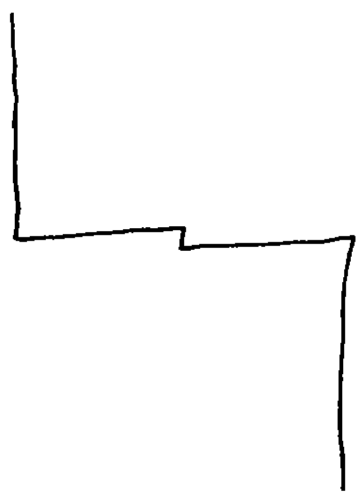
To show $\Omega(n^2)$ wire area is a little harder.



consider all bisections that ~~st~~ look like this



Need a little jog in the horizontal part to get exactly cut in half.



1) Each cut is $\Omega(n)$ wires
2) ~~all~~ the main vertical parts of the cuts don't intersect
3) The ~~avg~~ # of wires crossing the main vertical parts is

$$\Omega(n) + \Omega(n-1) + \dots + \Omega(1) = \Omega(n^2)$$

17. 11
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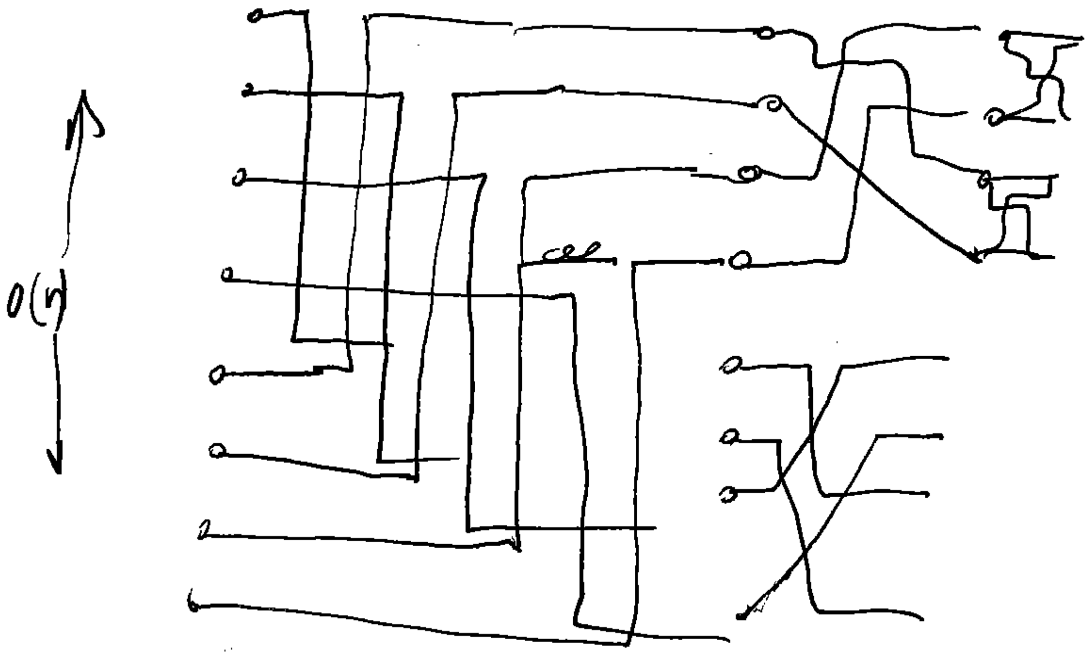
proof: butterfly is area $O(n^2)$



first stage is
$O(n)$

$O(n/2)$ $O(n/4)$ $\cdots$ $O(1)$

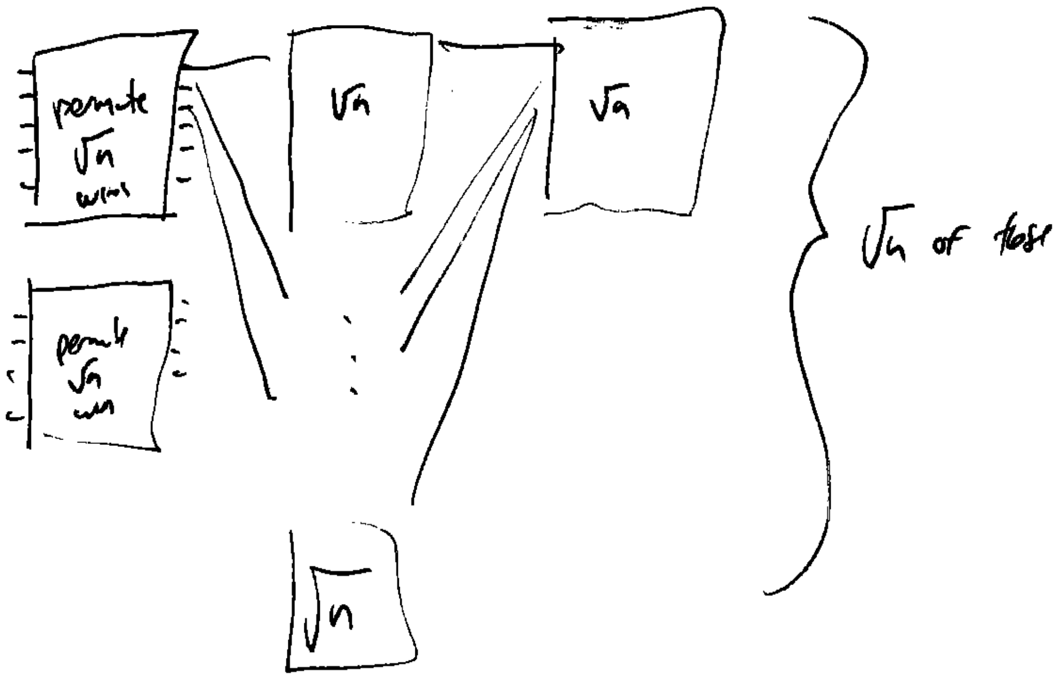
$\Rightarrow$ $O(n)$ wide

17.12 #12

Any permutation in 3D is $\Theta(n^{3/2})$

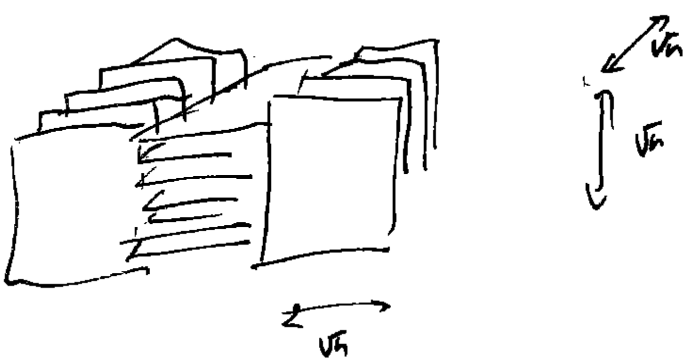
3D - similar wire model except wires take volume not area

Logical network



This is a Beneš network.

3 D layout



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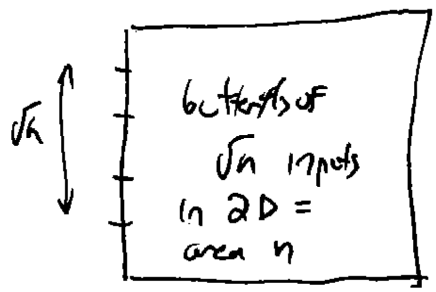
~~Consider a 3-D VLSI model.~~
~~wires can run → ↑ or ↗, but they take volume~~
~~proportional to their length.~~

~~Spec.~~
Butterfly layout in 3D is volume $\Theta(n^{3/2})$

Proof:



they touch at n^2 spots (every board touches every other board.)

1) It is a butterfly on n inputs

2) It has ~~area~~ volume $\underbrace{2\cdot\sqrt{n}}_{\text{number of boards}}\cdot\underbrace{n}_{\text{area per board}}$

$\Rightarrow$ volume is $O(n^{3/2})$

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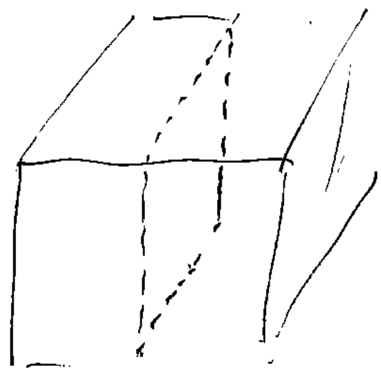
Claim volume of butterfly is $\Omega(n^{3/2})$

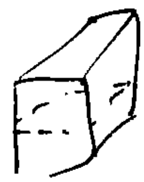
proof: Bisection argument

cut in half

that cut must cut $\Omega(n)$ wires.

So the cross section of the cut must be $\Omega(n)$





Similarly other planes cut $\Omega(n)$ wires.

does that prove bounding box is $\Omega(n^{3/2})$?

If it were square it would prove it.

~~to a HW qu~~

Can we assume it is square?

think about it.

~~The~~ $\boxed{\text{Homework}}$