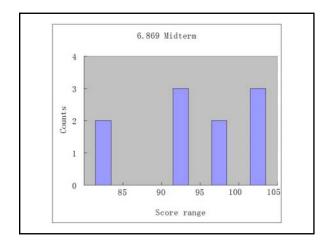
# Class logistics

- Exam results back today.
- This Thursday, your project proposals are due.
  - Feel free to ask Xiaoxu or me for feedback or ideas regarding the project.
  - Auditors are welcome to do a project, and we'll read them and give feedback.



#### A medley of project ideas

- Implement photographic vs photorealistic discrimination function
- Read and compare 3 papers on a computer vision/machine learning topic that excites you.
- Evaluate how well the "visual gist" (blurry texture representation) does at categorizing a large collection of images.
- Implement and evaluate example-based super-resolution.
- Implement and evaluate Soatto's temporal texture model (conceptually simple; neat results).
- Digitize a bird book, and make SVM classifiers for owls, pelicans, eagles, etc.
- Make a broken glass detector.
- Ask, and answer, what is the dimensionality of the manifold of image patches, of various sizes?
- Digitize tree identification books, and develop a texture-based classifier that will categorize trees from their leaf/needle textures.

#### Generative Models

Bill Freeman, MIT

6.869 March 29, 2005

Making probability distributions modular, and therefore tractable:

#### Probabilistic graphical models

Vision is a problem involving the interactions of many variables: things can seem hopelessly complex. Everything is made tractable, or at least, simpler, if we modularize the problem. That's what probabilistic graphical models do, and let's examine that.

Readings: Jordan and Weiss intro article—fantastic!

Kevin Murphy web page—comprehensive and with pointers to many advanced topics

 $P(a,b) = P(b|a) \ P(a)$ 

By the chain rule, for any probability distribution, we have:

 $P(x_1, x_2, x_3, x_4, x_5) = P(x_1)P(x_2, x_3, x_4, x_5 \mid x_1)$ 

 $= P(x_1)P(x_2 \mid x_1)P(x_3, x_4, x_5 \mid x_1, x_2)$ 

 $= P(x_1)P(x_2 \mid x_1)P(x_3 \mid x_1, x_2)P(x_4, x_5 \mid x_1, x_2, x_3)$ 

 $= P(x_1)P(x_2 \mid x_1)P(x_3 \mid x_1, x_2)P(x_4 \mid x_1, x_2, x_3)P(x_5 \mid x_1, x_2, x_3, x_4)$ 

But if we exploit the assumed modularity of the probability distribution over the 5 variables (in this case, the assumed Markov chain structure), then that expression simplifies:

 $= P(x_1)P(x_2 \mid x_1)P(x_3 \mid x_2)P(x_4 \mid x_3)P(x_5 \mid x_4)$ 

→ ₩ → ₩ → ₩ Now our marginalization summations distribute through those terms:

## Belief propagation

Performing the marginalization by doing the partial sums is called "belief propagation".

$$\sum_{x_1, x_2, x_3, x_4, x_5} P(x_1, x_2, x_3, x_4, x_5) = P(x_1) \sum_{x_1} P(x_2 \mid x_1) \sum_{x_2} P(x_3 \mid x_2) \sum_{x_3} P(x_4 \mid x_3) \sum_{x_4} P(x_5 \mid x_4)$$

In this example, it has saved us a lot of computation. Suppose each variable has 10 discrete states. Then, not knowing the special structure of P, we would have to perform 10000 additions (10^4) to marginalize over the four variables.

But doing the partial sums on the right hand side, we only need 40 additions (10\*4) to perform the same marginalization!

Another modular probabilistic structure, more common in vision problems, is an undirected graph:

$$(x_1)$$
  $(x_2)$   $(x_3)$   $(x_4)$   $(x_5)$ 

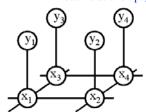
The joint probability for this graph is given by:

$$P(x_1, x_2, x_3, x_4, x_5) = \Phi(x_1, x_2)\Phi(x_2, x_3)\Phi(x_3, x_4)\Phi(x_4, x_5)$$

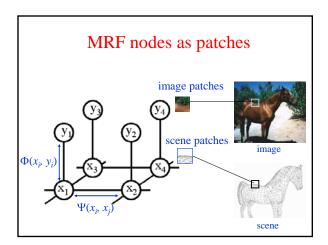
Where  $\Phi(x_1, x_2)$  is called a "compatibility function". We can define compatibility functions we result in the same joint probability as for the directed graph described in the previous slides; for that example, we could use either form.

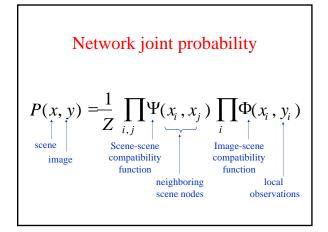
#### Markov Random Fields

- Allows rich probabilistic models for images.
- But built in a local, modular way. Learn local relationships, get global effects out.



# MRF nodes as pixels Fig. 2.3. Smoothing with the wrong prior. (a) Original, (b) degraded image, (c) MAP estimate $\beta = 1$ , (d) MAP estimate $\beta = 1$ , (d) MAP estimate $\beta = 0.3$ , (e) median filter





#### In order to use MRFs:

- Given observations y, and the parameters of the MRF, how infer the hidden variables, x?
- How <u>learn</u> the parameters of the MRF?

#### Outline of MRF section

- Inference in MRF's.
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  - Iterative proportional fitting (IPF)

#### Belief propagation messages

<u>A message</u>: can be thought of as a set of weights on each of your possible states

<u>To send a message</u>: Multiply together all the incoming messages, except from the node you're sending to, then multiply by the compatibility matrix and marginalize over the sender's states.

$$M_{i}^{j}(x_{i}) = \sum_{x_{j}} \psi_{ij}(x_{i}, x_{j}) \prod_{k \in N(j) \setminus i} M_{j}^{k}(x_{j})$$



#### **Beliefs**

<u>To find a node's beliefs</u>: Multiply together all the messages coming in to that node.

$$b_j(x_j) = \prod_{k \in N(j)} M_j^k(x_j)$$

# Belief, and message updates

$$b_j(x_j) = \prod_{k \in N(j)} M_j^k(x_j)$$

# Optimal solution in a chain or tree: Belief Propagation

- "Do the right thing" Bayesian algorithm.
- For Gaussian random variables over time: Kalman filter.
- For hidden Markov models: forward/backward algorithm (and MAP variant is Viterbi).

## No factorization with loops!

$$\begin{aligned} x_{\text{1MMSE}} &= \underset{x_1}{\text{mean}} \Phi(x_1, y_1) \\ &= \underset{x_2}{\text{sum}} \Phi(x_2, y_2) \, \Psi(x_1, x_2) \\ &= \underset{x_3}{\text{sum}} \Phi(x_3, y_3) \, \Psi(x_2, x_3) \, \, \Psi(x_1, x_3) \end{aligned}$$



#### Justification for running belief propagation in networks with loops

- Experimental results:
  - Error-correcting codes Kschischang and Frey, 1998;

McEliece et al., 1998

Freeman and Pasztor, 1999; - Vision applications

Frey, 2000

- Theoretical results:
  - For Gaussian processes, means are correct. 1999
  - Large neighborhood local maximum for MAP.
  - Weiss and Freeman, 200

     Equivalent to Bethe approx. in statistical physics.
  - Tree-weighted reparameterization

Wainwright, Willsky, Jaakkola, 200

#### Statistical mechanics interpretation

U - TS = Free energy

$$U = \text{avg. energy} = \sum_{\text{states}} p(x_1, x_2, ...) E(x_1, x_2, ...)$$

$$T = \text{temperature}$$

$$S = \text{entropy} = -\sum_{\text{states}} p(x_1, x_2, ...) \ln p(x_1, x_2, ...)$$

#### Free energy formulation

Defining

$$\Psi_{ii}(x_i, x_j) = e^{-E(x_i, x_j)/T}$$
  $\Phi_i(x_i) = e^{-E(x_i)/T}$ 

then the probability distribution  $P(x_1, x_2,...)$ 

that minimizes the F.E. is precisely

the true probability of the Markov network,

$$P(x_1, x_2, ...) = \prod_{ij} \Psi_{ij}(x_i, x_j) \prod_i \Phi_i(x_i)$$

# Approximating the Free Energy

 $F[p(x_1, x_2, ..., x_N)]$  $F[b_i(x_i)]$ Mean Field Theory:

 $F[b_i(x_i),b_{ii}(x_i,x_i)]$ Bethe Approximation:

Kikuchi Approximations:

 $F[b_i(x_i), b_{ij}(x_i, x_j), b_{ijk}(x_i, x_j, x_k),...]$ 

Mean field approximation to free energy

U - TS = Free energy

$$F_{MeanField}(b_i) = \sum_{(ij)} \sum_{x_i, x_j} b_i(x_i) b_j(x_j) E_{ij}(x_i, x_j) + \sum_i \sum_{x_i} b_i(x_i) T \ln b_i(x_i)$$

The variational free energy is, up to an additive constant, equal to the Kllback-Leibler divergence between b(x) and the true probability, P(x). KL divergence:

 $D_{KL}(b \parallel P) = \sum_{x_i, x_i, \dots} \prod_i b_i(x) \ln \frac{\prod_i b_i(x)}{P(x)}$ 

# Setting deriv w.r.t b<sub>i</sub>=0

U - TS = Free energy

Corresponds to eq. 18 in Jordan and Weiss ms

$$b_i(x_i) = \alpha \exp(-\sum_{(ij)} \sum_{x_j} b_j(x_j) E_{ij}(x_i, x_j) / T)$$

In words: "Set the probability of each state  $x_i$  at node i to be proportional to e to the minus expected energy corresponding to each state  $x_i$ , given the expected values of all the neighboring states."

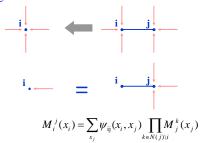
# Region marginal probabilities

$$b_{i}(x_{i}) = k \Phi(x_{i}) \prod_{k \in N(i)} M_{i}^{k}(x_{i})$$

$$b_{ij}(x_{i}, x_{j}) = k \Psi(x_{i}, x_{j}) \prod_{k \in N(i) \setminus j} M_{i}^{k}(x_{i}) \prod_{k \in N(j) \setminus i} M_{j}^{k}(x_{j})$$

## Belief propagation equations

Belief propagation equations come from the marginalization constraints.



#### Results from Bethe free energy analysis

- Fixed point of belief propagation equations iff. Bethe approximation stationary point.
- · Belief propagation always has a fixed point.
- Connection with variational methods for inference: both minimize approximations to Free Energy,
  - variational: usually use primal variables.
  - **belief propagation**: fixed pt. equs. for dual variables.
- Kikuchi approximations lead to more accurate belief propagation algorithms.
- Other Bethe free energy minimization algorithms— Yuille, Welling, etc.

#### References on BP and GBP

- J. Pearl, 1985
  - classic
- Y. Weiss, NIPS 1998
  - Inspires application of BP to vision
- W. Freeman et al learning low-level vision, IJCV 1999
   Applications in super-resolution, motion, shading/paint discrimination
- H. Shum et al, ECCV 2002
  - Application to stereo
- M. Wainwright, T. Jaakkola, A. Willsky
  - Reparameterization version
- J. Yedidia, AAAI 2000
  - The clearest place to read about BP and GBP.

Show program comparing some methods on a simple MRF

testMRF.m

## Graph cuts

- Algorithm: uses node label swaps or expansions as moves in the algorithm to reduce the energy.
   Swaps many labels at once, not just one at a time, as with ICM.
- Find which pixel labels to swap using min cut/max flow algorithms from network theory.
- Can offer bounds on optimality.
- See Boykov, Veksler, Zabih, IEEE PAMI 23 (11)
   Nov. 2001 (available on web).

# Comparison of graph cuts and belief propagation

Comparison of Graph Cuts with Belief Propagation for Stereo, using Identical MRF Parameters, ICCV 2003. Marshall F. Tappen William T. Freeman









(a) Tsukuba Imae

(b) Graph Cuts

) Synchronous BP

(d) Accelerated BP

Figure 3. Results produced by the three algorithms on the Tsukuba image. The parameters used to generate this field were  $\mathbf{s}=50, T=4, P=2$ . Again, Graph Cuts produces a much smoother solution. Belief Propagation does maintain some structures that are lost in the Graph Cuts solution, such as the camera and the face in the foreground.

# Ground truth, graph cuts, and belief propagation disparity solution energies

	Energy of MRF Labelling Returned (×10 <sup>3</sup> )			
Image	Ground-Truth	Graph Cuts	Synchronous Belief Prop	% Energy from Occluded Matching Costs
Map	757	383	442	61%
Sawtooth	6591	1652	1713	79%
Tsukuba	1852	663	775	61%
Morrow	5730	1442	1501	7696

Figure 2. Field Energies for the MRF labelled using ground-truth data compared to the energies for the fields labelled using Graph Cuts and Bellef Propagation. Notice that the solutions returned by the algorithms consistently have a much lower energy than the labellings produced from the groundtruth, showing a mismatch between the MRF formulation and the ground-truth. The final column contains the percentage of each ground-truth solution's energy that comes from matching costs of

#### Graph cuts versus belief propagation

- Graph cuts consistently gave slightly lower energy solutions for that stereo-problem MRF, although BP ran faster, although there is now a faster graph cuts implementation than what we used...
- However, here's why I still use Belief Propagation:
  - Works for any compatibility functions, not a restricted set like graph cuts.
  - I find it very intuitive.
  - Extensions: sum-product algorithm computes MMSE, and Generalized Belief Propagation gives you very accurate solutions, at a cost of time.

#### MAP versus MMSE





(a) MAP Estimate

(b) MMSE Estimate

Figure 7. Comparison of MAP and MMSE estimates on a different MRF formulation. The MAP estimate chooses the most likely discrete disparity level for each point, resulting in a depth-map with stair-stepping effects. Using the MMSE estimate assigns sub-pixel disparities, resulting in a smooth depth map.

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#### Joint probabilities for undirected graphs



$$P(x_1, x_2, x_3) = P(x_1, x_3 \mid x_2) P(x_2)$$
 By elementary probability

Use the conditional  $= P(x_1 \mid x_2)P(x_3 \mid x_2)P(x_2)$ independence assumption

Multiply top and bottom by P(x2)

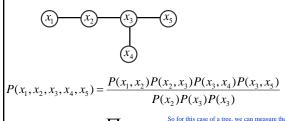
$$= \frac{P(x_1 \mid x_2)P(x_2)P(x_3 \mid x_2)P(x_2)}{P(x_2)}$$

Re-write conditionals as joint probabilities

$$= \frac{P(x_1, x_2)P(x_2, x_3)}{P(x_2)}$$

General result for separating clique x2

#### More complicated graph

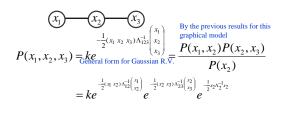


$$P(x_1, x_2, x_3, x_4, x_5) = \frac{P(x_1, x_2)P(x_2, x_3)P(x_3, x_4)P(x_3, x_5)}{P(x_2)P(x_3)P(x_3)}$$

$$= \prod_{(ij)} \phi(x_i, x_j)$$

 $=\prod_{(ij)}\phi(x_i,x_j) \begin{array}{l} \text{So for this case of a tree, we can measure the compatibility functions by measuring the joint statistics of neighboring nodes. For graphs with loops, we can use these functions as starting points for an iterative method (IPF) that handles the loops properly.} \end{array}$ 

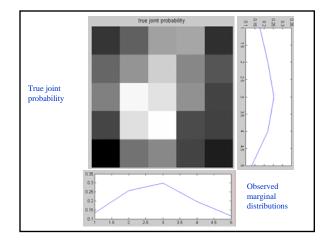
#### For jointly Gaussian random variables

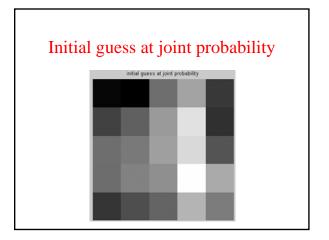


$$\Lambda_{123}^{-1} = \begin{pmatrix} a & b & 0 \\ b & c & d \\ 0 & d & e \end{pmatrix} \qquad \begin{array}{c} \text{Thus, for this graphical model,} \\ \text{the inverse covariance has this} \\ \text{particular structure.} \end{array}$$

#### Learning MRF parameters, labeled data

Iterative proportional fitting lets you make a maximum likelihood estimate of a joint distribution from observations of various marginal distributions.





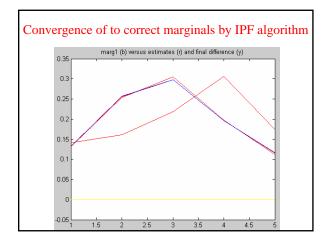
# IPF update equation

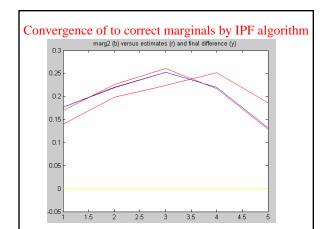
$$P(x_1, x_2, \dots, x_d)^{(t+1)} = P(x_1, x_2, \dots, x_d)^{(t)} \frac{P(x_i)^{\text{observed}}}{P(x_i)^{(t)}}$$

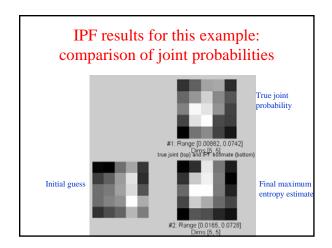
Scale the previous iteration's estimate for the joint probability by the ratio of the true to the predicted marginals.

Gives gradient ascent in the likelihood of the joint probability, given the observations of the marginals.

See: Michael Jordan's book on graphical models







# Application to MRF parameter estimation

 Can show that for the ML estimate of the clique potentials, φ<sub>c</sub>(x<sub>c</sub>), the empirical marginals equal the model marginals,

$$\tilde{p}(x_c) = p(x_c)$$

• This leads to the IPF update rule for  $\varphi_c(\boldsymbol{x}_c)$ 

$$\phi_C^{(t+1)}(x_c) = \phi_c^{(t)}(x_c) \frac{\tilde{p}(x_c)}{p^{(t)}(x_c)}$$

• Performs coordinate ascent in the likelihood of the MRF parameters, given the observed data.

Reference: unpublished notes by Michael Jordan

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- Vision applications of inference in MRF's.

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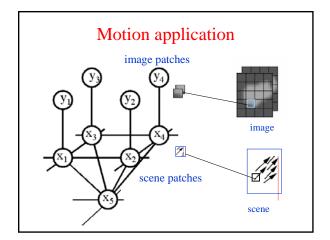
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# Vision applications of MRF's

- Stereo
- Motion estimation
- Super-resolution
- Many others...

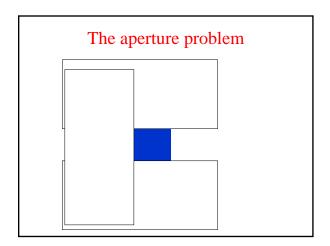
# Vision applications of MRF's

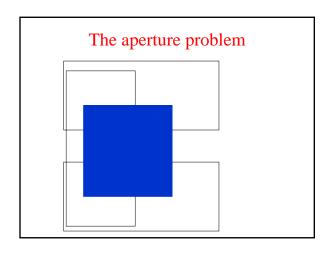
- Stereo
- Motion estimation
- Super-resolution
- Many others...

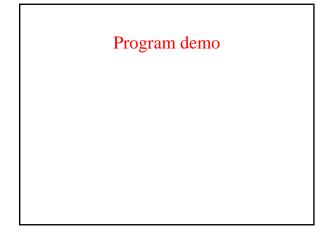


# What behavior should we see in a motion algorithm?

- Aperture problem
- Resolution through propagation of information
- Figure/ground discrimination

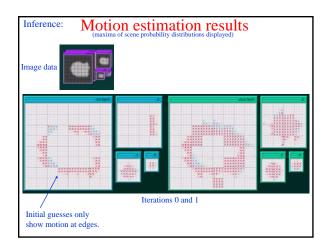


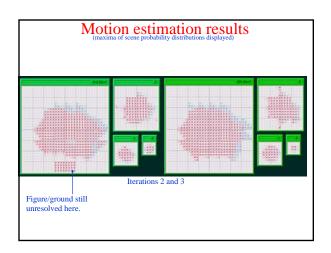


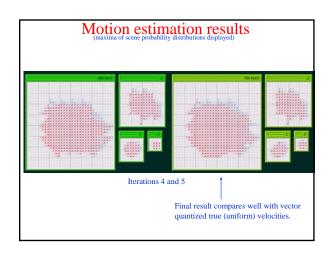


# Motion analysis: related work

- · Markov network
  - Luettgen, Karl, Willsky and collaborators.
- Neural network or learning-based
  - Nowlan & T. J. Senjowski; Sereno.
- · Optical flow analysis
  - Weiss & Adelson; Darrell & Pentland; Ju,
     Black & Jepson; Simoncelli; Grzywacz &
     Yuille; Hildreth; Horn & Schunk; etc.







# Vision applications of MRF's

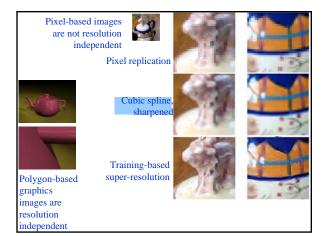
- Stereo
- Motion estimation
- Super-resolution
- Many others...

# Super-resolution

• Image: low resolution image

• Scene: high resolution image





# 3 approaches to perceptual sharpening

(1) Sharpening; boost existing high frequencies.



(2) Use multiple frames to obtain higher sampling rate in a still frame

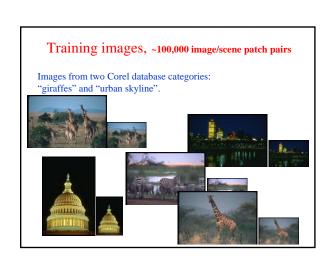
(3) Estimate high frequencies not present in image, although implicitly defined.

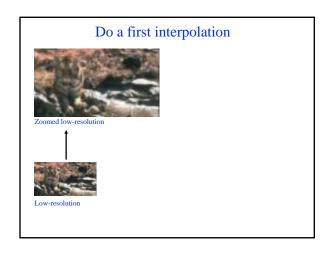
In this talk, we focus on (3), which

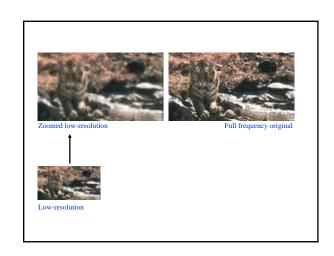
# we'll call "super-resolution".

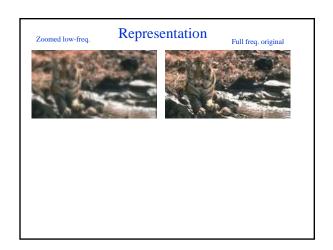
# Super-resolution: other approaches

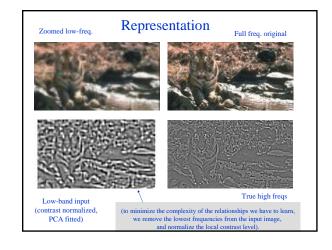
- Schultz and Stevenson, 1994
- Pentland and Horowitz, 1993
- fractal image compression (Polvere, 1998; Iterated Systems)
- astronomical image processing (eg. Gull and Daniell, 1978; "pixons" http://casswww.ucsd.edu/puetter.html)

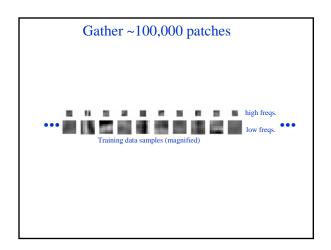


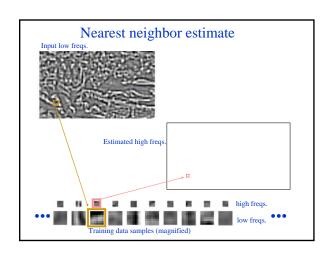


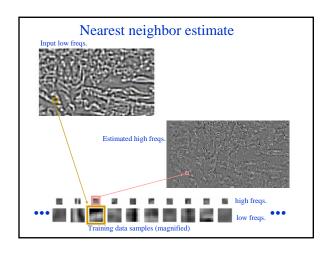


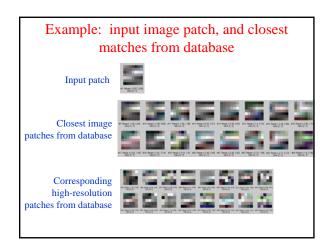


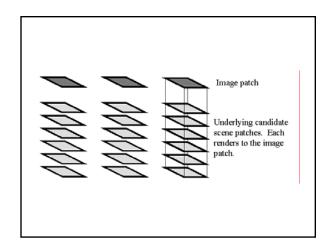


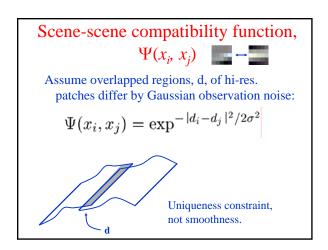


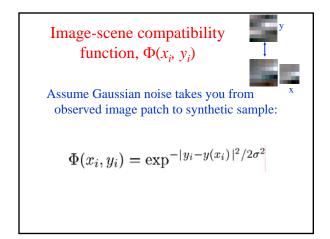


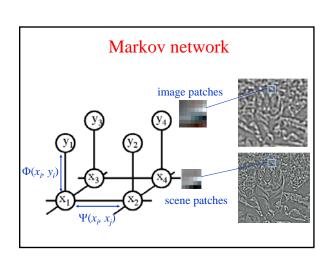


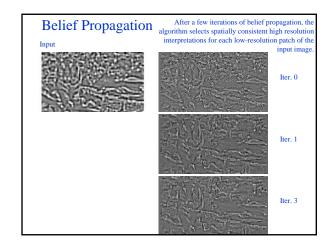


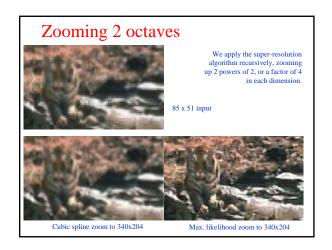


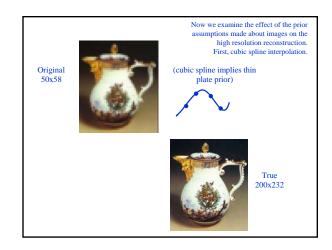


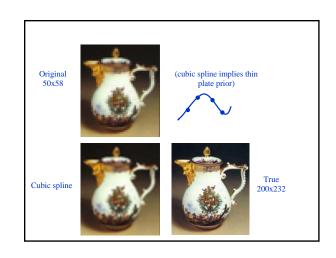


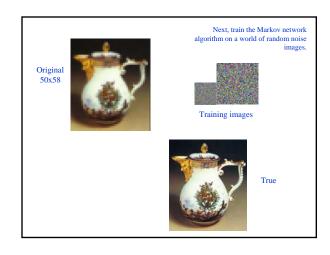


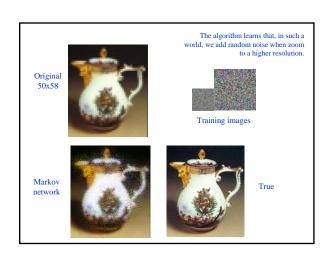


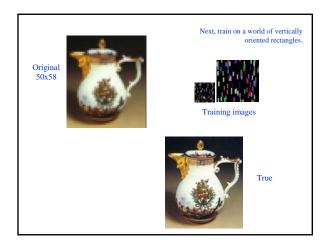


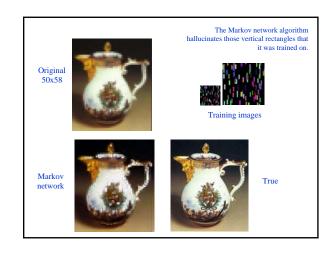




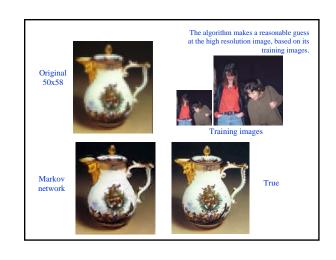


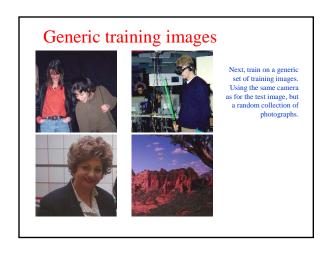


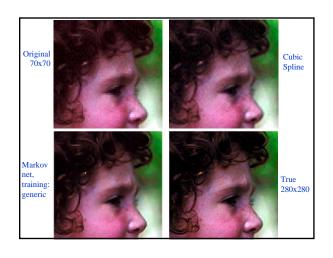












#### Kodak Imaging Science Technology Lab test.



3 test images, 640x480, to be zoomed up by 4 in each dimension.

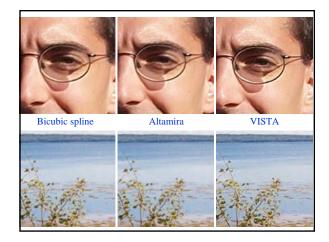
8 judges, making 2-alternative, forced-choice comparisons.





# Algorithms compared

- Bicubic Interpolation
- Mitra's Directional Filter
- Fuzzy Logic Filter
- Vector Quantization
- VISTA





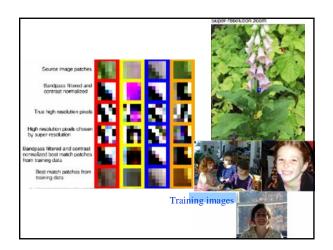
# User preference test results

"The observer data indicates that six of the observers ranked Freeman's algorithm as the most preferred of the five tested algorithms. However the other two observers rank Freeman's algorithm as the least preferred of all the algorithms....

Freeman's algorithm produces prints which are by far the sharpest out of the five algorithms. However, this sharpness comes at a price of artifacts (spurious detail that is not present in the original scene). Apparently the two observers who did not prefer Freeman's algorithm had strong objections to the artifacts. The other observers apparently placed high priority on the high level of sharpness in the images created by Freeman's algorithm."

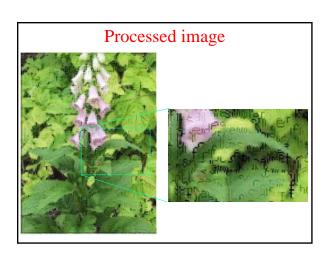






# Training image

angillegallyprended,or com anelvacatedarulingbythefer ystem,andsentitdowntoanew finedastandardforweighing eraproduct-bundlingdecisis softsaysthatthenewfeature: andpersonalidentification: psoft'sview,butusersandthicatedwithconsumerinnovation rePCindustryislookingforw



# More general graphical models than MRF grids

- In this course, we've studied Markov chains, and Markov random fields, but, of course, many other structures of probabilistic models are possible and useful in computer vision.
- For a nice on-line tutorial about Bayes nets, see Kevin Murphy's tutorial in his web page.

