## Class logistics

- Exam results back today.
- This Thursday, your project proposals are due.
- Feel free to ask Xiaoxu or me for feedback or ideas regarding the project.
- Auditors are welcome to do a project, and we'll read them and give feedback.



## A medley of project ideas

- Implement photographic vs photorealistic discrimination function.
- Read and compare 3 papers on a computer vision/machine learning topic that excites you.
- Evaluate how well the "visual gist" (blurry texture representation) does at categorizing a large collection of images.
- Implement and evaluate example-based super-resolution.
- Implement and evaluate Soatto's temporal texture model (conceptually simple; neat results).
- Digitize a bird book, and make SVM classifiers for owls, pelicans, eagles, etc.
- Make a broken glass detector.
- Ask, and answer, what is the dimensionality of the manifold of image patches, of various sizes?
- Digitize tree identification books, and develop a texture-based classifier that will categorize trees from their leaf/needle textures.


## Generative Models

Bill Freeman, MIT
6.869 March 29, 2005

Making probability distributions modular, and therefore tractable:
Probabilistic graphical models

Vision is a problem involving the interactions of many variables: things can seem hopelessly complex. Everything is made tractable, or at least, simpler, if we modularize the problem. That's what probabilistic graphical models do, and let's examine that.

Readings: Jordan and Weiss intro article—fantastic!
Kevin Murphy web page-comprehensive and with pointers to many advanced topics

## $\mathrm{P}(\mathrm{a}, \mathrm{b})=\mathrm{P}(\mathrm{b} \mid \mathrm{a}) \mathrm{P}(\mathrm{a})$

By the chain rule, for any probability distribution, we have:

$$
\begin{aligned}
P\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\right) & =P\left(x_{1}\right) P\left(x_{2}, x_{3}, x_{4}, x_{5} \mid x_{1}\right) \\
& =P\left(x_{1}\right) P\left(x_{2} \mid x_{1}\right) P\left(x_{3}, x_{4}, x_{5} \mid x_{1}, x_{2}\right) \\
& =P\left(x_{1}\right) P\left(x_{2} \mid x_{1}\right) P\left(x_{3} \mid x_{1}, x_{2}\right) P\left(x_{4}, x_{5} \mid x_{1}, x_{2}, x_{3}\right) \\
& =P\left(x_{1}\right) P\left(x_{2} \mid x_{1}\right) P\left(x_{3} \mid x_{1}, x_{2}\right) P\left(x_{4} \mid x_{1}, x_{2}, x_{3}\right) P\left(x_{5} \mid x_{1}, x_{2}, x_{3}, x_{4}\right)
\end{aligned}
$$

But if we exploit the assumed modularity of the probability distribution over the 5 variables (in this case, the assumed Markov chain structure), then that expression simplifies:
$=P\left(x_{1}\right) P\left(x_{2} \mid x_{1}\right) P\left(x_{3} \mid x_{2}\right) P\left(x_{4} \mid x_{3}\right) P\left(x_{5} \mid x_{4}\right)$
$0 \rightarrow 0 \rightarrow 0 \rightarrow 0 \rightarrow 0$
Now our marginalization summations distribute through those terms:
$\sum_{x_{2}, x_{3}, x_{4}, x_{5}} P\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\right)=\quad P\left(x_{1}\right) \sum_{x_{2}} P\left(x_{2} \mid x_{1}\right) \sum_{x_{3}} P\left(x_{3} \mid x_{2}\right) \sum_{x_{4}} P\left(x_{4} \mid x_{3}\right) \sum_{x_{5}} P\left(x_{5} \mid x_{4}\right)$

## Belief propagation

Performing the marginalization by doing the partial sums is called "belief propagation".
$\sum_{x_{2}, x_{3}, x_{4}, x_{5}} P\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\right)=\quad P\left(x_{1}\right) \sum_{x_{2}} P\left(x_{2} \mid x_{1}\right) \sum_{x_{3}} P\left(x_{3} \mid x_{2}\right) \sum_{x_{4}} P\left(x_{4} \mid x_{3}\right) \sum_{x_{5}} P\left(x_{5} \mid x_{4}\right)$

In this example, it has saved us a lot of computation. Suppose each variable has 10 discrete states. Then, not knowing the special structure of $P$, we would have to perform 10000 additions $(10 \wedge 4)$ to marginalize over the four variables.
But doing the partial sums on the right hand side, we only need 40 additions $(10 * 4)$ to perform the same marginalization!

Another modular probabilistic structure, more common in vision problems, is an undirected graph:


The joint probability for this graph is given by:
$P\left(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\right)=\Phi\left(x_{1}, x_{2}\right) \Phi\left(x_{2}, x_{3}\right) \Phi\left(x_{3}, x_{4}\right) \Phi\left(x_{4}, x_{5}\right)$

Where $\Phi\left(X_{1}, X_{2}\right)$ is called a "compatibility function". We can define compatibility functions we result in the same joint probability as for the directed graph described in the previous slides; for that example, we could use either form


## Network joint probability



## In order to use MRFs:

- Given observations y, and the parameters of the MRF, how infer the hidden variables, x ?
- How learn the parameters of the MRF?


## Outline of MRF section

- Inference in MRF's.
- Gibbs sampling, simulated annealing
- Iterated condtional modes (ICM)
- Variational methods
- Belief propagation
- Graph cuts
- Vision applications of inference in MRF's.
- Learning MRF parameters.
- Iterative proportional fitting (IPF)


## Belief propagation messages

A message: can be thought of as a set of weights on each of your possible states
To send a message: Multiply together all the incoming messages, except from the node you're sending to, then multiply by the compatibility matrix and marginalize over the sender's states.
$M_{i}^{j}\left(x_{i}\right)=\sum_{x_{j}} \psi_{\mathrm{ij}}\left(x_{i}, x_{j}\right) \prod_{k \in N(j) i} M_{j}^{k}\left(x_{j}\right)$


Belief, and message updates

$$
b_{j}\left(x_{j}\right)=\prod_{k \in N(j)} M_{j}^{k}\left(x_{j}\right)
$$

$$
M_{i}^{j}\left(x_{i}\right)=\sum_{x_{j}} \psi_{\mathrm{ij}}\left(x_{i}, x_{j}\right) \prod_{k \in N(j) i} M_{j}^{k}\left(x_{j}\right)
$$

$$
{ }^{\mathrm{i} \cdot}=\mathrm{i}_{\bullet} \quad \mathrm{j}_{\bullet}
$$

## Beliefs

To find a node's beliefs: Multiply together all the messages coming in to that node.

$$
b_{j}\left(x_{j}\right)=\prod_{k \in N(j)} M_{j}^{k}\left(x_{j}\right)
$$

## Optimal solution in a chain or tree:

 Belief Propagation- "Do the right thing" Bayesian algorithm.
- For Gaussian random variables over time: Kalman filter.
- For hidden Markov models: forward/backward algorithm (and MAP variant is Viterbi).


Justification for running belief propagation in networks with loops

- Experimental results:


## Statistical mechanics interpretation

U-TS = Free energy
$\mathrm{U}=$ avg. energy $=\sum_{\text {states }} p\left(x_{1}, x_{2}, \ldots\right) E\left(x_{1}, x_{2}, \ldots\right)$
$\mathrm{T}=$ temperature

## Free energy formulation

Defining

$$
\Psi_{i j}\left(x_{i}, x_{j}\right)=e^{-E\left(x_{i}, x_{j}\right) / T} \quad \Phi_{i}\left(x_{i}\right)=e^{-E\left(x_{i}\right) / T}
$$

then the probability distribution $P\left(x_{1}, x_{2}, \ldots\right)$ that minimizes the F.E. is precisely the true probability of the Markov network,

$$
P\left(x_{1}, x_{2}, \ldots\right)=\prod_{i j} \Psi_{i j}\left(x_{i}, x_{j}\right) \prod_{i} \Phi_{i}\left(x_{i}\right)
$$

- Error-correcting codes Kschischang and Frey, 1998; McEliece et al., 1998
- Vision applications $\begin{aligned} & \text { Freeman and Pasztor, 1999; } \\ & \text { Frey, } 2000\end{aligned}$
- Theoretical results:
- For Gaussian processes, means are correct.
- Large neighborhood local maximum for MAP.
- Equivalent to Bethe approx in statistical physics.
- Tree-weighted reparameterization Freeman, and Weiss, 2000

Wainwright, Willsky, Jaakkola, 2001
$\mathrm{S}=$ entropy $=\quad-\sum_{\text {salas }} p\left(x_{1}, x_{2}, \ldots\right) \ln p\left(x_{1}, x_{2}, \ldots\right)$

## Approximating the Free Energy

$\begin{array}{ll}\text { Exact: } & F\left[p\left(x_{1}, x_{2}, \ldots, x_{N}\right)\right] \\ \text { Mean Field Theory: } & F\left[b_{i}\left(x_{i}\right)\right] \\ \text { Bethe Approximation: } & F\left[b_{i}\left(x_{i}\right), b_{i j}\left(x_{i}, x_{j}\right)\right] \\ \text { Kikuchi Approximations: }\end{array}$
Kikuchi Approximations:

$$
F\left[b_{i}\left(x_{i}\right), b_{i j}\left(x_{i}, x_{j}\right), b_{i j k}\left(x_{i}, x_{j}, x_{k}\right), \ldots .\right]
$$

| Approximating the Free Energy |
| :--- |
| Exact: $\quad F\left[p\left(x_{1}, x_{2}, \ldots, x_{N}\right)\right]$ <br> Mean Field Theory: $\quad F\left[b_{i}\left(x_{i}\right)\right]$ <br> Bethe Approximation: $\quad F\left[b_{i}\left(x_{i}\right), b_{i j}\left(x_{i}, x_{j}\right)\right]$ <br> Kikuchi Approximations: <br> $\qquad F\left[b_{i}\left(x_{i}\right), b_{i j}\left(x_{i}, x_{j}\right), b_{i j k}\left(x_{i}, x_{j}, x_{k}\right), \ldots.\right]$ |



## Region marginal probabilities



## Results from Bethe free energy analysis

- Fixed point of belief propagation equations iff. Bethe approximation stationary point.
- Belief propagation always has a fixed point.
- Connection with variational methods for inference: both minimize approximations to Free Energy,
- variational: usually use primal variables.
- belief propagation: fixed pt. equs. for dual variables.
- Kikuchi approximations lead to more accurate belief propagation algorithms.
- Other Bethe free energy minimization algorithmsYuille, Welling, etc.

Show program comparing some methods on a simple MRF testMRF.m

## Graph cuts

- Algorithm: uses node label swaps or expansions as moves in the algorithm to reduce the energy. Swaps many labels at once, not just one at a time, as with ICM.
- Find which pixel labels to swap using min cut/max flow algorithms from network theory.
- Can offer bounds on optimality.
- See Boykov, Veksler, Zabih, IEEE PAMI 23 (11) Nov. 2001 (available on web).

Ground truth, graph cuts, and belief propagation disparity solution energies


Figure 2. Field Energies for the MRF labelled using ground-truth data compared to the energies for the fiedds labelled using Graph Cuts and Beiler Propagation. Notice that the solutions returned by
the algorithms consistently have a much lower energy than the labellings produced from the ground truth, showing a mismatch between the MRF formulation and the ground-truth. The final column contains the percentage of each ground-truth solution's energy that comes from matching costs of occluded pixels.

Graph cuts versus belief propagation

- Graph cuts consistently gave slightly lower energy solutions for that stereo-problem MRF, although BP ran faster, although there is now a faster graph cuts implementation than what we used...
- However, here's why I still use Belief Propagation:
- Works for any compatibility functions, not a restricted set like graph cuts.
- I find it very intuitive.
- Extensions: sum-product algorithm computes MMSE, and Generalized Belief Propagation gives you very accurate solutions, at a cost of time.


## MAP versus MMSE


(a) MAP Lsimate
(b) MMSE Sainuma

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Joint probabilities for undirected graphs


$$
\begin{aligned}
& P\left(x_{1}, x_{2}, x_{3}\right)=P\left(x_{1}, x_{3} \mid x_{2}\right) P\left(x_{2}\right) \text { by elementary } \\
& \text { Use the conditional }=P\left(x_{1} \mid x_{2}\right) P\left(x_{3} \mid x_{2}\right) P\left(x_{2}\right) \\
& \text { independence assumption } \\
& \begin{array}{l}
\begin{array}{l}
\text { Multiply top and } \\
\text { botom by } P(x 2)
\end{array} \\
\text { (x) }
\end{array}=\frac{P\left(x_{1} \mid x_{2}\right) P\left(x_{2}\right) P\left(x_{3} \mid x_{2}\right) P\left(x_{2}\right)}{P\left(x_{2}\right)} \\
& \text { Re-write conditionals } \\
& \text { as joint probabilities } \\
& =\frac{P\left(x_{1}, x_{2}\right) P\left(x_{2}, x_{3}\right)}{P\left(x_{2}\right)} \quad \begin{array}{r}
\text { General result for } \\
\text { separating clique } \mathrm{x} 2
\end{array}
\end{aligned}
$$



Learning MRF parameters, labeled data
Iterative proportional fitting lets you make a maximum likelihood estimate of a joint distribution from observations of various marginal distributions.


Initial guess at joint probability


## IPF update equation

$$
P\left(x_{1}, x_{2}, \ldots, x_{d}\right)^{(t+1)}=P\left(x_{1}, x_{2}, \ldots, x_{d}\right)^{(t)} \frac{P\left(x_{i}\right)^{\text {observed }}}{P\left(x_{i}\right)^{(t)}}
$$

Scale the previous iteration's estimate for the joint probability by the ratio of the true to the predicted marginals.

Gives gradient ascent in the likelihood of the joint probability, given the observations of the marginals.

See: Michael Jordan's book on graphical models


[^0]

IPF results for this example: comparison of joint probabilities


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Vision applications of MRF's

- Stereo
- Motion estimation
- Super-resolution
- Many others...


## Vision applications of MRF's

- Stereo
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- Many others...

What behavior should we see in a motion algorithm?

- Aperture problem
- Resolution through propagation of information
- Figure/ground discrimination




## Motion analysis: related work

- Markov network
- Luettgen, Karl, Willsky and collaborators.
- Neural network or learning-based
- Nowlan \& T. J. Senjowski; Sereno.
- Optical flow analysis
- Weiss \& Adelson; Darrell \& Pentland; Ju, Black \& Jepson; Simoncelli; Grzywacz \& Yuille; Hildreth; Horn \& Schunk; etc.



## Vision applications of MRF's

- Stereo
- Motion estimation
- Super-resolution
- Many others...
resolution independent


## Super-resolution: other approaches

- Schultz and Stevenson, 1994
- Pentland and Horowitz, 1993
- fractal image compression (Polvere, 1998; Iterated Systems)
- astronomical image processing (eg. Gull and Daniell, 1978; "pixons" http://casswww.ucsd.edu/puetter.html)


## Super-resolution

- Image: low resolution image
- Scene: high resolution image


Images from two Corel database categories:
"giraffes" and "urban skyline".




## Scene-scene compatibility function, <br> 

Assume overlapped regions, d, of hi-res. patches differ by Gaussian observation noise:

$$
\Psi\left(x_{i}, x_{j}\right)=\exp ^{-\left|d_{i}-d_{j}\right|^{2} / 2 \sigma^{2} \mid}
$$






## Algorithms compared

- Bicubic Interpolation
- Mitra's Directional Filter
- Fuzzy Logic Filter
- Vector Quantization
- VISTA



## User preference test results

"The observer data indicates that six of the observers ranked Freeman's algorithm as the most preferred of the five tested algorithms. However the other two observers rank Freeman's algorithm as the least preferred of all the algorithms....

Freeman's algorithm produces prints which are by far the sharpest out of the five algorithms. However, this sharpness comes at a price of artifacts (spurious detail that is not present in the original scene). Apparently the two observers who did not prefer Freeman's algorithm had strong objections to the artifacts. The other observers apparently placed high priority on the high level of sharpness in the images created by Freeman's algorithm."


## Training image

ariylifeyaliyuteriueu, ur cull anelvacatedarul ingbythefer jstem, andsent itdowntoaneun finedastandardforweighing' sraproduct-bundlingdecisi, softsaysthat thenewfeature: andpersonal identification: jsoft'suiew, butusersandthı adedwi thconsumerinnovatiol ^ePCindustryislookingforw.

More general graphical models than MRF grids

- In this course, we've studied Markov chains, and Markov random fields, but, of course, many other structures of probabilistic models are possible and useful in computer vision.
- For a nice on-line tutorial about Bayes nets, see Kevin Murphy's tutorial in his web page.




[^0]:    Application to MRF parameter estimation

    - Can show that for the ML estimate of the clique potentials, $\phi_{\mathrm{c}}\left(\mathrm{x}_{\mathrm{c}}\right)$, the empirical marginals equal the model marginals,

    $$
    \tilde{p}\left(x_{c}\right)=p\left(x_{c}\right)
    $$

    - This leads to the IPF update rule for $\phi_{\mathrm{c}}\left(\mathrm{x}_{\mathrm{c}}\right)$

    $$
    \phi_{C}^{(t+1)}\left(x_{c}\right)=\phi_{c}^{(t)}\left(x_{c}\right) \frac{\tilde{p}\left(x_{c}\right)}{p^{(t)}\left(x_{c}\right)}
    $$

    - Performs coordinate ascent in the likelihood of the MRF parameters, given the observed data.

    Reference: unpublished notes by Michael Jordan

