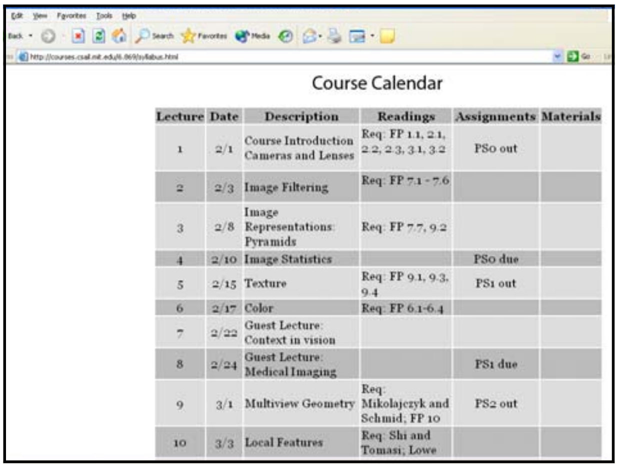
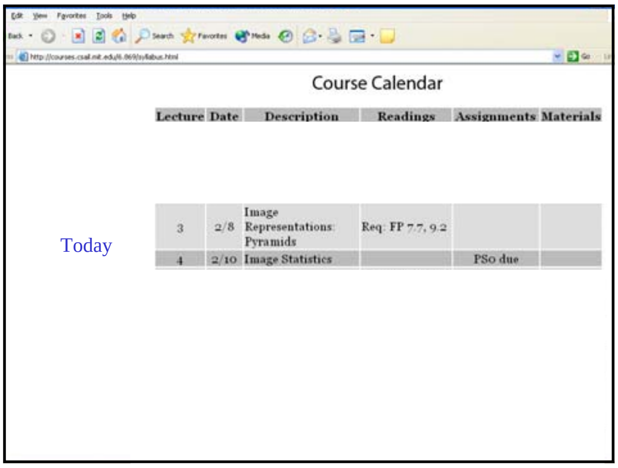


Course Calendar

Lecture	Date	Description	Readings	Assignments	Materials
1	2/1	Course Introduction Cameras and Lenses	Req: FP 1.1, 2.1, 2.2, 2.3, 3.1, 3.2	PS0 out	
2	2/3	Image Filtering	Req: FP 7.1 - 7.6		
3	2/8	Image Representations: Pyramids	Req: FP 7.7, 9.2		
4	2/10	Image Statistics		PS0 due	
5	2/15	Texture	Req: FP 9.1, 9.3, 9.4	PS1 out	
6	2/17	Color	Req: FP 6.1-6.4		
7	2/22	Guest Lecture: Context in vision			
8	2/24	Guest Lecture: Medical Imaging		PS1 due	
9	3/1	Multiview Geometry	Req: Mikolajczyk and Schmid; FP 10	PS2 out	
10	3/3	Local Features	Req: Shi and Tomasi; Lowe		



Course Calendar

Today

Lecture	Date	Description	Readings	Assignments	Materials
3	2/8	Image Representations: Pyramids	Req: FP 7.7, 9.2		
4	2/10	Image Statistics		PS0 due	

Reading

- Related to today’s lecture:
  - Adelson article on pyramid representations, posted on web site.
  - Farid paper posted on web site.

Image pyramids

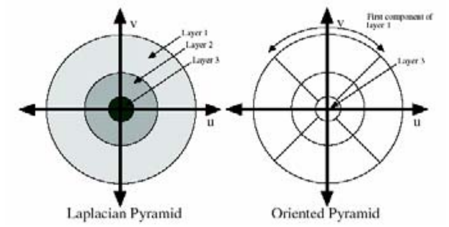
- Gaussian
- Laplacian
- Wavelet/QMF
- Steerable pyramid

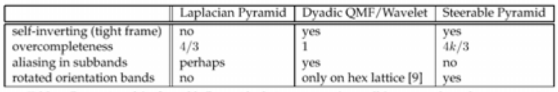
Steerable pyramids

- Good:
  - Oriented subbands
  - Non-aliased subbands
  - Steerable filters
- Bad:
  - Overcomplete
  - Have one high frequency residual subband, required in order to form a circular region of analysis in frequency from a square region of support in frequency.

Oriented pyramids

- Laplacian pyramid is orientation independent
- Apply an oriented filter to determine orientations at each layer
  - by clever filter design, we can simplify synthesis
  - this represents image information at a particular scale and orientation





	Laplacian Pyramid	Dyadic QMF/Wavelet	Steerable Pyramid
self-inverting (tight frame)	no	yes	yes
overcompleteness	$4/3$	1	$4k/3$
aliasing in subbands	perhaps	yes	no
rotated orientation bands	no	only on hex lattice [9]	yes

Table 1: Properties of the Steerable Pyramid relative to two other well-known multi-scale representations.

http://www.cns.nyu.edu/ftp/eero/simoncelli95b.pdf Simoncelli and Freeman, ICIP 1995

But we need to get rid of the corner regions before starting the recursive circular filtering

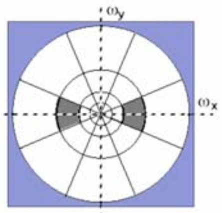
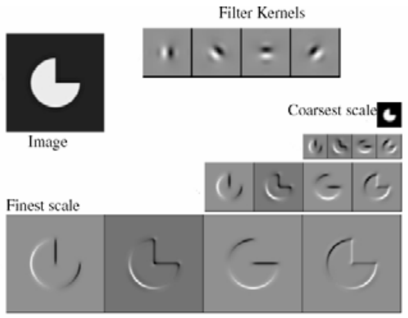


Figure 1. Idealized illustration of the spectral decomposition performed by a steerable pyramid with $k = 4$. Frequency axes range from $-\pi$ to π. The basis functions are related by translations, dilations and *rotations* (except for the initial highpass subband and the final lowpass subband). For example, the shaded region corresponds to the spectral support of a single (vertically-oriented) subband.

http://www.cns.nyu.edu/ftp/eero/simoncelli95b.pdf Simoncelli and Freeman, ICIP 1995



Reprinted from "Shiftable MultiScale Transforms," by Simoncelli et al., IEEE Transactions on Information Theory, 1992, copyright 1992, IEEE

- Summary of pyramid representations

Image pyramids

- Gaussian

Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

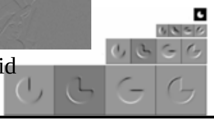
- Laplacian

Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

- Wavelet/QMF

Bandpassed representation, complete, but with aliasing and some non-oriented subbands.

- Steerable pyramid



Shows components at each scale and orientation separately. Non-aliased subbands. Good for texture and feature analysis.

Linear image transformations

- In analyzing images, it’s often useful to make a change of basis.

transformed image

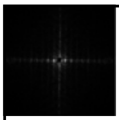
$$\vec{F} = U\vec{f}$$

Vectorized image

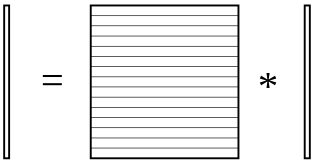
Fourier transform, or
Wavelet transform, or
Steerable pyramid transform

Schematic pictures of each matrix transform

- Shown for 1-d images
- The matrices for 2-d images are the same idea, but more complicated, to account for vertical, as well as horizontal, neighbor relationships.



Fourier transform

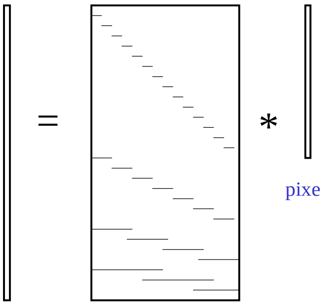


Fourier transform

Fourier bases are global: each transform coefficient depends on all pixel locations.

pixel domain image

Gaussian pyramid

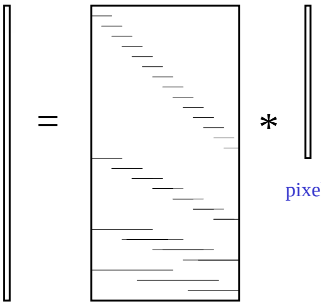


Gaussian pyramid

pixel image

Overcomplete representation.
Low-pass filters, sampled appropriately for their blur.

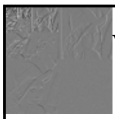
Laplacian pyramid



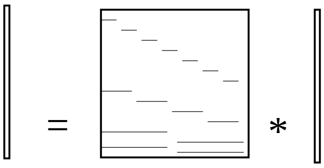
Laplacian pyramid

pixel image

Overcomplete representation.
Transformed pixels represent bandpassed image information.



Wavelet (QMF) transform

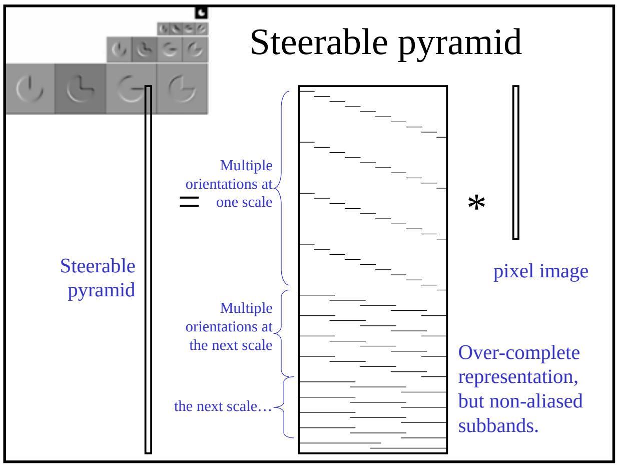


Wavelet pyramid

Ortho-normal transform (like Fourier transform), but with localized basis functions.

pixel image

Steerable pyramid



Steerable pyramid

=

Multiple orientations at one scale

Multiple orientations at the next scale

the next scale…

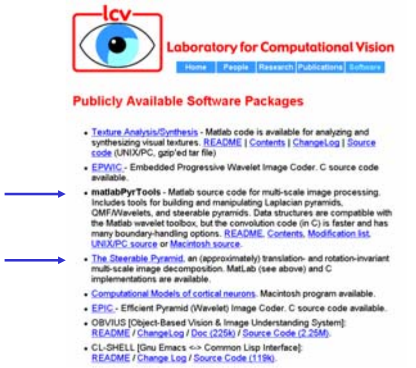
*

pixel image

Over-complete representation, but non-aliased subbands.

Matlab resources for pyramids (with tutorial)

http://www.cns.nyu.edu/~eero/software.html



Publicly Available Software Packages

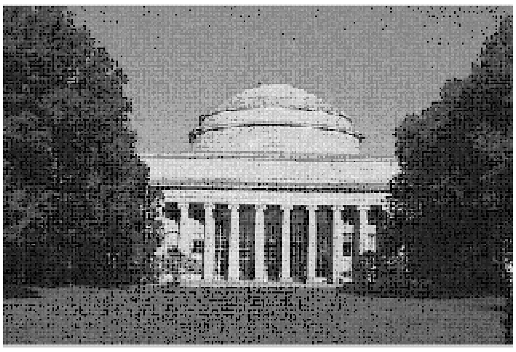
- Texture Analysis/Synthesis - Matlab code is available for analyzing and synthesizing visual textures. README | Contents | ChangeLog | Source code (UNIX/PC, gzip'ed tar file)
- EPWIC - Embedded Progressive Wavelet Image Coder. C source code available.
- **matlabPyrTools** - Matlab source code for multi-scale image processing. Includes tools for building and manipulating Laplacian pyramids, QMF/Wavelets, and steerable pyramids. Data structures are compatible with the Matlab wavelet toolbox, but the convolution code (in C) is faster and has many boundary-handling options. README, Contents, Modification list, UNIX/PC source or Macintosh source.
- The Steerable Pyramid, an (approximately) translation- and rotation-invariant multi-scale image decomposition. MatLab (see above) and C implementations are available.
- Computational Models of cortical neurons. Macintosh program available.
- EPIC - Efficient Pyramid (Wavelet) Image Coder. C source code available.
- OBVIUS [Object-Based Vision & Image Understanding System]: README / ChangeLog / Doc (225k) / Source Code (2.25M).
- CL-SHELL [Gnu Emacs <-> Common Lisp Interface]: README / Change Log / Source Code (119k).

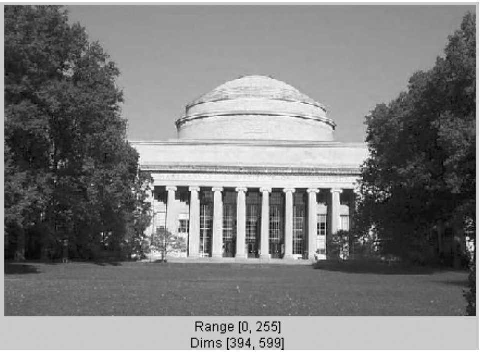
Why use these representations?

- Handle real-world size variations with a constant-size vision algorithm.
- Remove noise
- Analyze texture
- Recognize objects
- Label image features

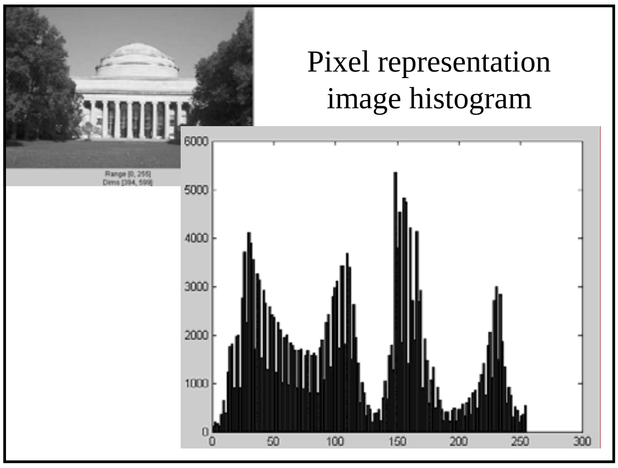
An application of image pyramids: noise removal

Image statistics (or, mathematically, how can you tell image from noise?)

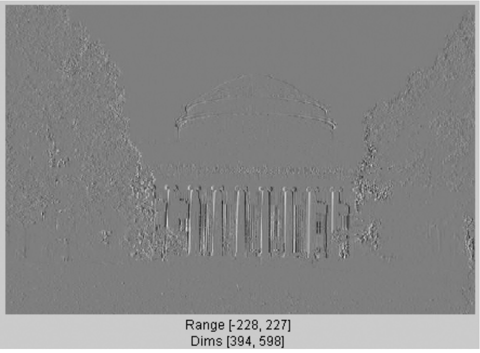




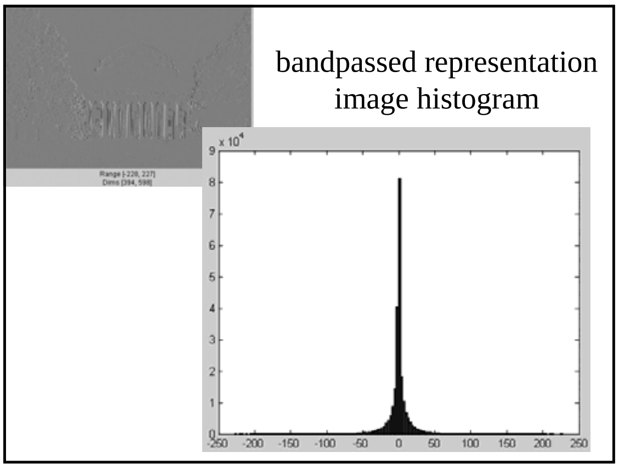
Pixel representation image histogram



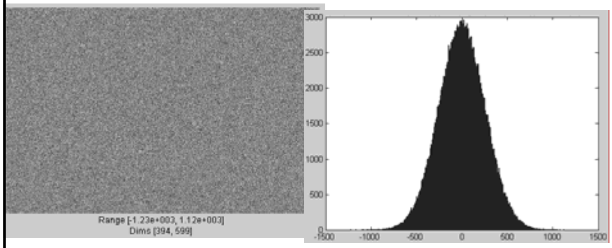
bandpass filtered image



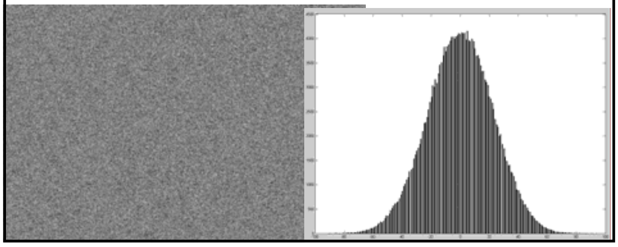
bandpassed representation image histogram



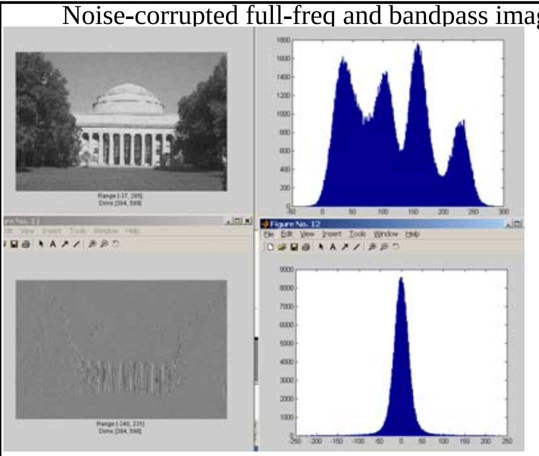
Pixel domain noise image and histogram



Bandpass domain noise image and histogram



Noise-corrupted full-freq and bandpass images



Bayes theorem

P(x, y) = P(x|y) P(y)

so

P(x|y) P(y) = P(y|x) P(x)

and

P(x|y) = P(y|x) P(x) / P(y)

The parameters you want to estimate

What you observe

Likelihood function

Prior probability

Constant w.r.t. parameters x.

Bayesian MAP estimator for clean bandpass coefficient values

Let x = bandpassed image value before adding noise.
Let y = noise-corrupted observation.

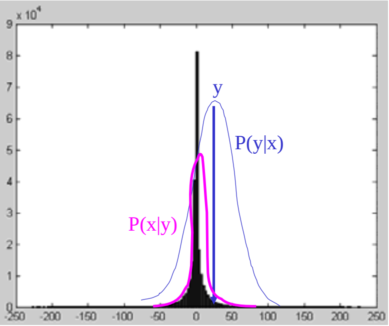
By Bayes theorem

P(x|y) = k P(y|x) P(x)

P(x)

P(y|x)

P(x|y)



Bayesian MAP estimator

Let x = bandpassed image value before adding noise.
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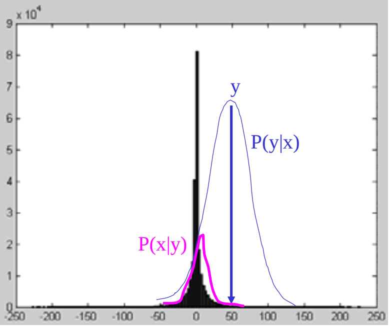
P(x)

P(y|x)

P(x|y)



Bayesian MAP estimator

Let x = bandpassed image value before adding noise.
Let y = noise-corrupted observation.

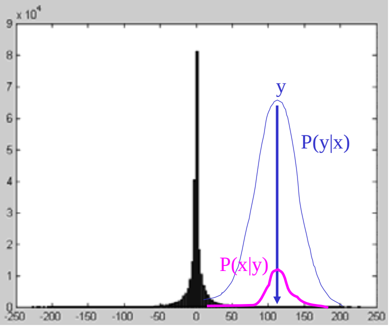
By Bayes theorem

P(x|y) = k P(y|x) P(x)

P(x)

P(y|x)

P(x|y)



MAP estimate, $\hat{x}$, as function of observed coefficient value, y

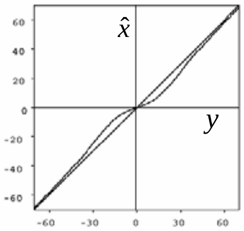


Figure 2: Bayesian estimator (symmetrized) for the signal and noise histograms shown in figure 1. Superimposed on the plot is a straight line indicating the identity function.

http://www-bcs.mit.edu/people/adelson/pub_pdfs/simoncelli_noise.pdf **Simoncelli and Adelson, Noise Removal via Bayesian Wavelet Coring**

Noise removal results

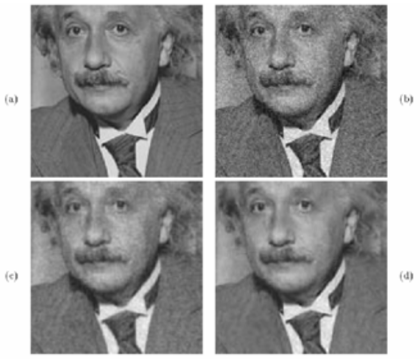


Figure 4: Noise reduction example. (a) Original image (cropped). (b) Image contaminated with additive Gaussian white noise (SNR = 9.00dB). (c) Image restored using (semi-blind) Wiener filter (SNR = 11.88dB). (d) Image restored using (semi-blind) Bayesian estimator (SNR = 13.82dB).

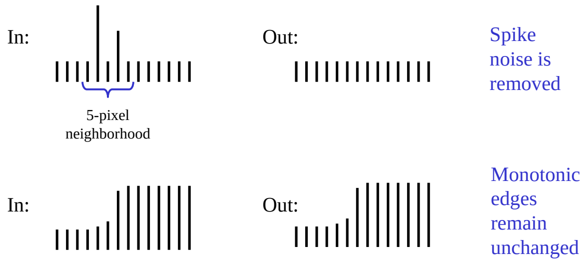
http://www-bcs.mit.edu/people/adelson/pub_pdfs/simoncelli_noise.pdf **Simoncelli and Adelson, Noise Removal via Bayesian Wavelet Coring**

Insert hany farid slides

Non-linear filtering example

Median filter

Replace each pixel by the median over N pixels (5 pixels, for these examples). Generalizes to “rank order” filters.

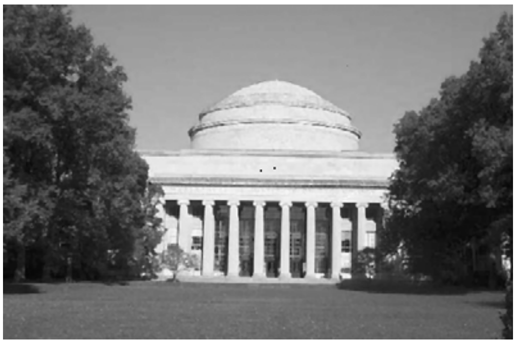


Spike noise is removed

Monotonic edges remain unchanged

Degraded image

Radius 1 median filter



Radius 2 median filter

CCD color sampling

Color sensing, 3 approaches

- Scan 3 times (temporal multiplexing)
- Use 3 detectors (3-ccd camera, and color film)
- Use offset color samples (spatial multiplexing)

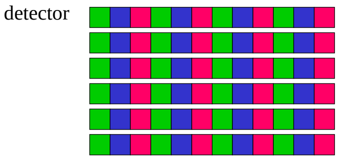
Typical errors in temporal multiplexing approach

- Color offset fringes

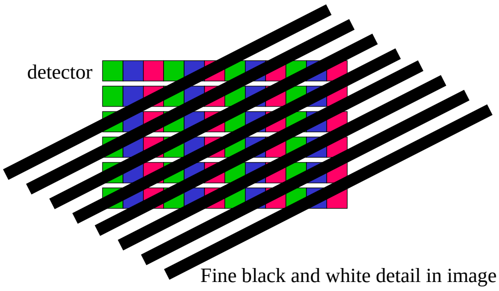
Typical errors in spatial multiplexing approach.

- Color fringes.

CCD color filter pattern

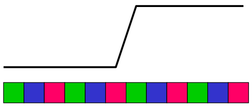


The cause of color moire



Fine black and white detail in image mis-interpreted as color information.

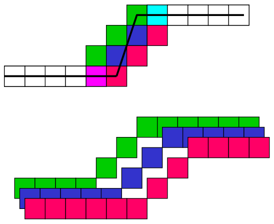
Black and white edge falling on color CCD detector



Black and white image (edge)

Detector pixel colors

Color sampling artifact



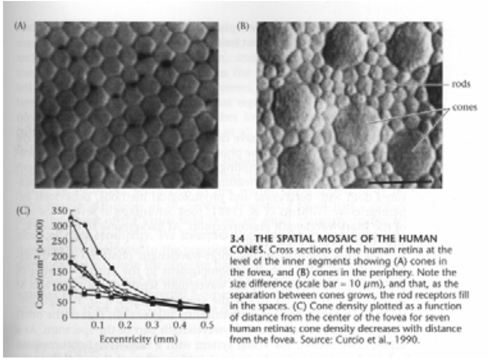
Interpolated pixel colors,
for grey edge falling on colored
detectors (linear interpolation).

Typical color moire patterns

Blow-up of electronic camera image. Notice spurious colors in the regions of fine detail in the plants.

Color sampling artifacts

Human Photoreceptors

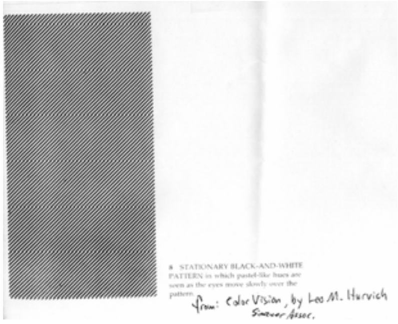


3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.

(From Foundations of Vision, by Brian Wandell, Sinauer Assoc.)

Brewster’s colors example (subtle).

Scale relative to human photoreceptor size: each line covers about 7 photoreceptors.



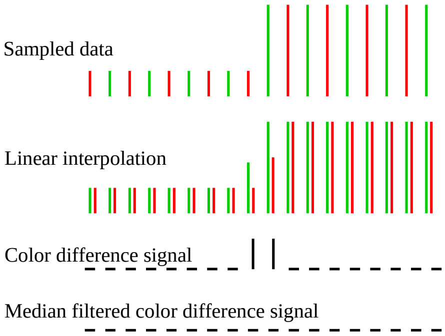
8 STATIONARY BLACK-AND-WHITE PATTERN in which pastel-like hues are seen as the eyes move slowly over the pattern.

from: Color Vision, by Leo M. Hurvich
Sinauer Assoc.

Median Filter Interpolation

- Perform first interpolation on isolated color channels.
- Compute color difference signals.
- Median filter the color difference signal.
- Reconstruct the 3-color image.

Two-color sampling of BW edge



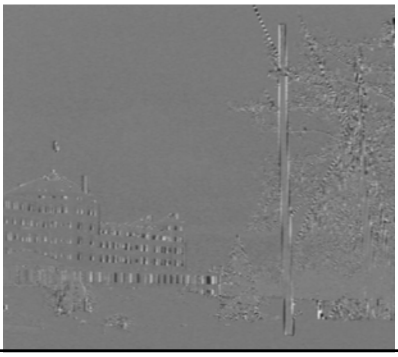
Sampled data

Linear interpolation

Color difference signal

Median filtered color difference signal

R-G, after linear interpolation



R – G, median filtered (5x5)

Recombining the median filtered colors

Linear interpolation

Median filter interpolation