Problem Set 2

This problem set is due in class on *Tuesday, March 2*.

Problem 2-1. Randomized selection

The following program computes the k-th smallest element of a sequence s of integers:

```
function kth_smallest(s, k) =
let
    pivot = s[rand(#s)];
    lesser = { e in s | e < pivot }
in
    if (k < #lesser) then kth_smallest(lesser, k)
    else
        let
            greater = { e in s | e > pivot }
            in
                if (k >= #s - #greater) then
                    kth_smallest(greater, k - (#s - #greater))
                else pivot;
```

(There is a similar program on page 7 of the NESL manual. However, our program selects the pivot uniformly at random, rather than picking the pivot deterministically.)

Analyze the expected work and the expected critical path of the algorithm. *Hint:* You need a high-probability analysis in order to prove that the expected critical path of the algorithm is $O(\log n)$, where n is the size of the input sequence s.

Problem 2-2. Connected components

Do Problem 30-3 on page 727 of CLR.