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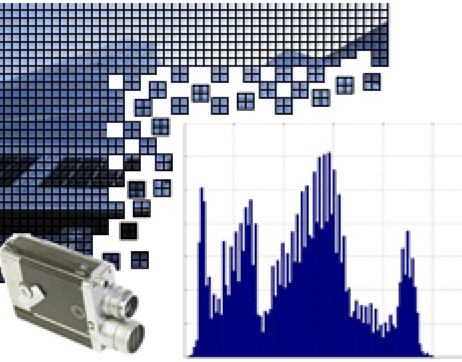

6.869 Advances in Computer Vision: Learning and Interfaces

Spring 2005

Tuesday and Thursday; 2:30 to 4:00pm in 36-153

Announcements

Course Information

- Syllabus
- Problem Sets and Exams
- Grading and Requirements
- Internet Resources

Contacts

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Contacts

Instructor	Professor William T. Freeman billf at mit dot edu 32-D476 (617) 253- 8828
Office Hours	By Appointment
Teaching Assistant	Xiaoxu Ma xiaoxuma at mit dot edu 32-D542 (617) 258-5485
Office Hours	Monday, Wed. 4-5pm in 32-D451

All offices are located on the fourth and fifth floor of the Dreyfoos building (Stata Center).

If you cannot attend our normally scheduled office hours, please send e-mail to schedule an alternate appointment.

Administration

- Syllabus
- Grading
- Collaboration Policy
- Project

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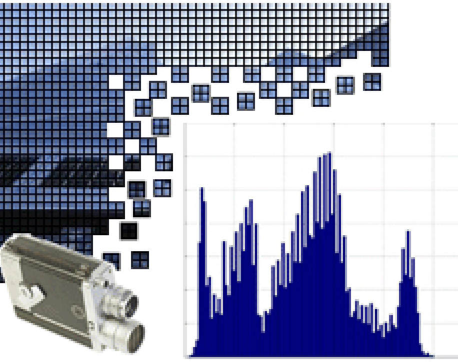

6.869 Advances in Computer Vision: Learning and Interfaces

Spring 2005

Syllabus

The topics studied in this course will include:

- Image statistics, image representations, and texture models
- Color Vision
- Graphical models, Bayesian methods
- Markov Random Fields, applications to low-level vision
- Approximate inference methods
- Statistical classifiers
- Clustering & Segmentation
- Object recognition
- Tracking and Density Propagation
- Visual Surveillance and Activity Monitoring

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Course Calendar

Lecture	Date	Description	Readings	Assignments	Materials
1	2/1	Course Introduction Cameras and Lenses	Req: FP 1.1, 2.1, 2.2, 2.3, 3.1, 3.2	PS0 out	
2	2/3	Image Filtering	Req: FP 7.1 - 7.6		
3	2/8	Image Representations: Pyramids	Req: FP 7.7, 9.2		
4	2/10	Image Statistics		PS0 due	
5	2/15	Texture	Req: FP 9.1, 9.3, 9.4	PS1 out	
6	2/17	Color	Req: FP 6.1-6.4		
7	2/22	Guest Lecture: Context in vision			
8	2/24	Guest Lecture: Medical Imaging		PS1 due	
9	3/1	Multiview Geometry	Req: Mikolajczyk and Schmid; FP 10	PS2 out	
10	3/3	Local Features	Req: Shi and Tomasi; Lowe		

11	3/8	Bayesian Analysis			
12	3/10	Markov Random Fields Belief Propagation		PS2 due	
13	3/15	Model Based Recognition	Req: FP 18.1-18.5, Lowe	EX1 out	
14	3/17	Discriminative Models		EX1 due	
	3/22-3/24	Spring Break (NO LECTURE)			
15	3/29	Face Detection and Recognition I	Req: FP 22		
16	3/31	Face Detection and Recognition II		Project proposal due	
17	4/5	Segmentation and Clustering	Req: FP 14, 15.1-15.2, Comaniciu and Meer	PS3 out	
18	4/7	Segmentation and Fitting	Req: FP 15.3-15.5, 16		
19	4/12	Tracking I	Req: FP 17		
20	4/14	Articulated Tracking and Shape Inference	Req: FP Extra Chapter	PS3 due	
	4/19	No class (Patriot's Day Holiday)			
21	4/21	Approximate Inference Methods		PS4 out	

Course requirements

- Two take-home exams
- Five problem sets with lab exercises in Matlab
- No final exam
- Final project

Grading

- Problem sets are graded check, check-plus, check-minus
- Contribution to grade:
  - 5 problem sets: 30 %
  - 2 take-home exams: 40%
  - final project: 30%

Collaboration Policy

Problem sets may be discussed, but all written work and coding must be done individually. Take-home exams may not be discussed. Individuals found submitting duplicate or substantially similar materials due to inappropriate collaboration may get an F in this class and other sanctions.

Project

The final project may be

- An original implementation of a new or published idea
- A detailed empirical evaluation of an existing implementation of one or more methods
- A paper comparing three or more papers not covered in class, or surveying recent literature in a particular area

A project proposal not longer than two pages must be submitted and approved by April 1st. I can provide ideas or suggestions for projects.

Problem Set 0

- Out today, due 2/12
- Matlab image exercises
  - load, display images
  - pixel manipulation
  - RGB color interpolation
  - image warping / morphing with `interp2`
  - simple background subtraction
- All psets graded loosely: *check*, *check*-, *0*.
- (Outstanding solutions get extra credit.)

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William T. Freeman

William T. Freeman
Associate Professor
Dept. of Electrical Engineering and Computer Science
Massachusetts Institute of Technology

Computer Science and Artificial Intelligence Laboratory
Stata Center , D32-476
(maps for building or office).
Cambridge, MA 02139

Phone: 617-253-8828
Email: billf at mit dot edu

Publications: **Selected, grouped by topic**

All publications

Patents

Biography, CV, and Research Statement

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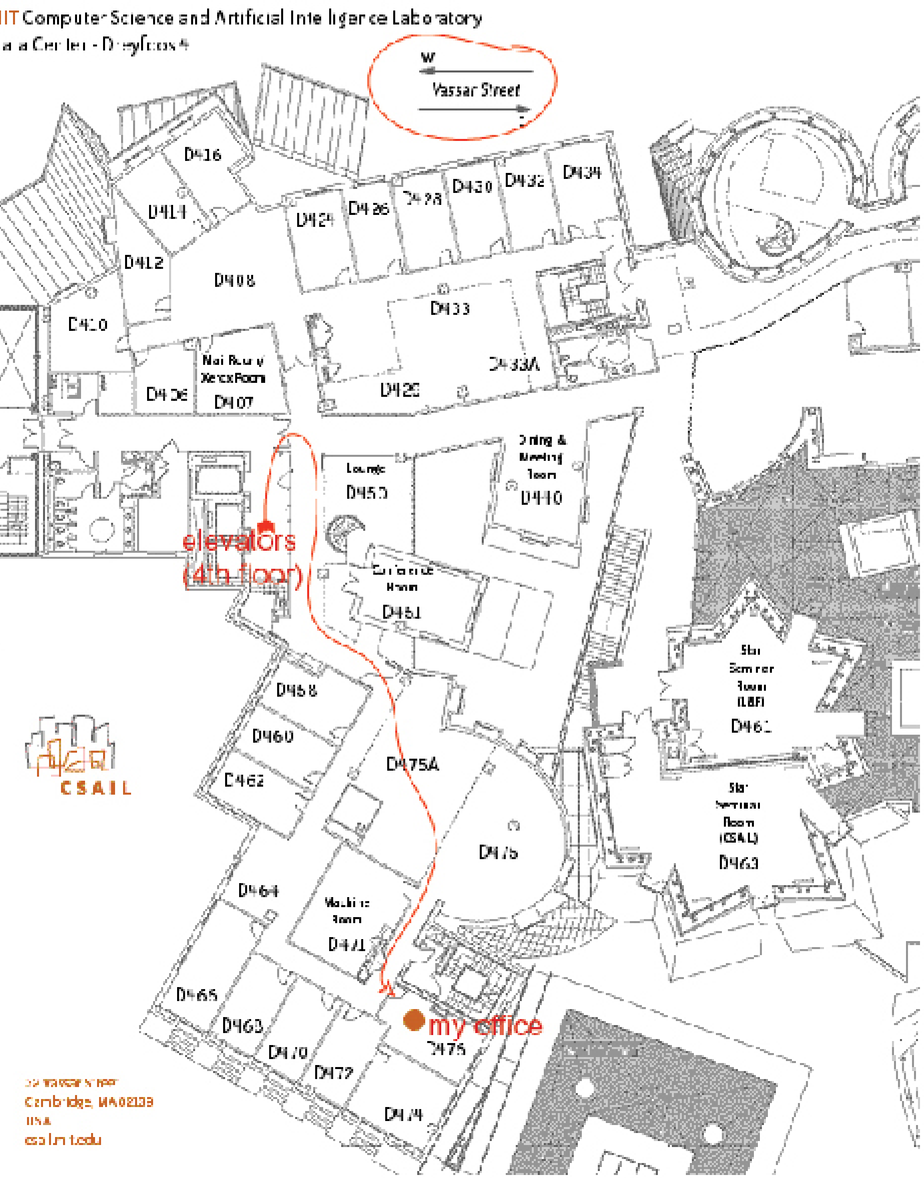
http://people.csail.mit.edu/people/billf/images/D476.jpg



Vision

- What does it mean, to see? "to know what is where by looking".
- How to discover from images what is present in the world, where things are, what actions are taking place.

from Marr, 1982

Vision

- What does it mean, to see? "to know what is where by looking".
- How to discover from images what is present in the world, where things are, what actions are taking place.

from Marr, 1982

Why study Computer Vision?

- One can "predict the future" (and avoid bad things…)!
- Images and movies are everywhere; fast-growing collection of useful applications
  - building representations of the 3D world from pictures
  - automated surveillance (who's doing what)
  - movie post-processing
  - face finding
- Greater understanding of human vision
- Various scientific questions
  - how does object recognition work?

What is object recognition?

- People draw distinctions between what is seen
  - This could mean “is this a fish or a bicycle?”
  - It could mean “is this George Washington?”
  - It could mean “is this poisonous or not?”
  - It could mean “is this slippery or not?”
  - It could mean “will this support my weight?”
  - Area of research:
    - How to build programs that can draw useful distinctions based on image properties.

The course, in broad categories

- Images and image formation
- Low-level vision
- High-level vision
- Implementations and applications

Computer vision class, fast-forward

Images and image formation

Cameras, lenses, and sensors

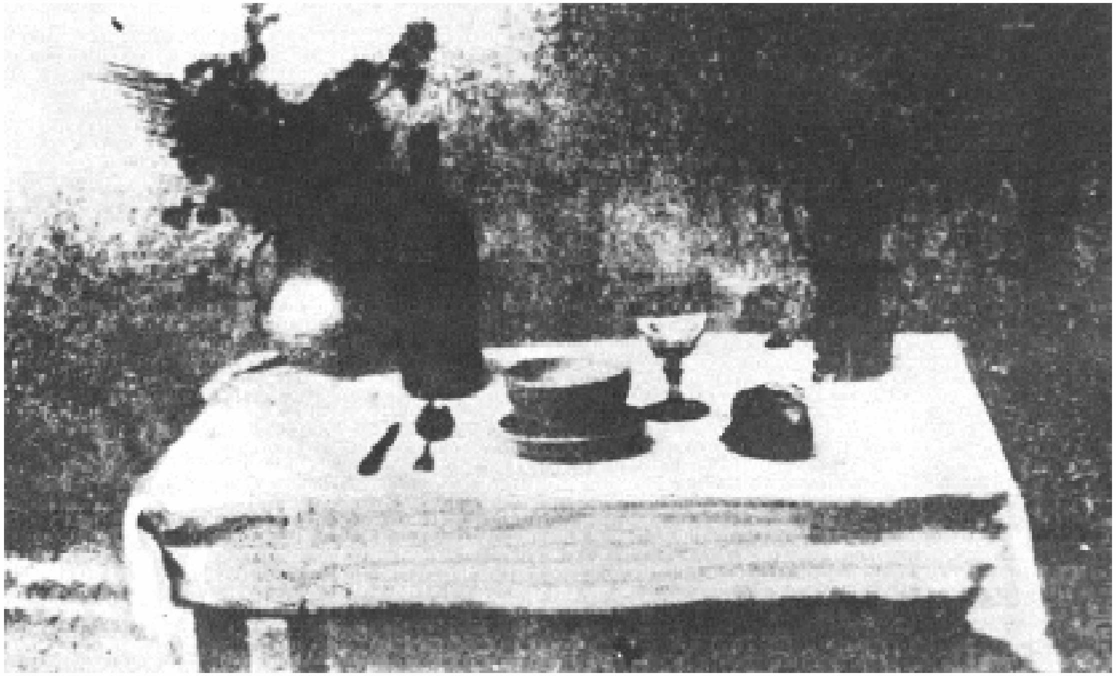
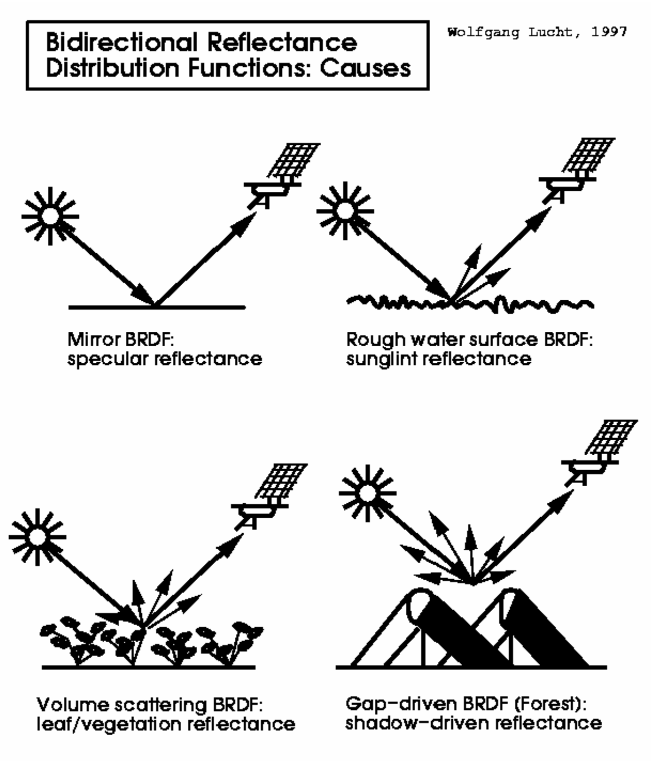


Figure 1.16 The first photograph on record, *la table servie*, obtained by Nicéphore Niepce in 1822. *Collection Harlinge–Viollet.*

- Pinhole cameras
- Lenses
- Projection models
- Geometric camera parameters

From Computer Vision, Forsyth and Ponce, Prentice-Hall, 2002.

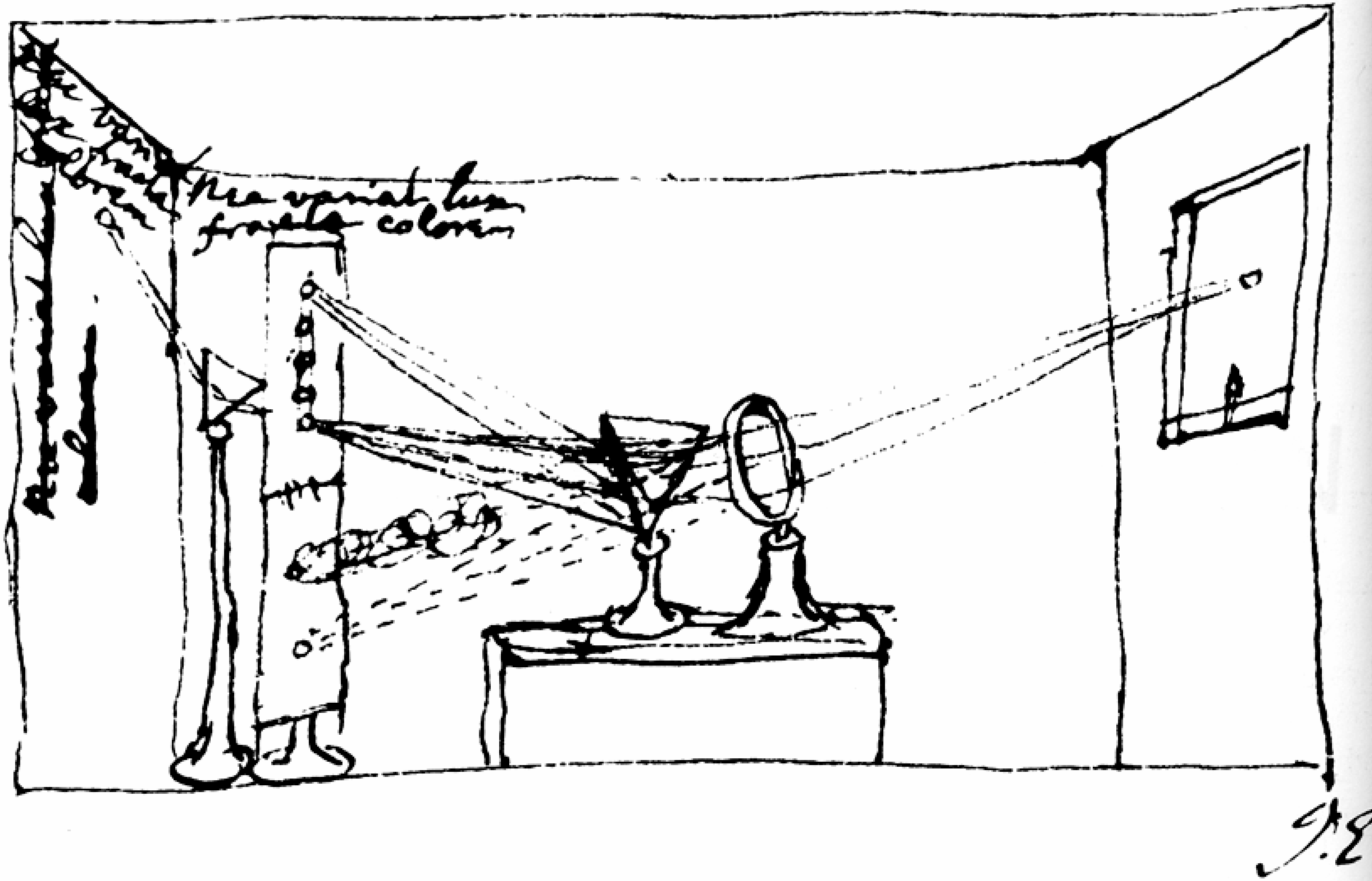
Radiometry…not covered (see 6.801)



Wolfgang Lucht

http://geography.bu.edu/brdf/brdfexpl.html

Color



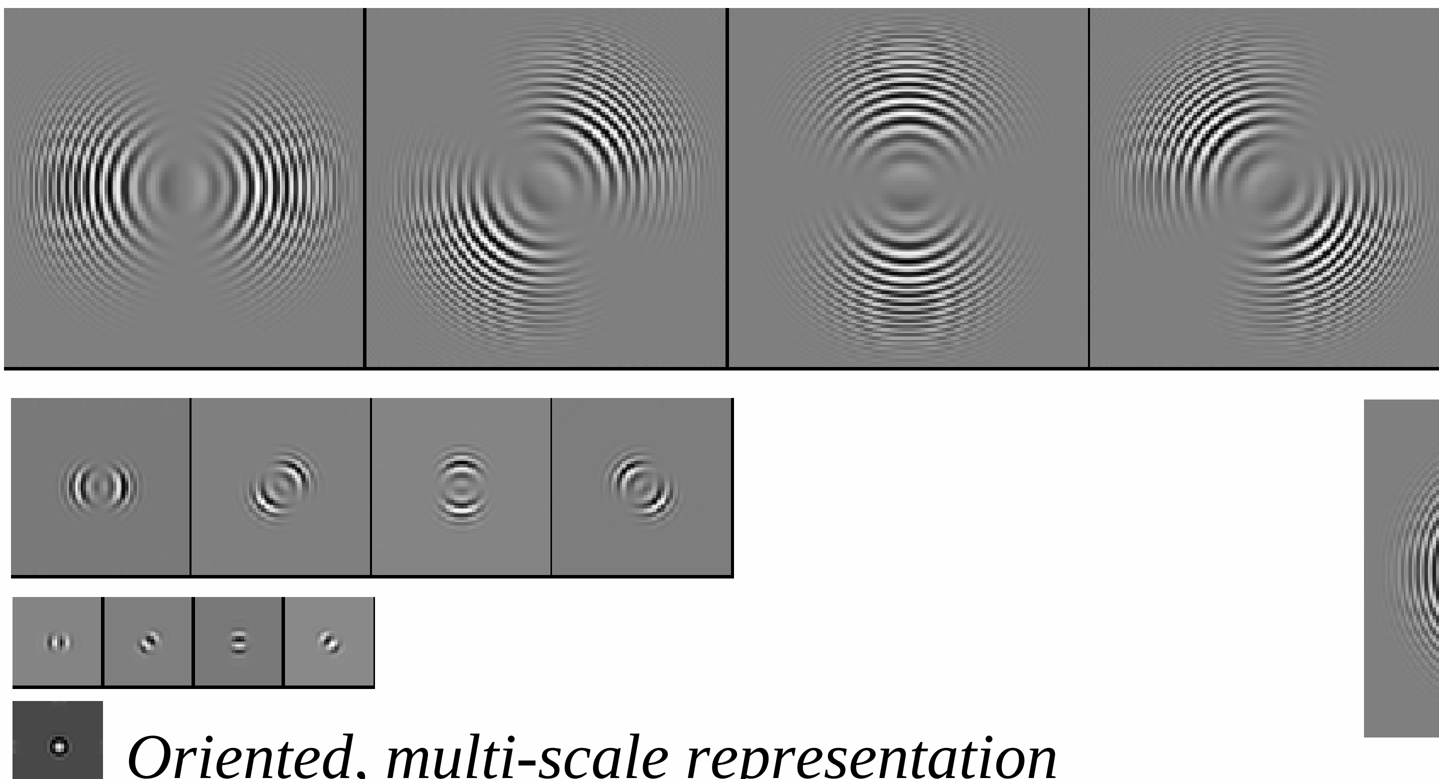
4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

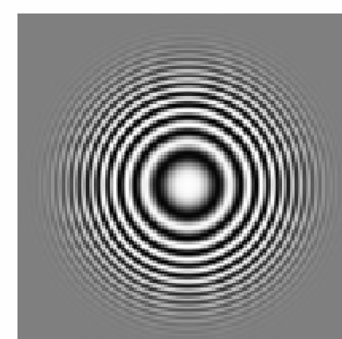
Low-level vision

Image filtering

- Review of linear systems, convolution
- Bandpass filter-based image representations
- Probabilistic models for images

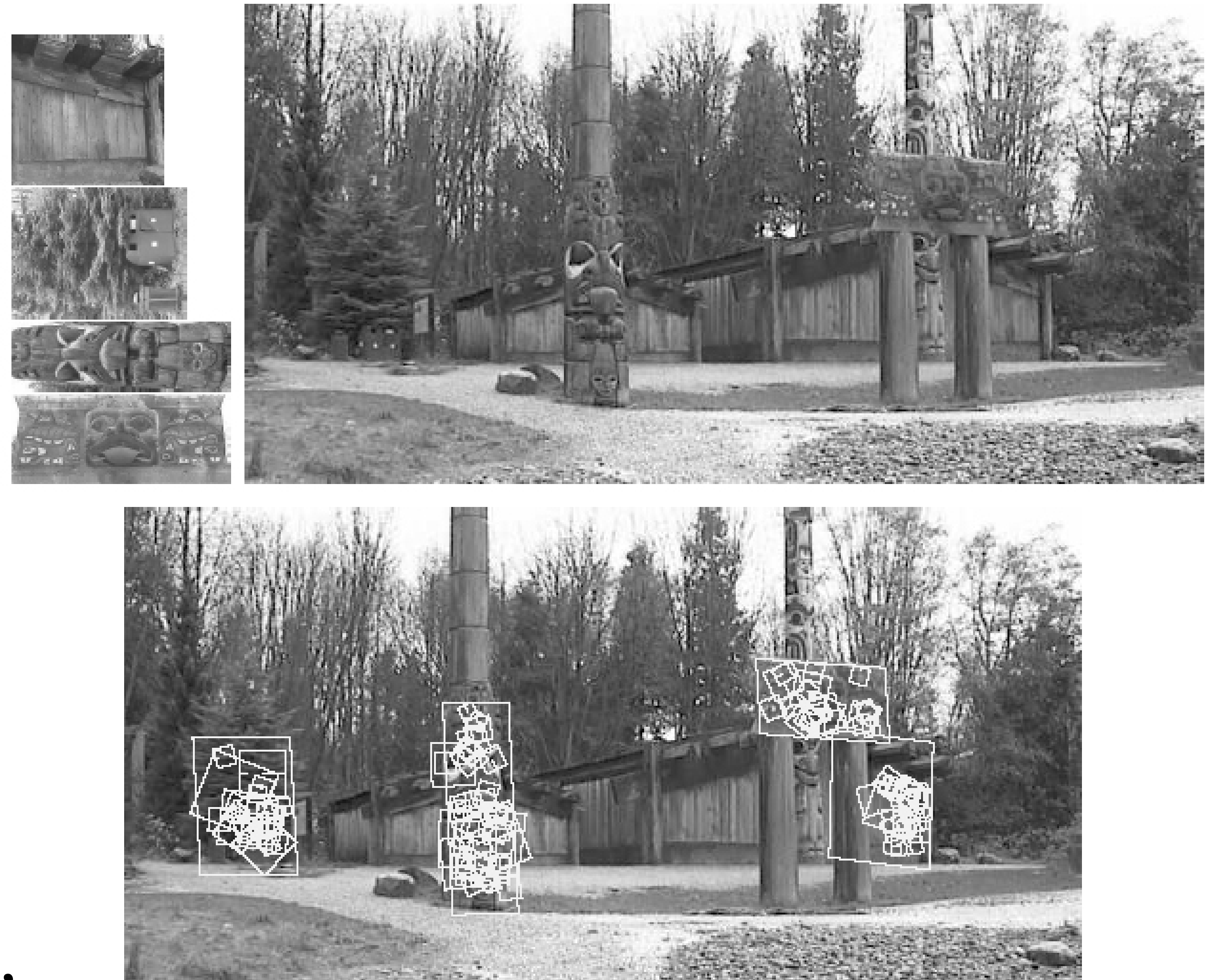


Oriented, multi-scale representation



Image

SIFT (scale invariant feature transforms)

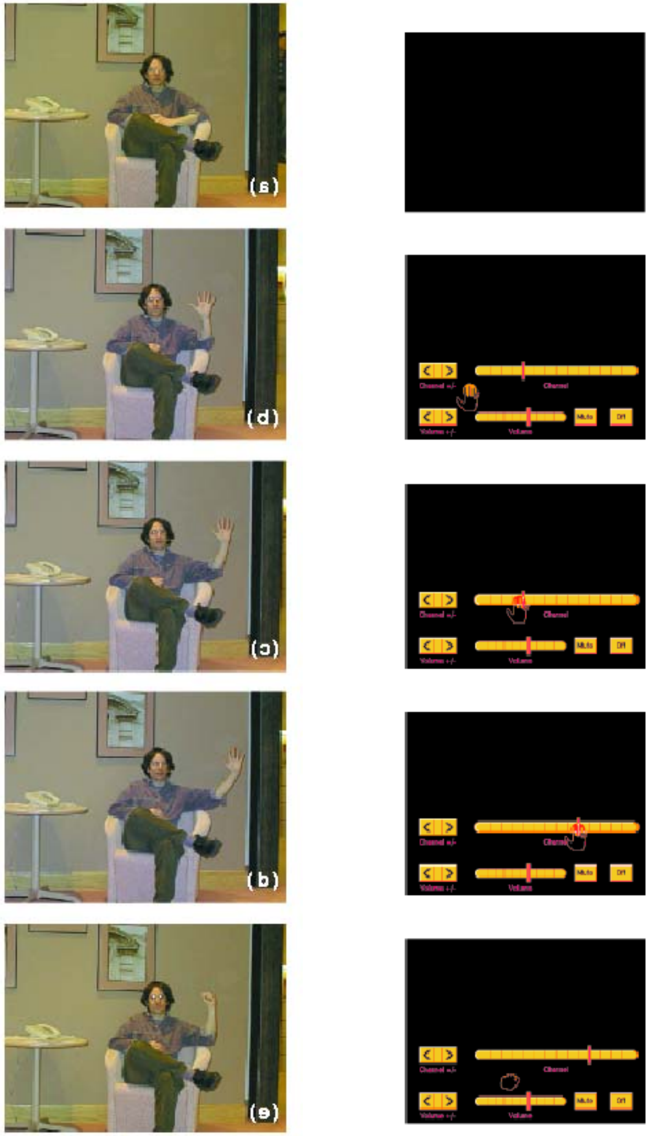


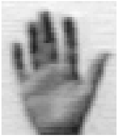
David Lowe,
IJCV 2004

Figure 13: This example shows location recognition within a complex scene. The training images for locations are shown at the upper left and the 640x315 pixel test image taken from a different viewpoint is on the upper right. The recognized regions are shown on the lower image, with keypoints shown as squares and an outer parallelogram showing the boundaries of the training images under the affi ne transform used for recognition.

Non-linear filtering, and applications

viewer **television display**





template

image

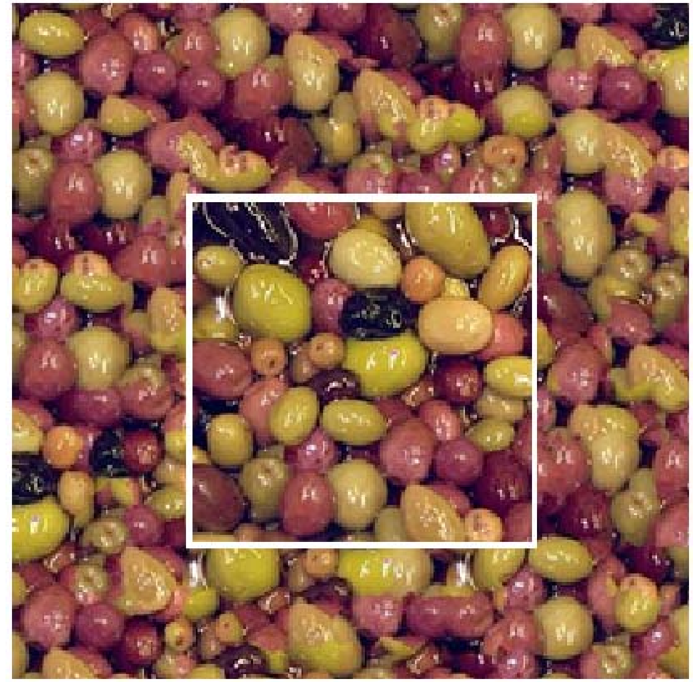
Normalized correlation

12 Sample session of television viewing. (a) Television is off, but searching for the trigger gesture. (b) Viewer shows trigger gesture (open hand). Television set turns on and hand icon and graphics overlays appear. (c) The hand icon tracks the user's hand movement. User changes controls as with a mouse. (d) User has moved hand icon to change channel. (e) User closes hand to leave control mode. After one second, the hand icon and controls then disappear.

IEEE Computer Graphics and Applications, 18, no. 3, 1998

Models of texture

Parametric model



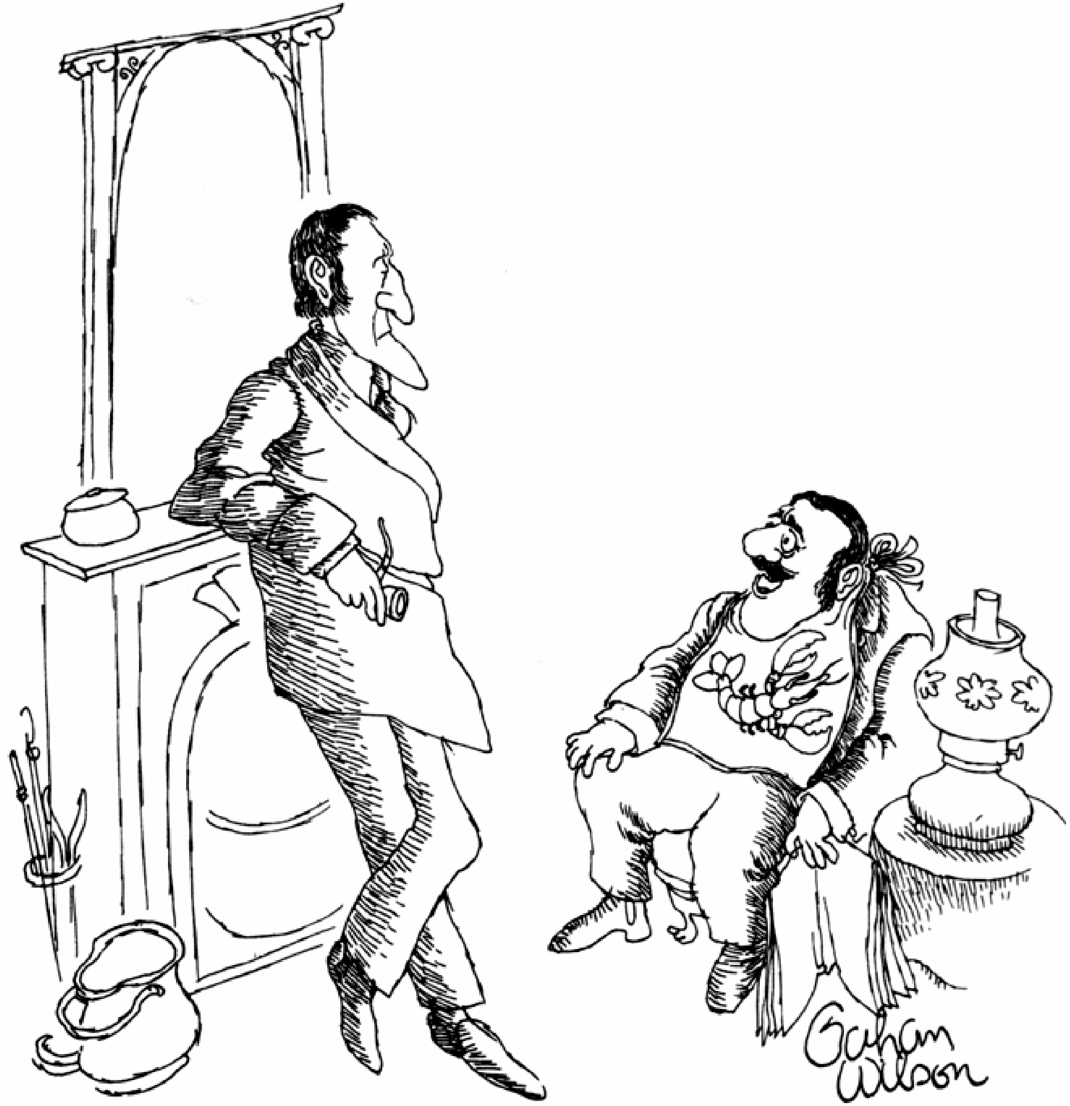
Non-parametric model

A Parametric Texture Model based on Joint Statistics of Complex Wavelet Coefficients
J. Portilla and E. Simoncelli, *International Journal of Computer Vision 40(1): 49-71, October 2000.*

A. Efros and W. T Freeman, **Image quilting for texture synthesis and transfer**, SIGGRAPH 2001

Learning and vision

Bayesian framework for vision



"Good lord, Holmes! How did you come to know I'd seafood for lunch?"

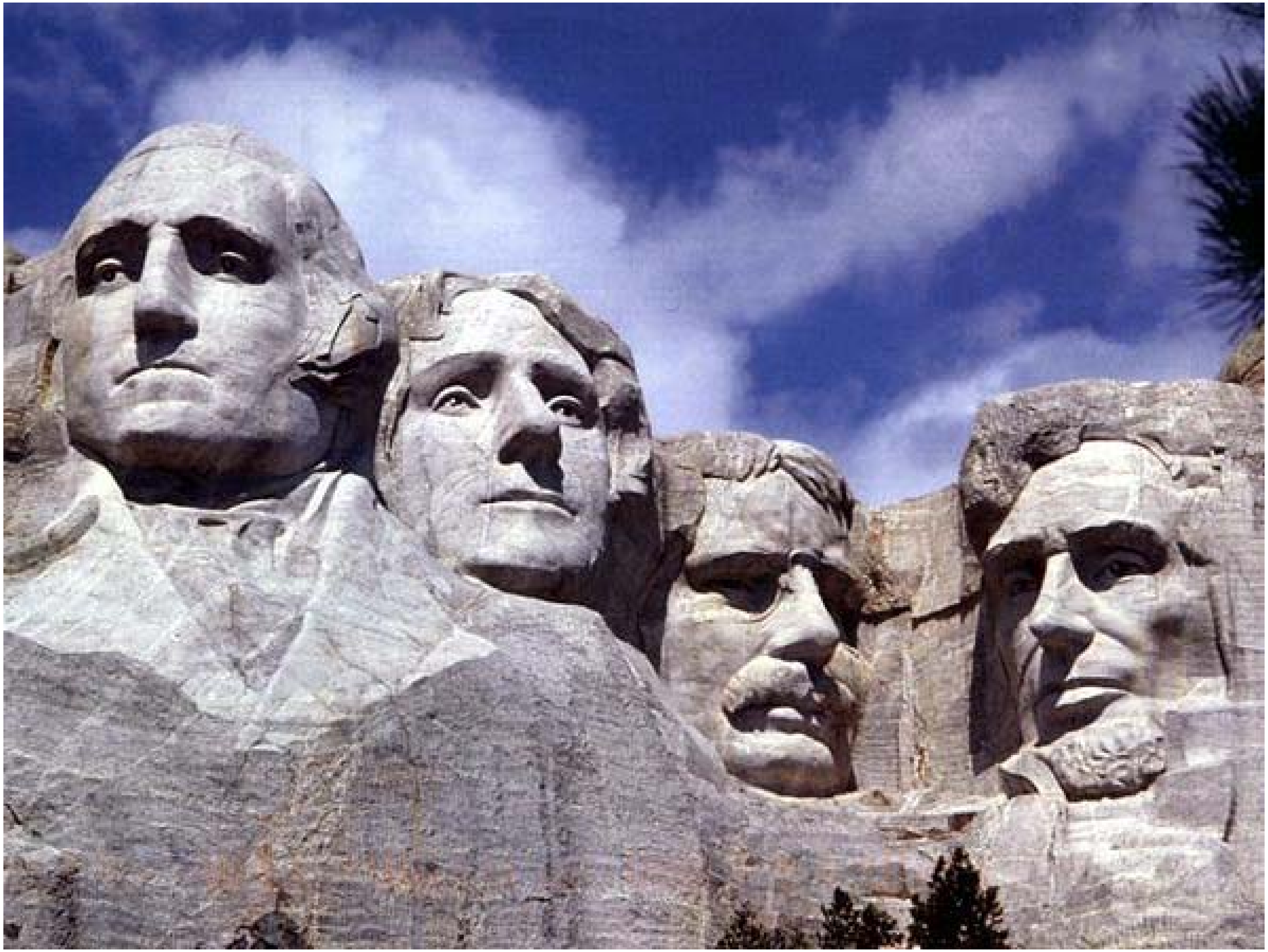
Gahan Wilson's Still Weird, Forge, 1994

Bayesian framework for vision

Coincidental appearance of face profile in rock?

http://www.cs.dartmouth.edu/whites/old_man.html

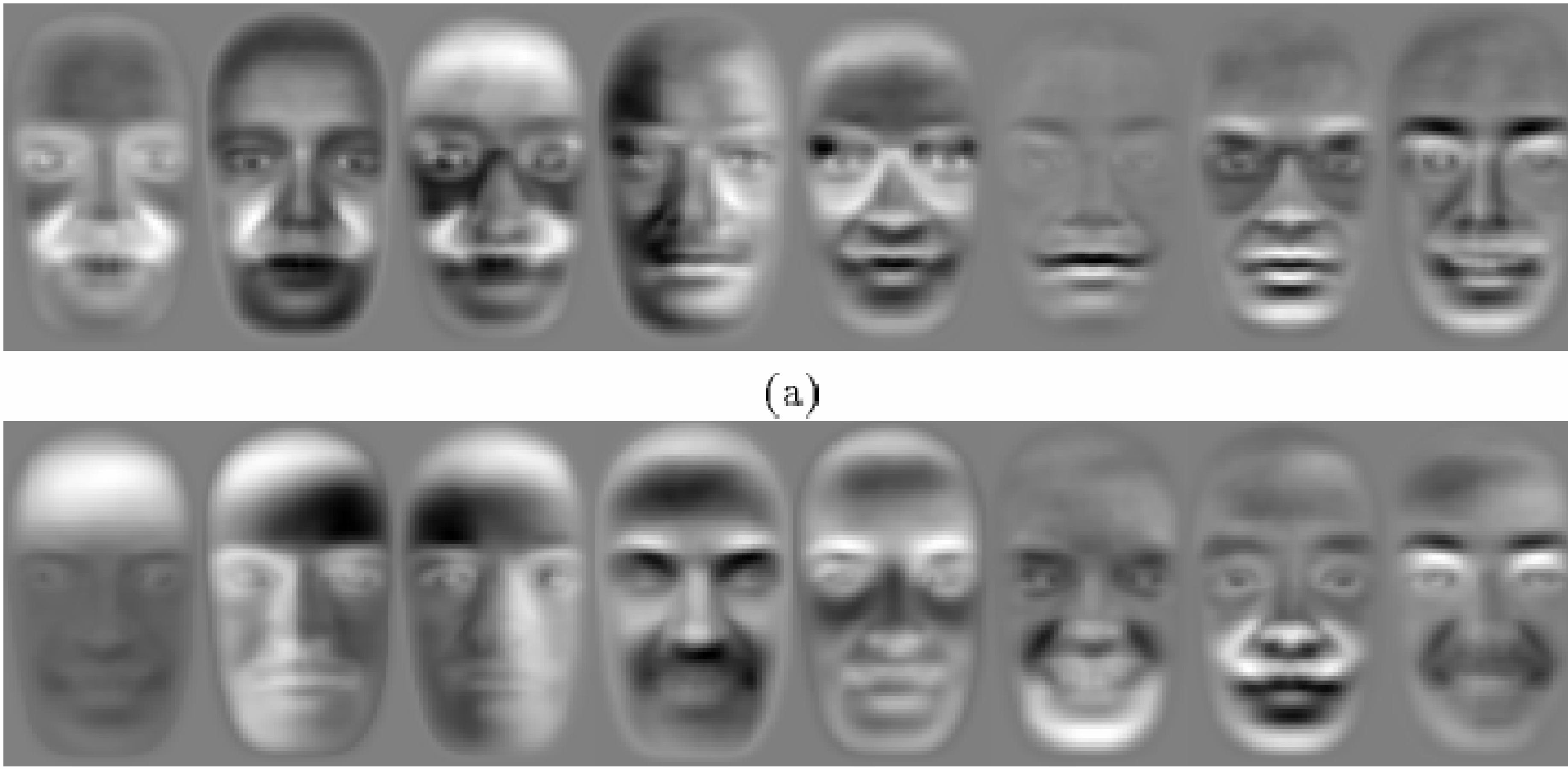
Bayesian framework for vision



Coincidental appearance of faces in rock?

http://bensguide.gpo.gov/3-5/symbols/print/mountrushmore.html

Eigenfaces: linear bases for faces



(a)

(b)

Figure 6: "Dual" Eigenfaces: (a) Intrapersonal, (b) Extrapersonal

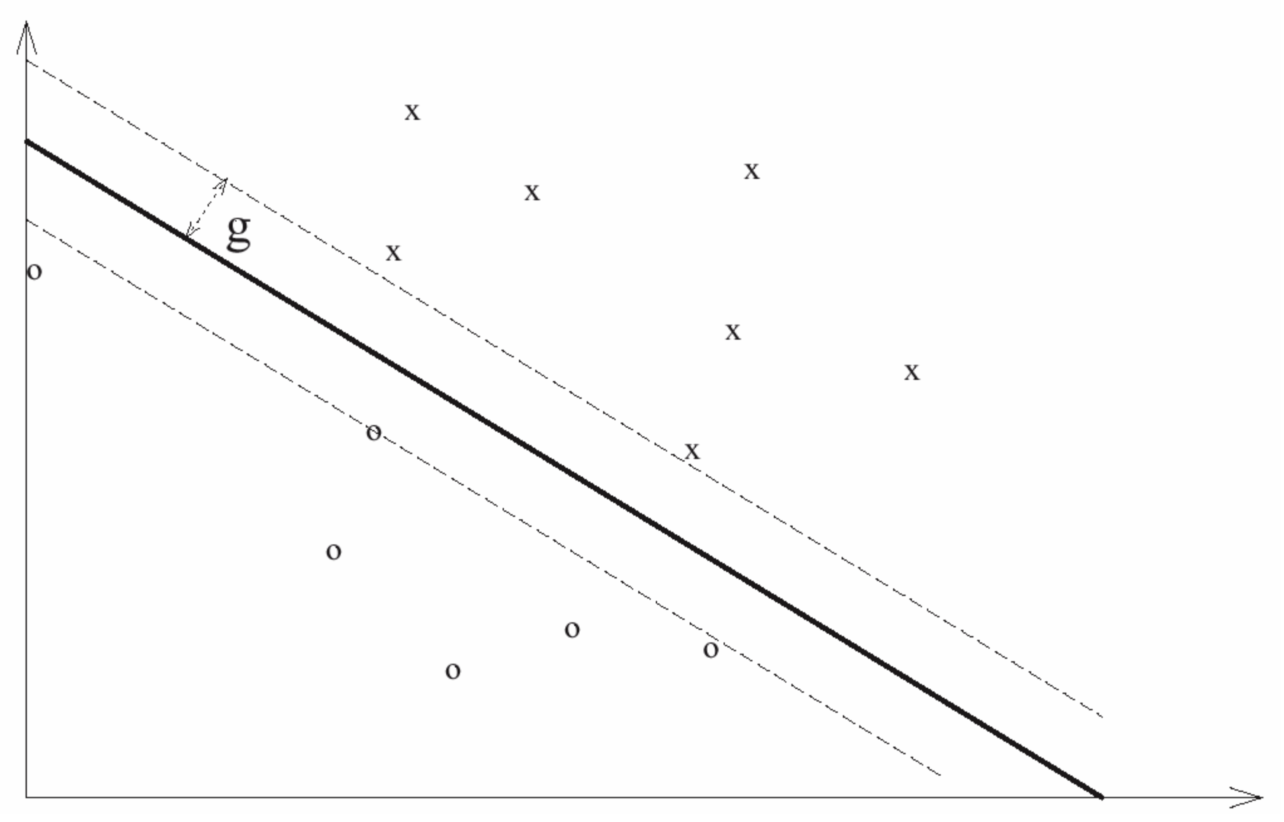
Moghaddam, B.; Jebara, T.; Pentland, A., "Bayesian Face Recognition", *Pattern Recognition*, Vol 33, Issue 11, pps 1771-1782, November 2000

Statistical classifiers

– MIT Media Lab face localization results.

– Applications: database search, human machine interaction, video conferencing.

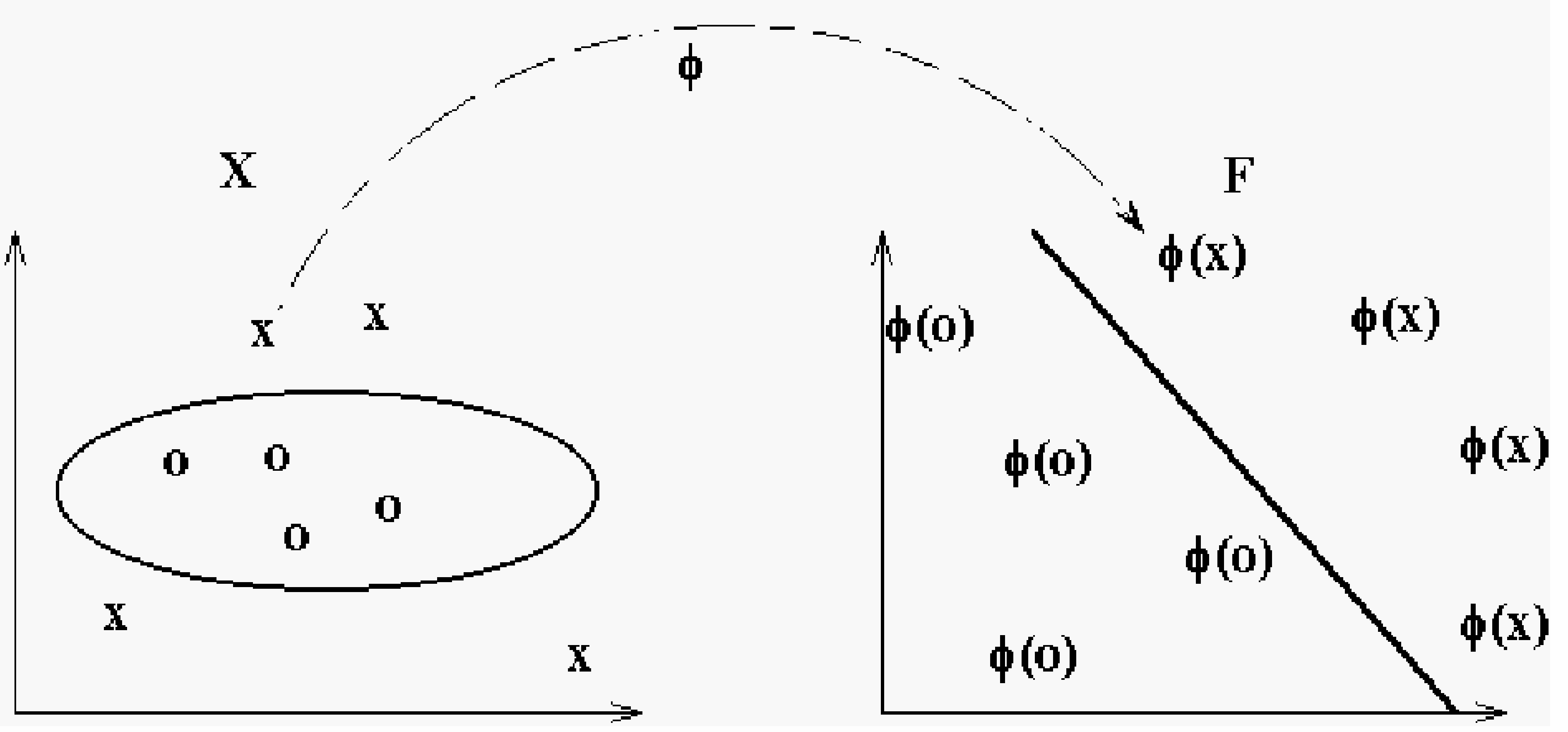
Support vector machines and boosting



Large-margin classifier

www.support-vector.net/nello.html

Support vector machines and boosting



“The kernel trick”

www.support-vector.net/nello.html

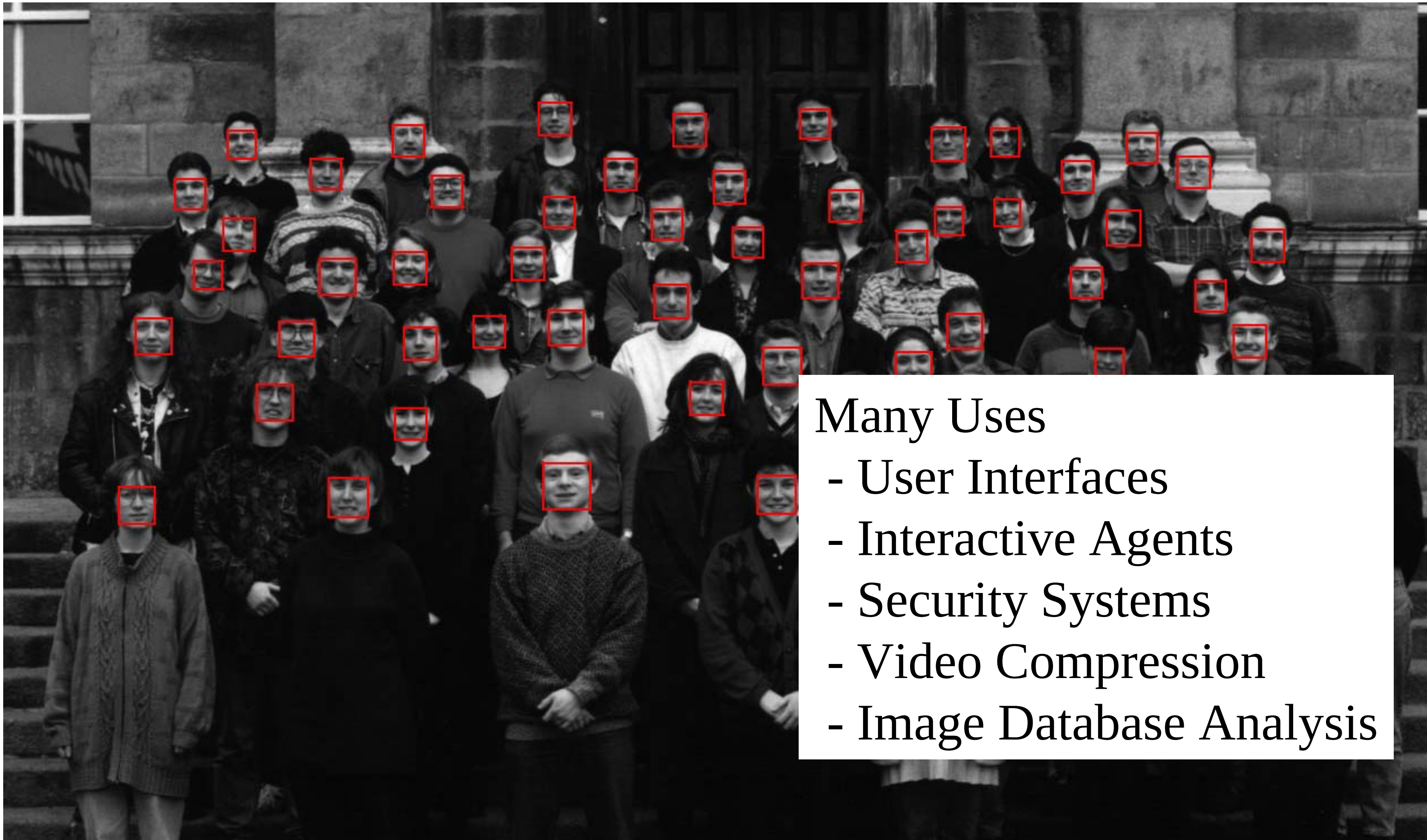
Recent, now classic, paper on face detection:

Rapid Object Detection Using a Boosted Cascade of Simple Features

Paul Viola   Michael J. Jones
Mitsubishi Electric Research Laboratories (MERL)
Cambridge, MA

Most of this work was done at Compaq CRL before the authors moved to MERL

Face Detection Goal



Many Uses
- User Interfaces
- Interactive Agents
- Security Systems
- Video Compression
- Image Database Analysis

Viola and Jones, Robust object detection using a boosted cascade of simple features, CVPR 2001

Use of context for object detection

car

pedestrian

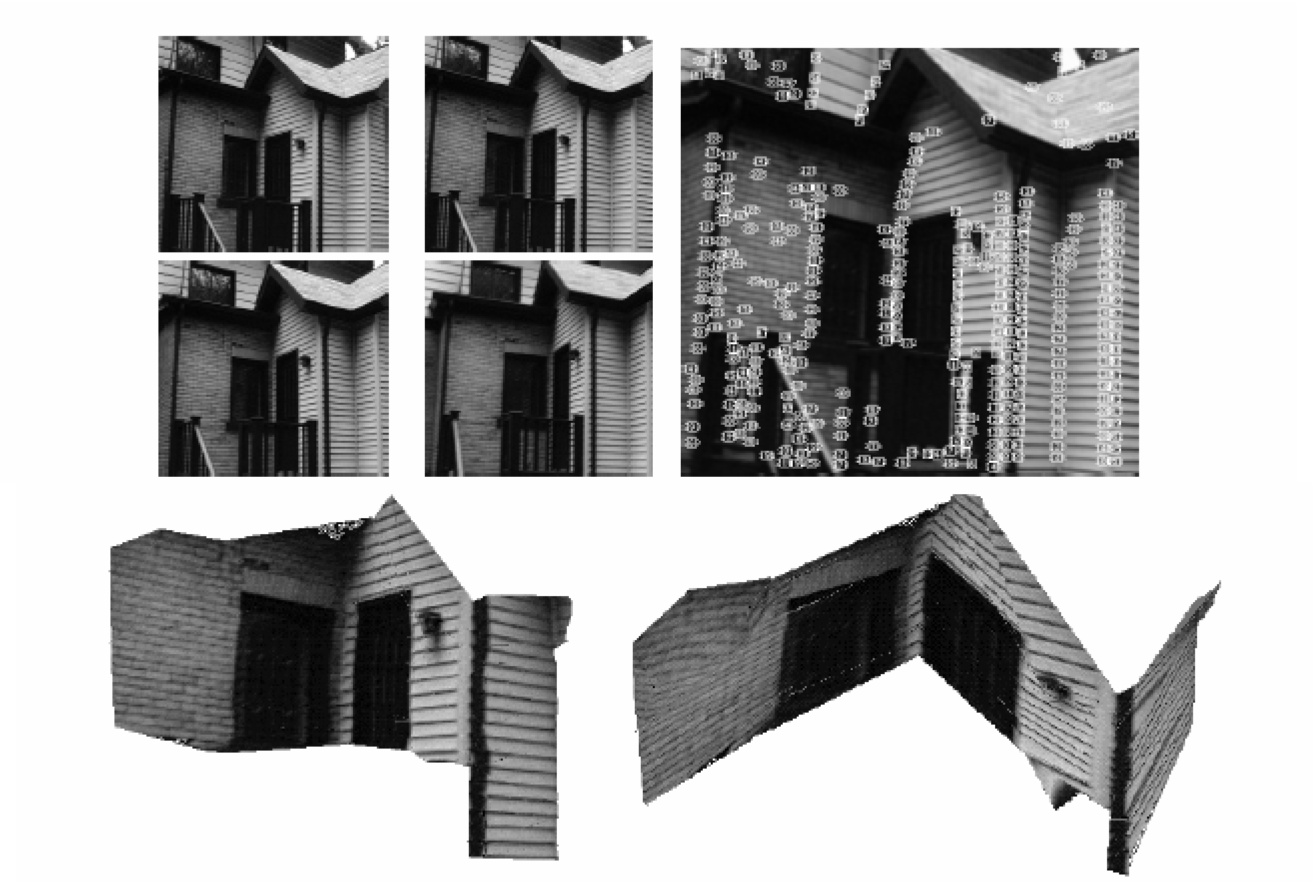
Identical local image features!

Images by Antonio Torralba

The world, to a face detector

Structure from Motion

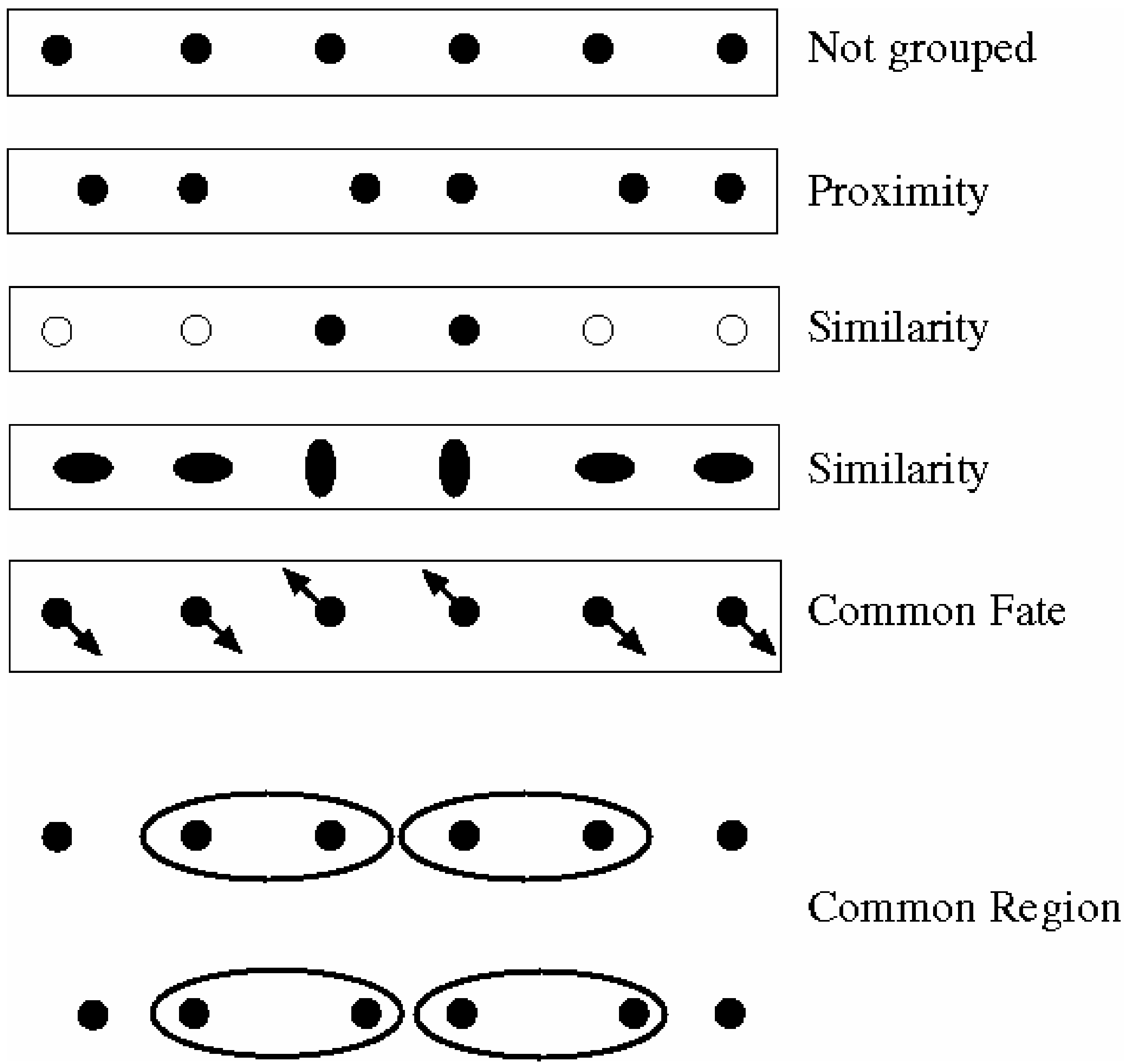
What is the shape of the scene?

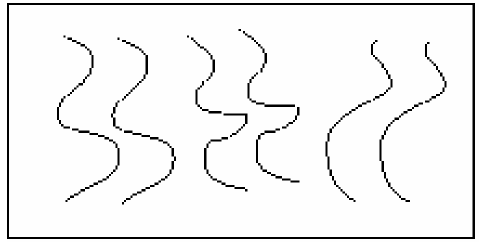


Segmentation (perceptual grouping)

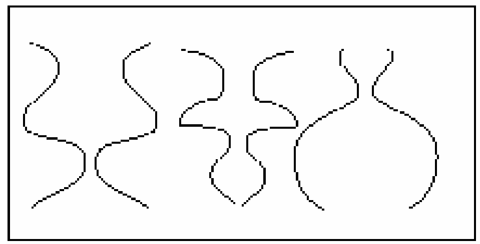
How many ways can you segment six points?

(or curves)

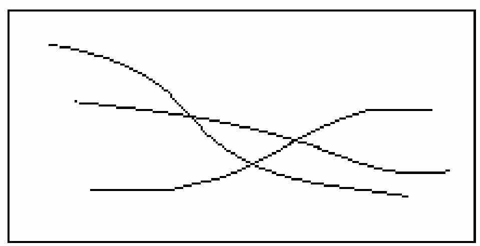

Not grouped
Proximity
Similarity
Similarity
Common Fate
Common Region



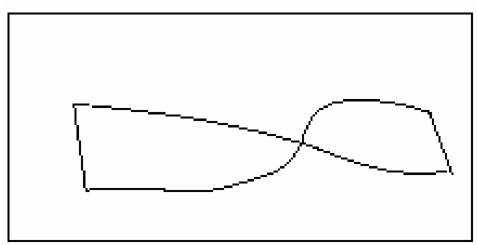
Parallelism



Symmetry



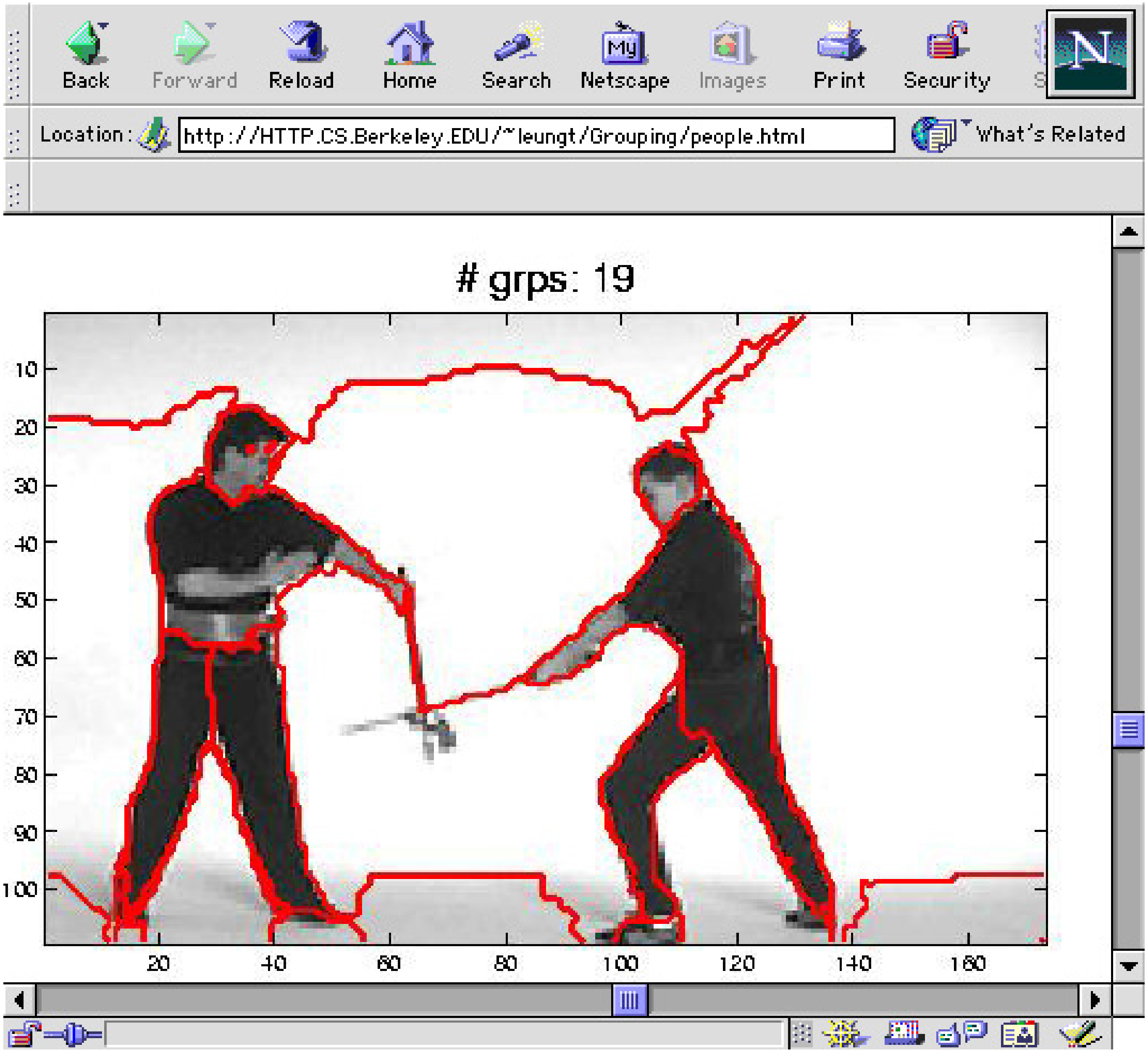
Continuity



Closure

Segmentation

- Which image components “belong together”?
- Belong together=lie on the same object
- Cues
  - similar colour
  - similar texture
  - not separated by contour
  - form a suggestive shape when assembled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What's Related
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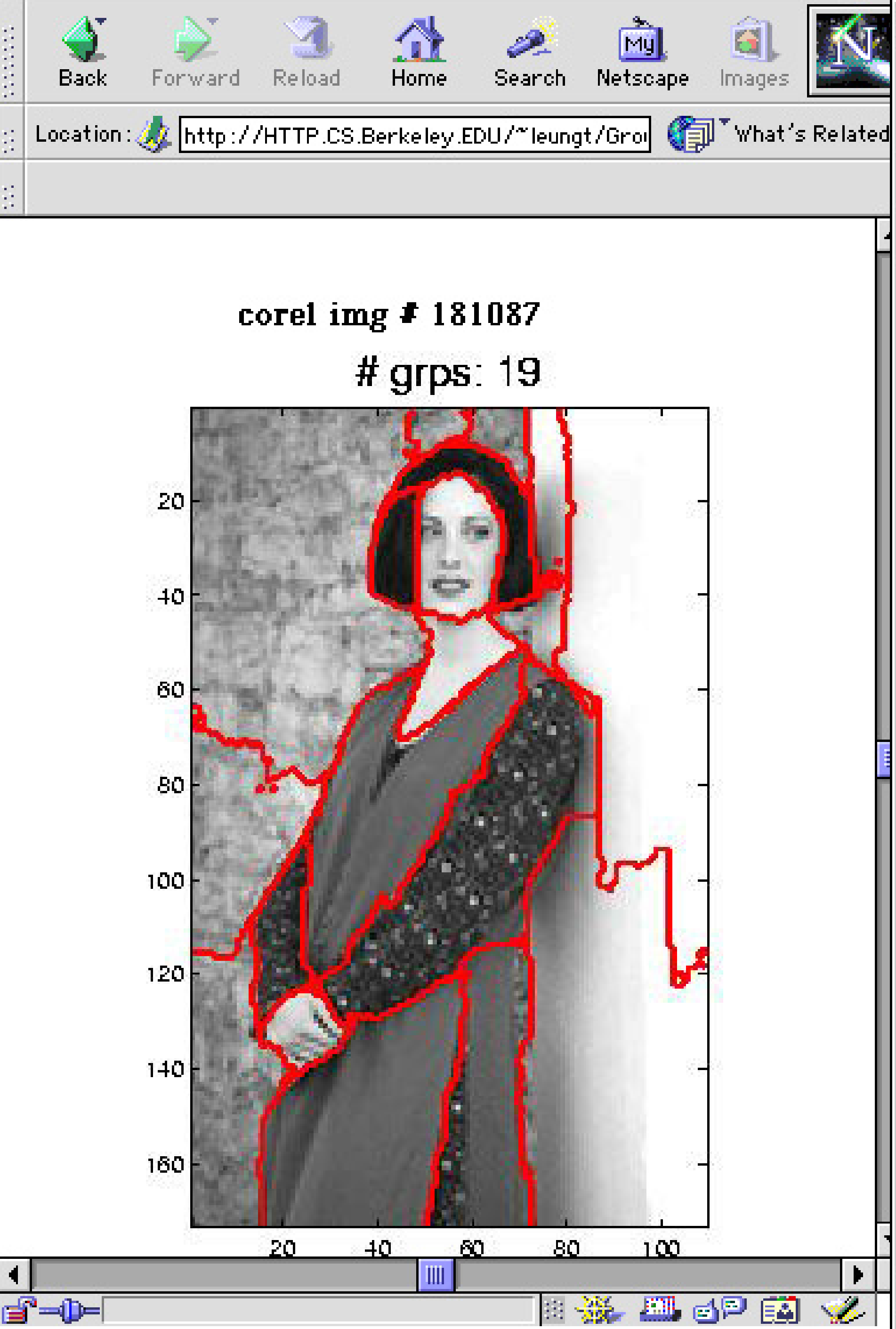
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http://HTTP.CS.Berkeley.EDU/~leungt/Gro
What's Related
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grps: 19
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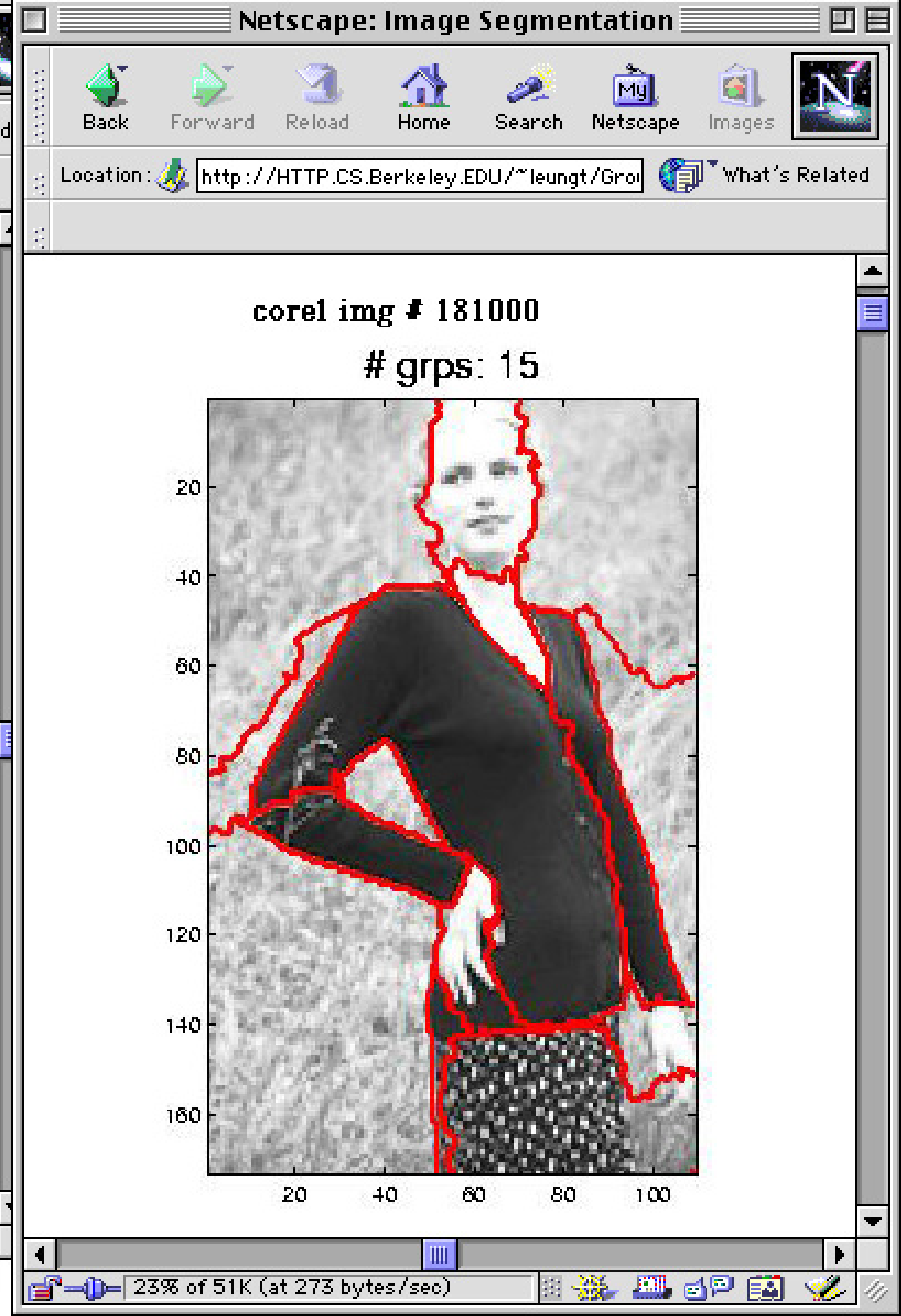
Netscape: Image Segmentation
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Location:
http://HTTP.CS.Berkeley.EDU/~leungt/Gro
What's Related
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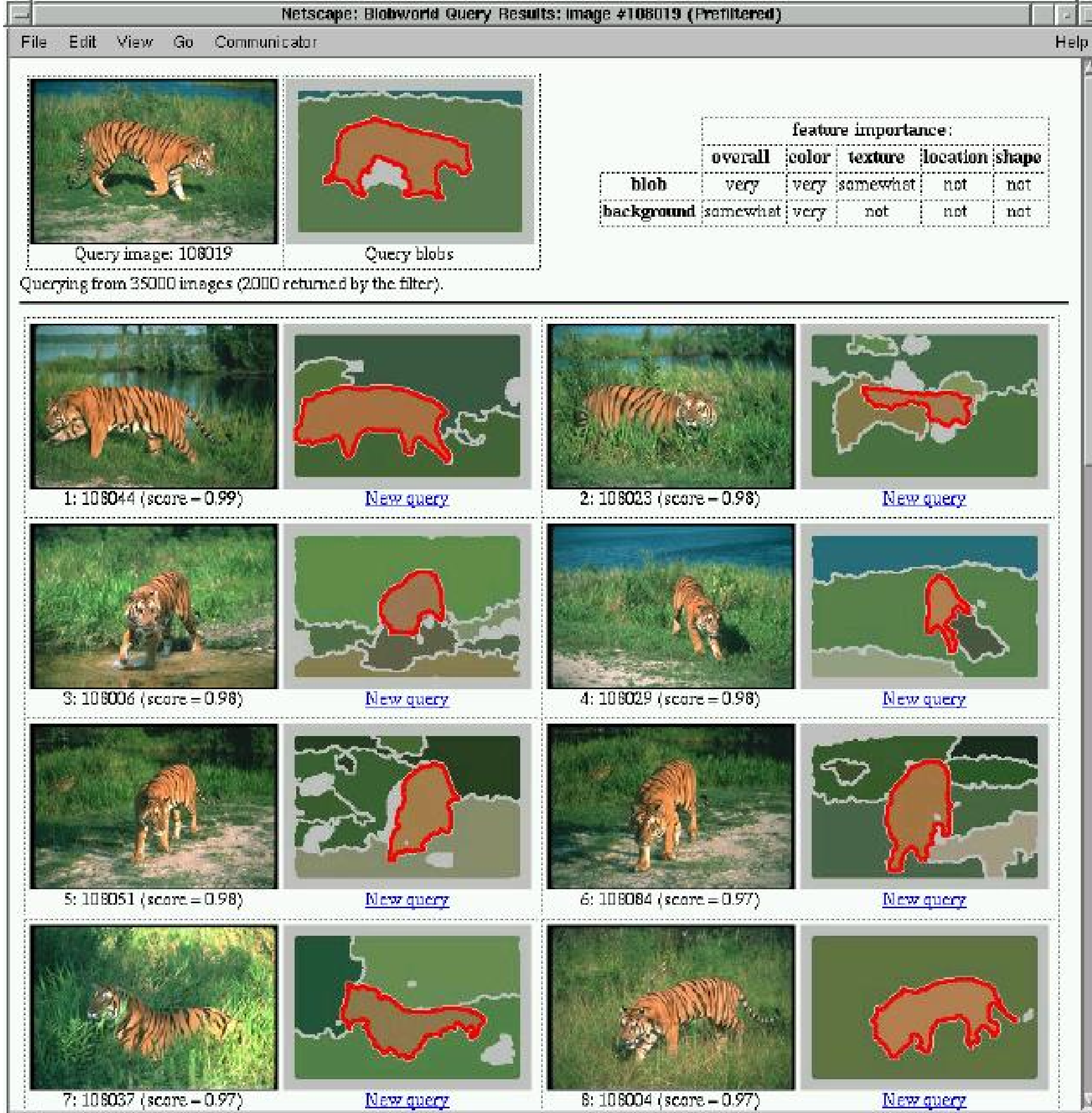
23% of 51K (at 273 bytes/sec)


Netscape: Blobworld Query Results: image #108019 (Prefiltered)
File Edit View Go Communicator Help
feature importance:
overall color texture location shape
blob very very somewhat not not
background somewhat very not not not
Query image: 108019
Query blobs
Querying from 35000 images (2000 returned by the filter).
1: 108044 (score = 0.99)
New query
2: 108023 (score = 0.98)
New query
3: 108006 (score = 0.98)
New query
4: 108029 (score = 0.98)
New query
5: 108051 (score = 0.98)
New query
6: 108084 (score = 0.97)
New query
7: 108037 (score = 0.97)
New query
8: 108004 (score = 0.97)
New query

Applications

Tracking

Follow objects and estimate location..

- radar / planes
- pedestrians
- cars
- face features / expressions

Many ad-hoc approaches…

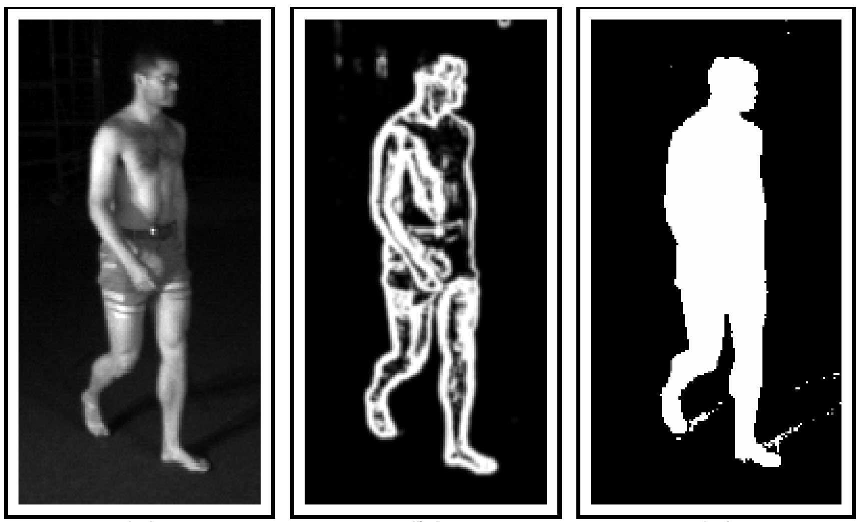
General probabilistic formulation: model density over time.

Tracking

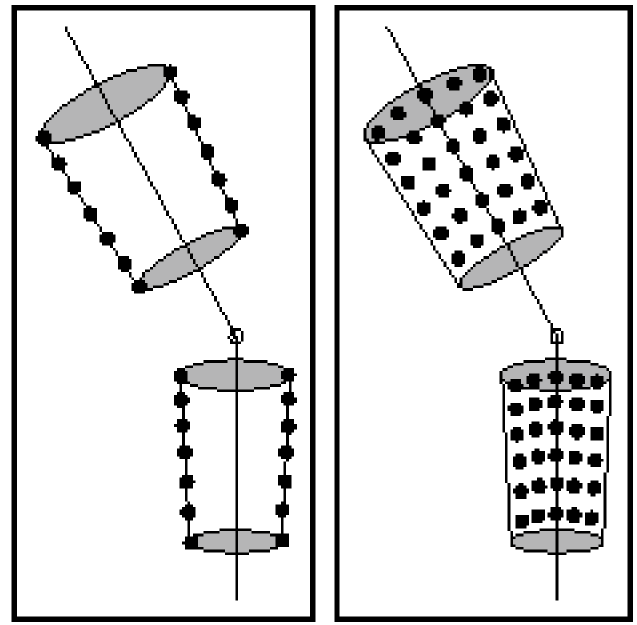
- Use a model to predict next position and refine using next image
- Model:
  - simple dynamic models (second order dynamics)
  - kinematic models
  - etc.
- Face tracking and eye tracking now work rather well


Hi, David!

Articulated Models



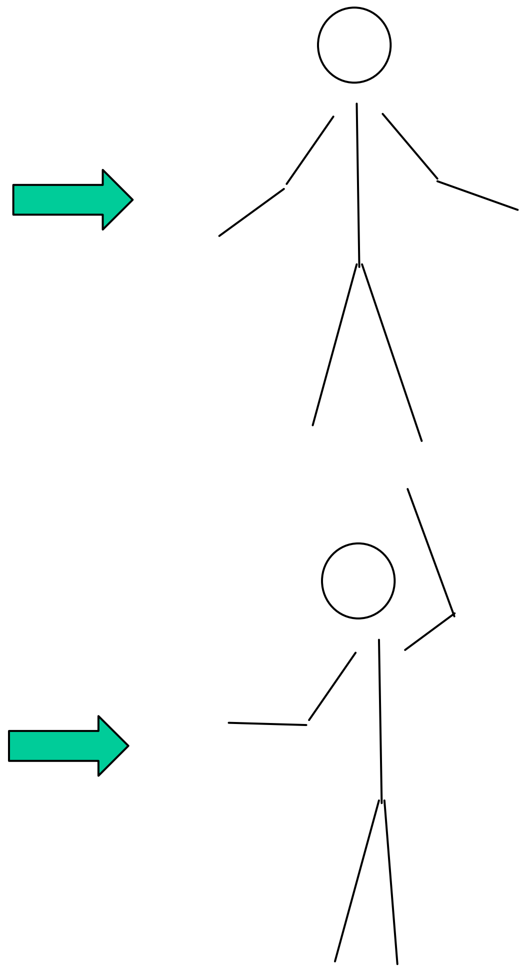
(a) (b) (c)



(a) (b)

Find most likely model consistent with observations….(and previous configuration)

Articulated tracking



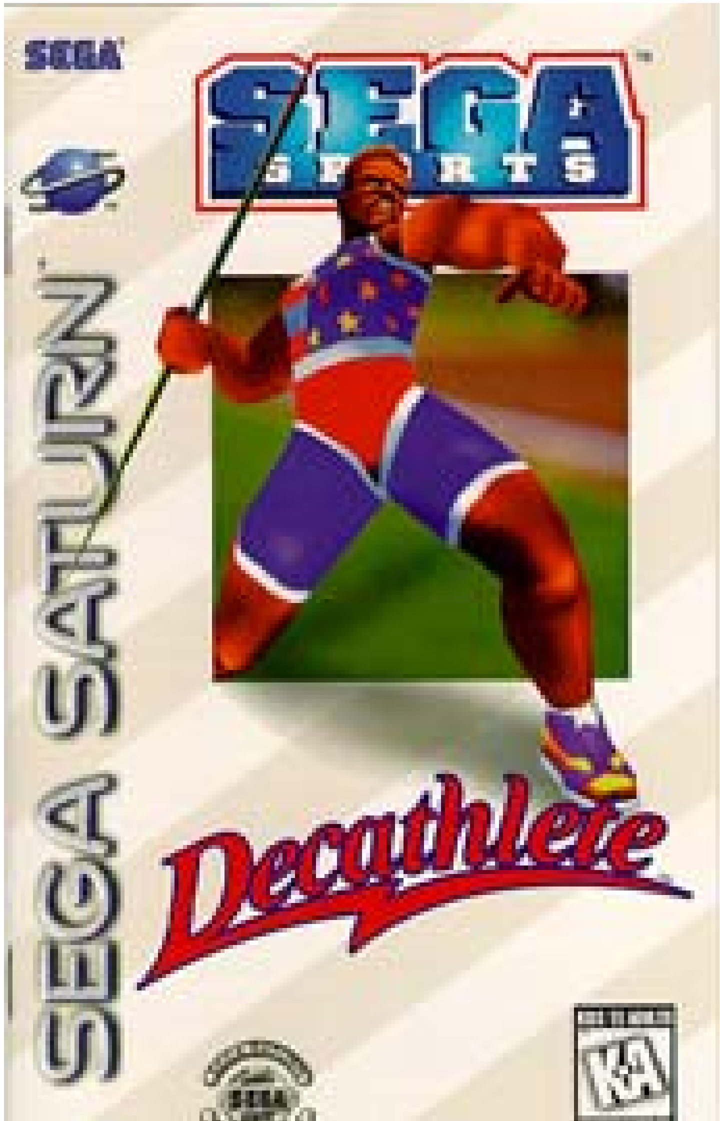
- Constrained optimization
- Coarse-to-fine part iteration
- Propagate joint constraints through each limb
- Real-time on Ghz pentium…


slow

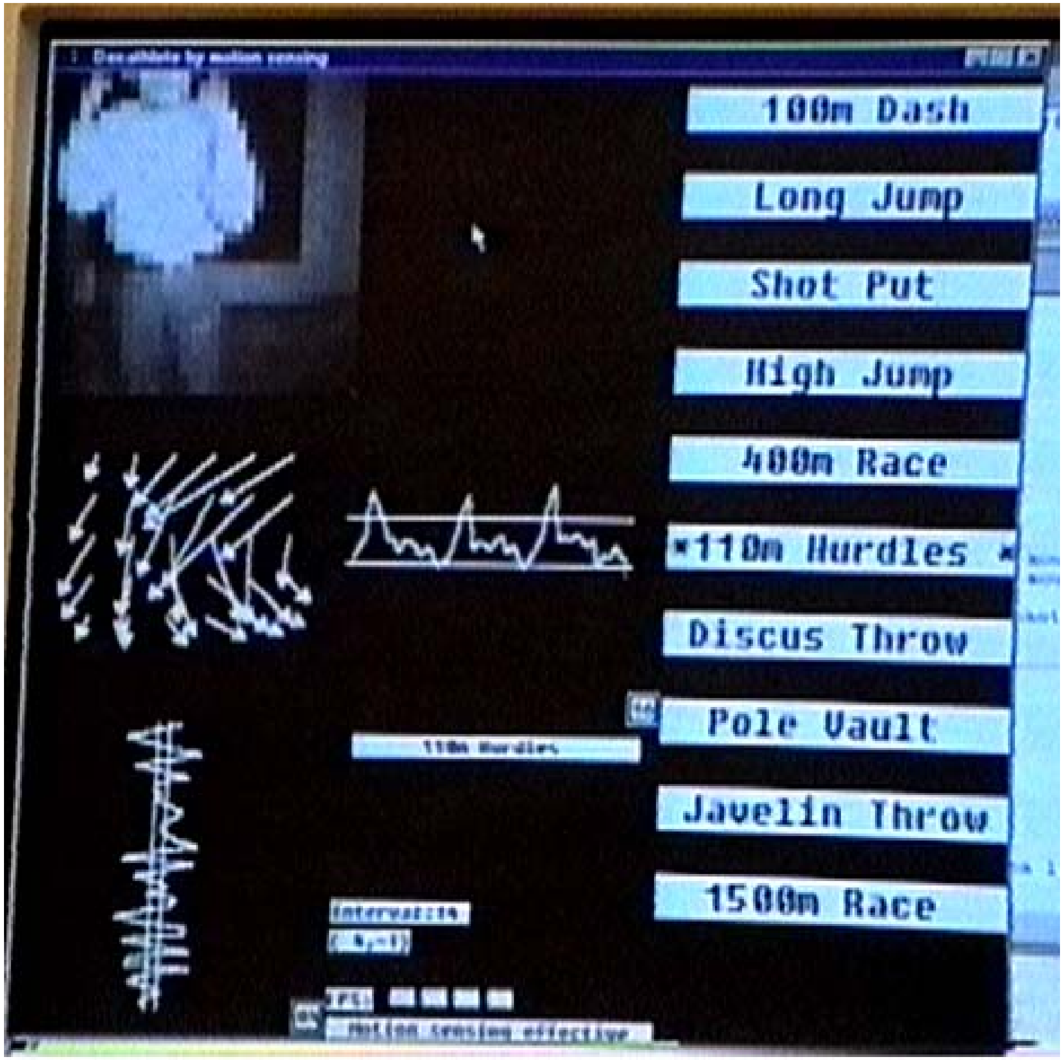
Computer vision applications as ocean-going vessels

this application ←

Game: Decathlete

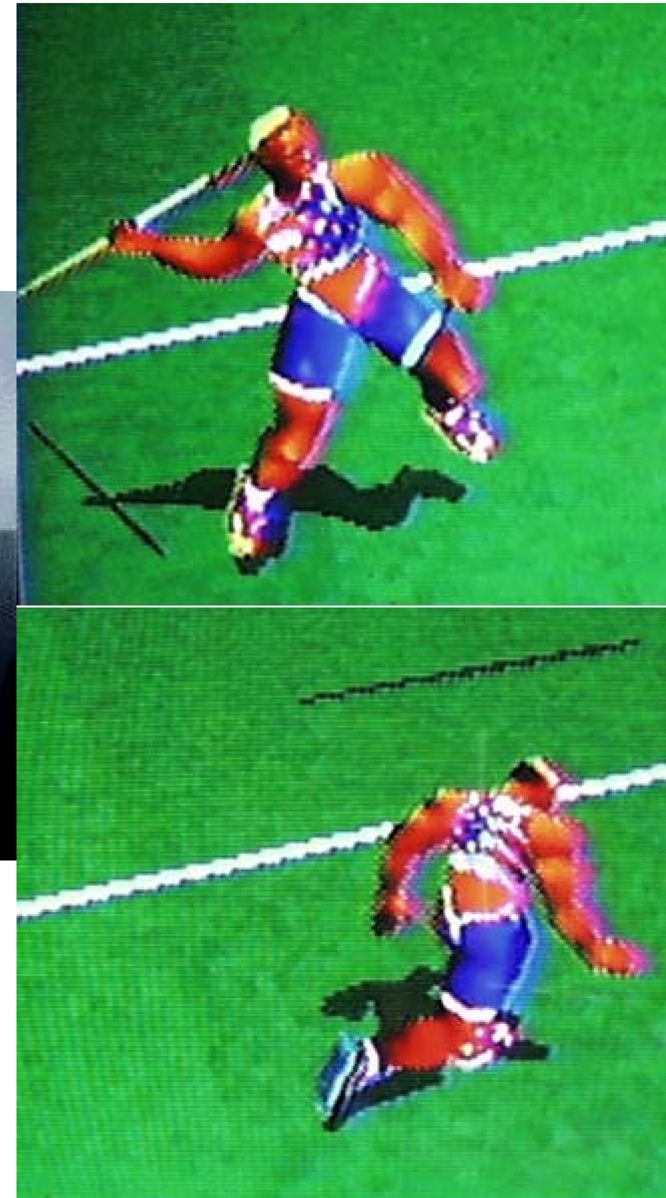


Optical-flow-based Decathlete figure motion analysis



Decathlete 100m hurdles

Decathlete javelin throw



Companies and applications

- Cognex
- Reactrix
- Poseidon
- Mobileye
- Eyetoy
- Identix
- Roomba

Aquatic safety - Drowning problem - Poseidon - Microsoft Internet Explorer

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Address http://www.poseidon-tech.com/us/drowning.html Go Links

Poséidon

THE LIFEGUARD'S THIRD EYE

Aquatic Safety
The Drowning Problem

Drowning problem | Facts & Statistics | Lifeguarding challenges | Stages of a drowning

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Select your country

According to the Centers for Disease Control, **9 people drown per day in the U.S**. For every person who drowns, **four times as many people nearly drown**. Many of these incidents happen **in pools staffed with certified professional lifeguards**.

If you've been to a pool recently, you've witnessed firsthand the **challenges that lifeguards face** in monitoring activity within a pool. Not only is it warm, but there are usually lots of swimmers, glare from the sun in some cases, and other distractions. The toughest part of a lifeguard's job is maintaining constant vigilance, and no human being can see everything all the time. But it only takes a second for someone to get into trouble and start to drown. Contrary to what most people think, drowning victims don't yell or wave their arms to alert someone that they are in trouble. They are in a state of shock, and are often silent.

It's vital that lifeguards reach a drowning victim before it's too late, and **every second counts**. To prevent death or lifelong injury, the resuscitation of drowning victims must be initiated as quickly as possible – ideally within 30 seconds.

The solution isn't just more lifeguards or better training. It's a better means of surveillance and detection. It's Poseidon. Poseidon helps lifeguards monitor what is happening in the pool, maintaining vigilance, and alerting them in seconds to a swimmer in trouble. Poseidon does not rescue drowning victims – lifeguards do – but it can help them more quickly initiate a rescue and save a life.

vimming pools with Poseidon - Microsoft Internet Explorer

Edit View Favorites Tools Help

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http://www.poseidon-tech.com/us/sites.html Go

Here is a partial list of our installed sites:

USA

- McCoy Natatorium, Penn State University
  University Park
  Pennsylvania, USA

McCoy
Natatorium
The Aquatics Facilities of Penn State University

- YMCA Southcoast, New Bedford Division
  New Bedford
  Massachusetts, USA
- Medina Community Recreation Center
  Medina
  Ohio , USA
- Metro Atlanta YMCA - Carl E. Sanders Family YMCA
  Buckhead
  Georgia , USA

Metro Atlanta YMCA
We build strong kids, strong families, strong communities.

- Heritage YMCA Group - 95th St. Family Center
  Naperville
  Illinois, USA

- The Sarasota Family YMCA Pools
  Florida, USA
- The Fort Wayne Community Schools' Pool
  Indiana, USA
- The St. Cloud School District Pools
  Minnesota, USA

EUROPE

Poseidon saved a life here!

oséidon
E LIFEGUARD'S THIRD EYE

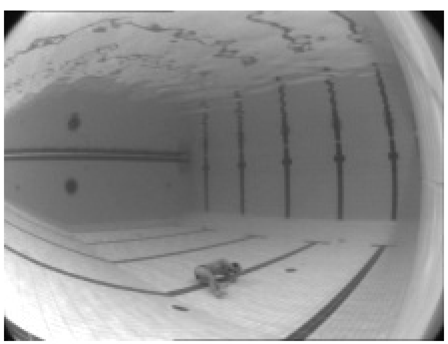
Poseidon saves a life

Back

Ancenis – November 28, 2000

A young French teenager was out for his usual swim one night. Suddenly something went wrong. He sank to the bottom of the pool, unseen by the lifeguards on duty. Poseidon detected him, alerted the lifeguards, and he was saved. In less than thirty seconds.

Some images from the drowning:



Download what the cameras saw (Animated gif: 7MB)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Motion magnification

And…

- Visual Category Learning
- Image Databases
- Image-based Rendering
- Medical Imaging

Skills learned from this class

- Goal: You'll be able to go to a computer vision conference and understand what's going on in most of the presentations.
- You'll have the skills and awareness of the literature to start building the vision systems you want.

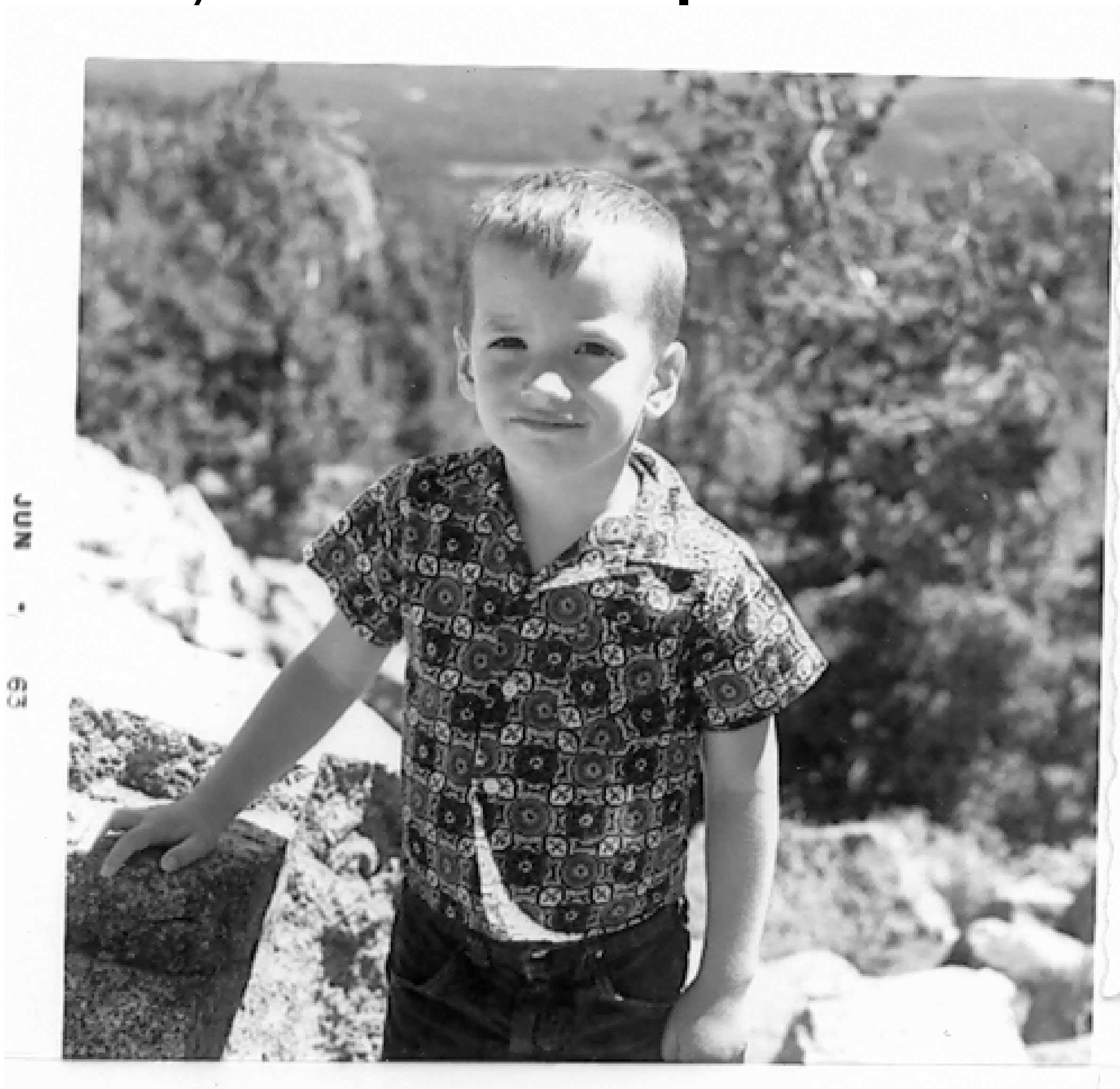
Cameras, lenses, and calibration

Today:

- Camera models
- Projection equations
- Calibration methods

Images are projections of the 3-D world onto a 2-D plane…

7-year old's question

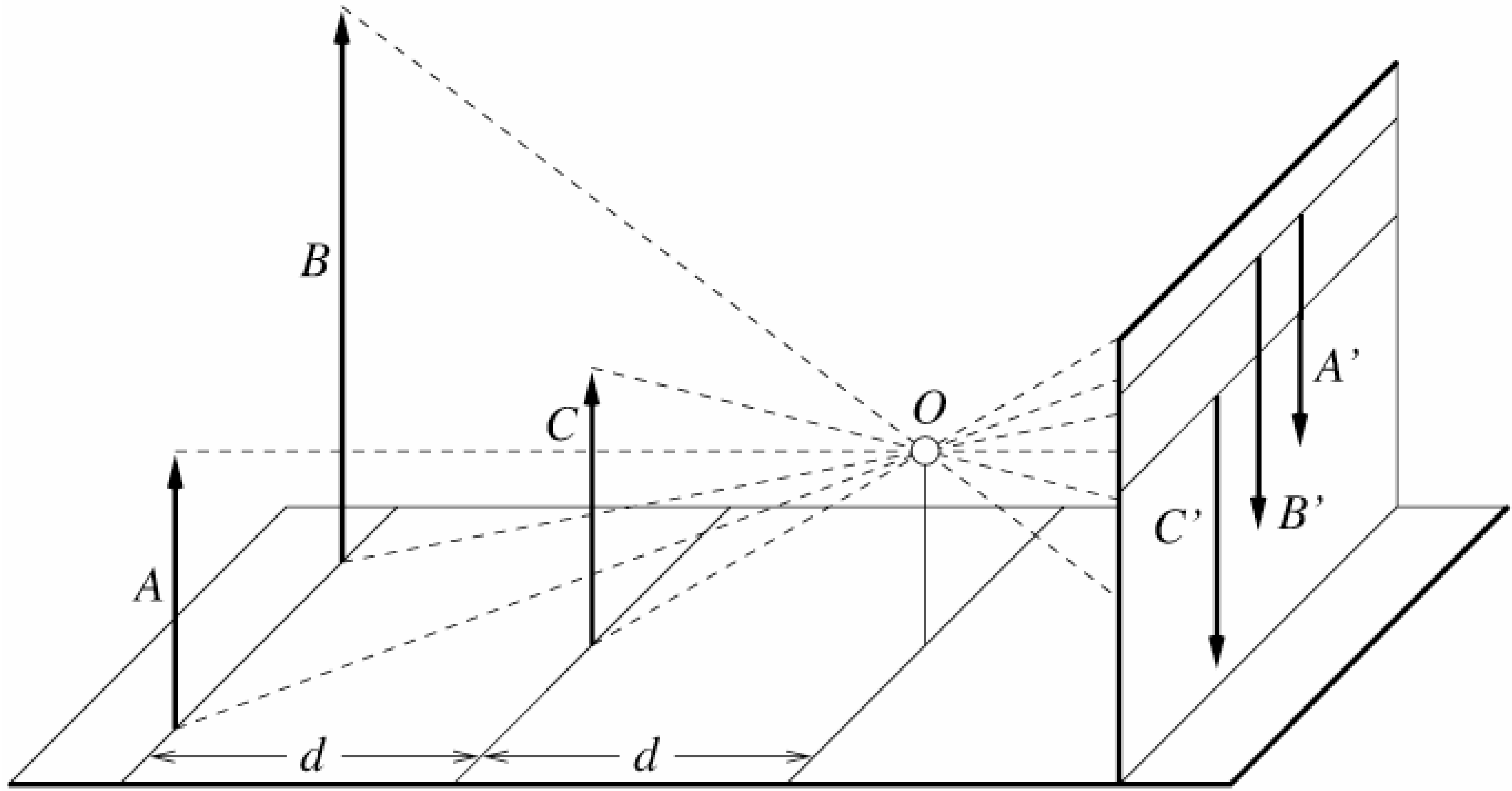


Why is there no image on a white piece of paper?

Pinhole cameras

- Geometry

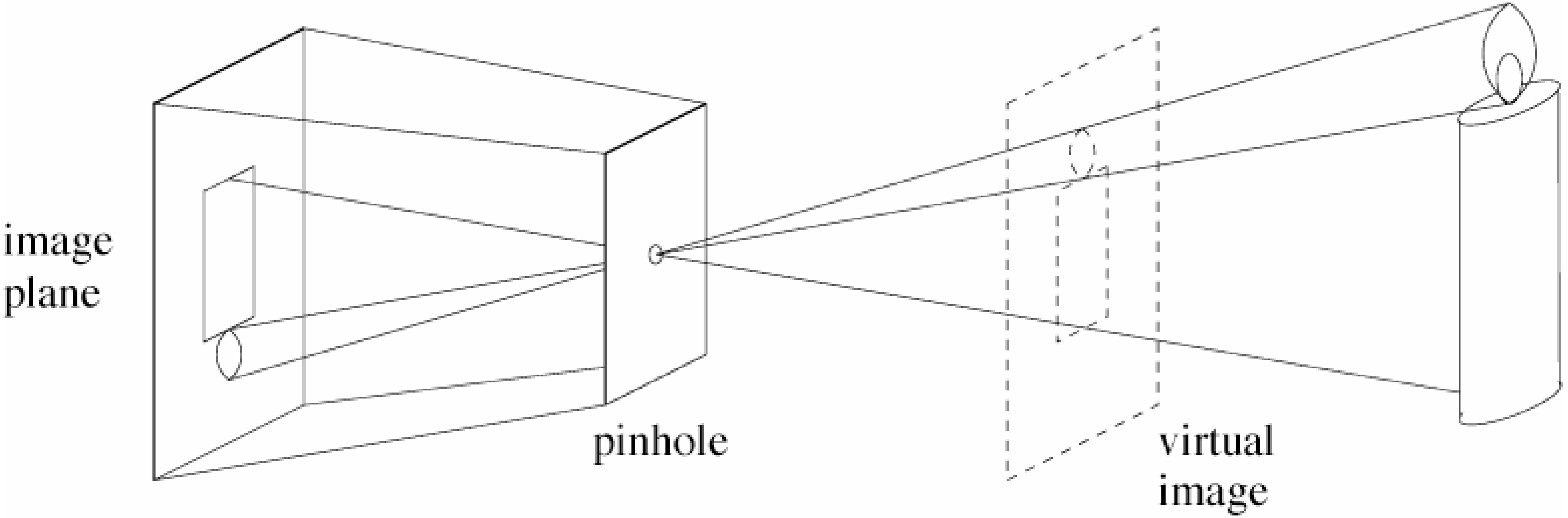
Distant objects are smaller



Forsyth&Ponce

Virtual image, perspective projection

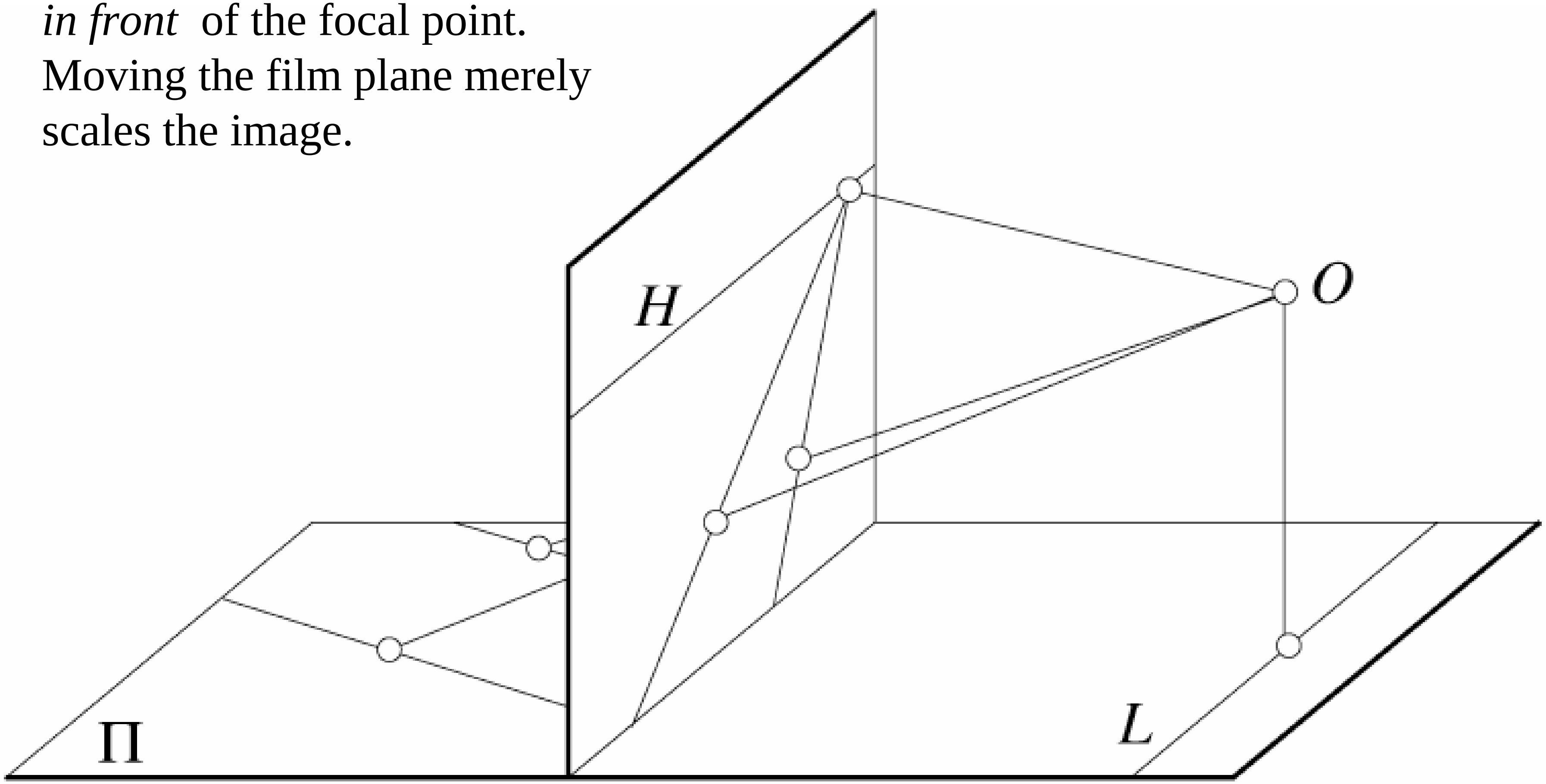
- Abstract camera model - box with a small hole in it



Forsyth&Ponce

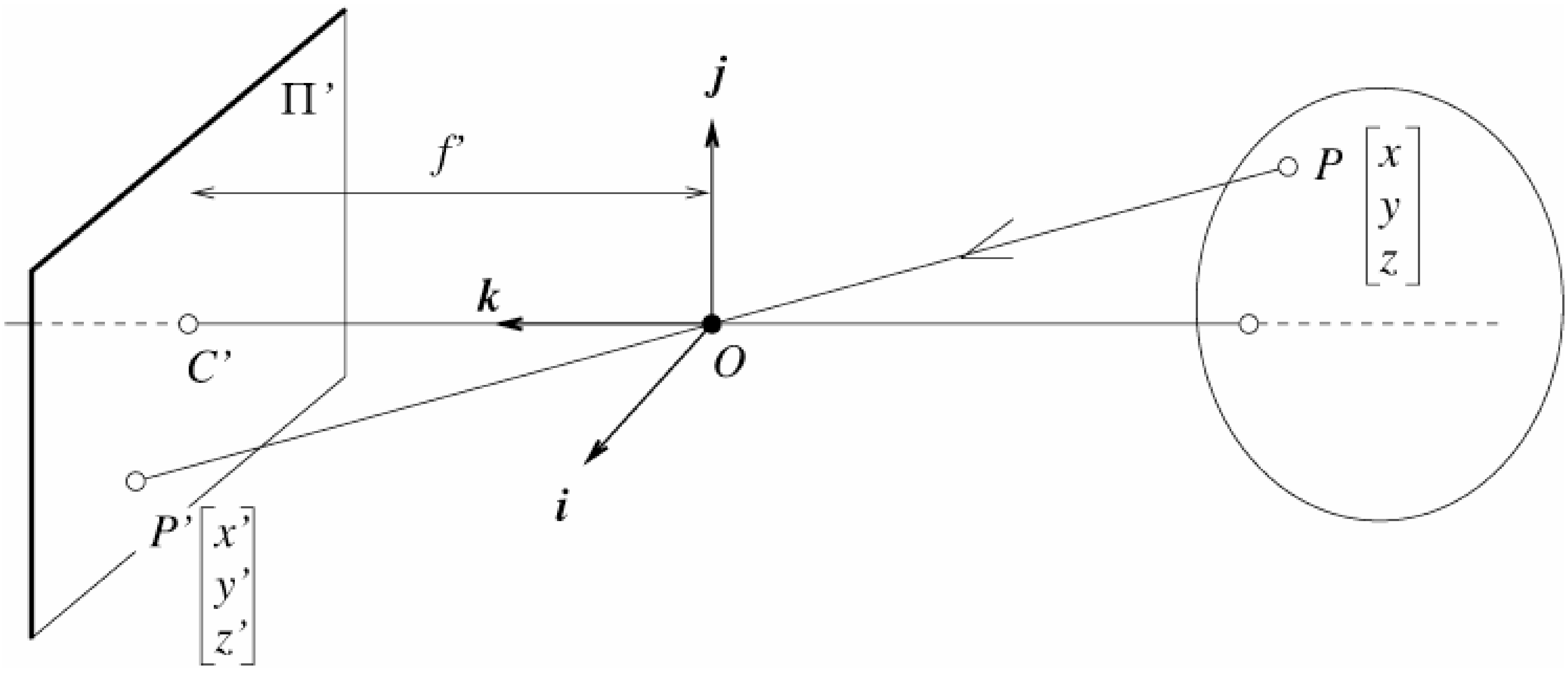
Parallel lines meet

Common to draw film plane *in front* of the focal point. Moving the film plane merely scales the image.



Forsyth&Ponce

The equation of projection



The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that (x, y, z) -> (f x/z, f y/z, -f)
  - Ignore the third coordinate, and get

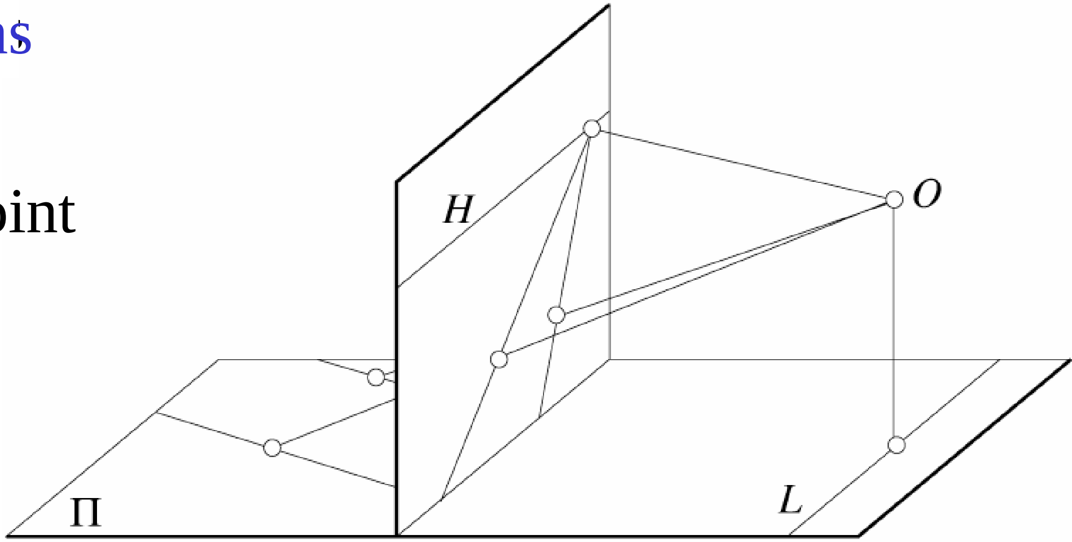
$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Vanishing points

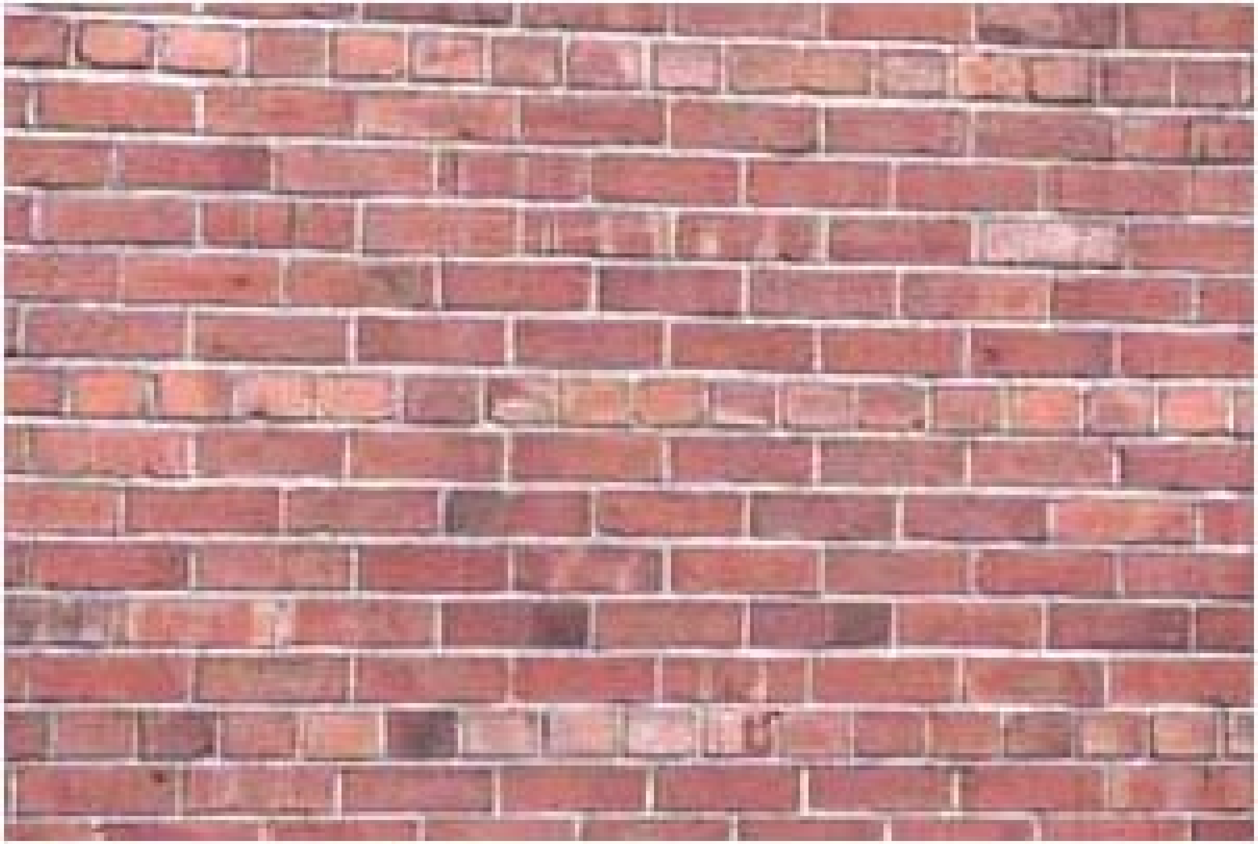
- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane
- We show this on the board…

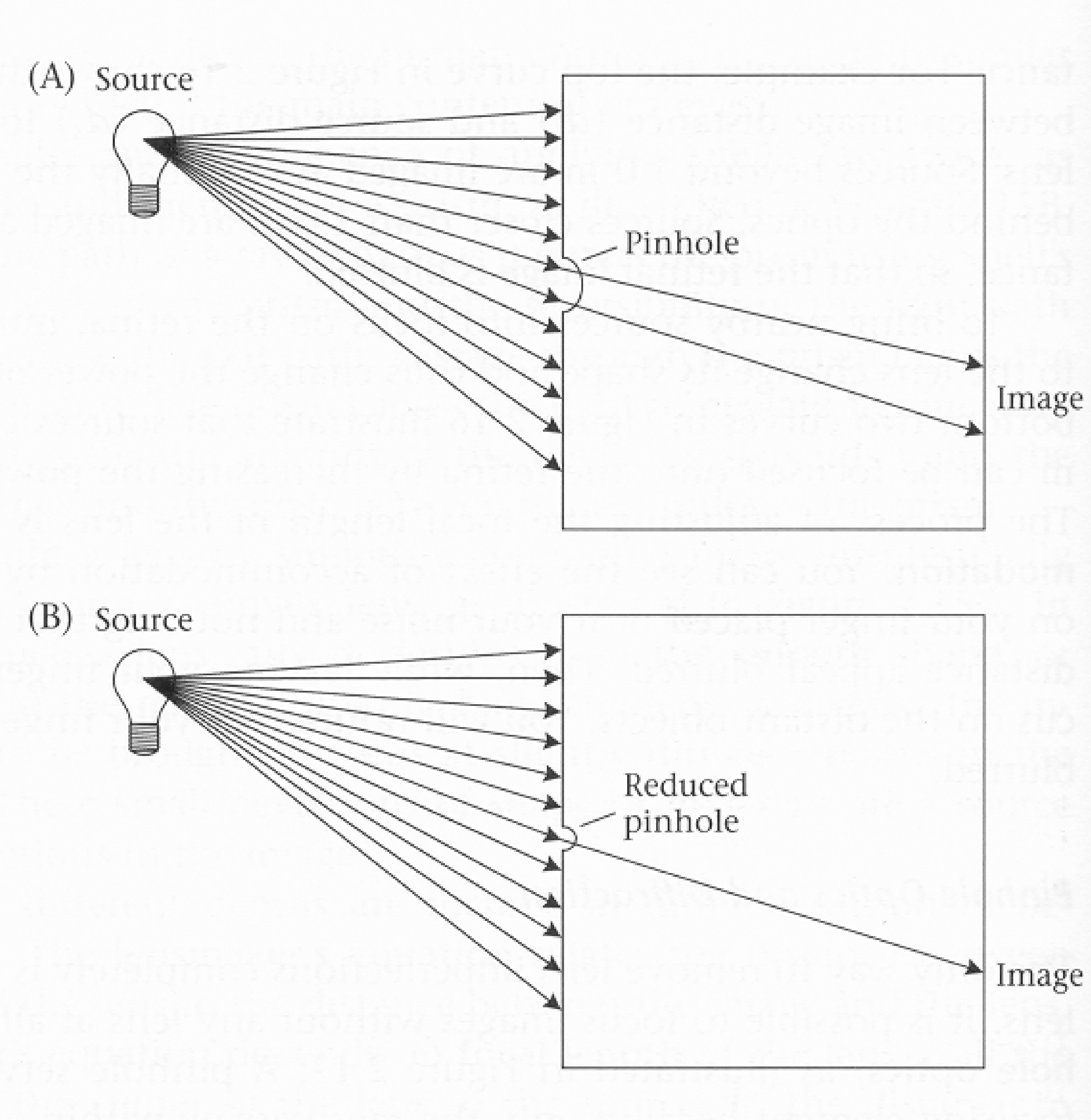
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to the whole image or a half-plane
- Polygons go to polygons
- Degenerate cases
  - line through focal point to point
  - plane through focal point to line



What if you photograph a brick wall head-on?

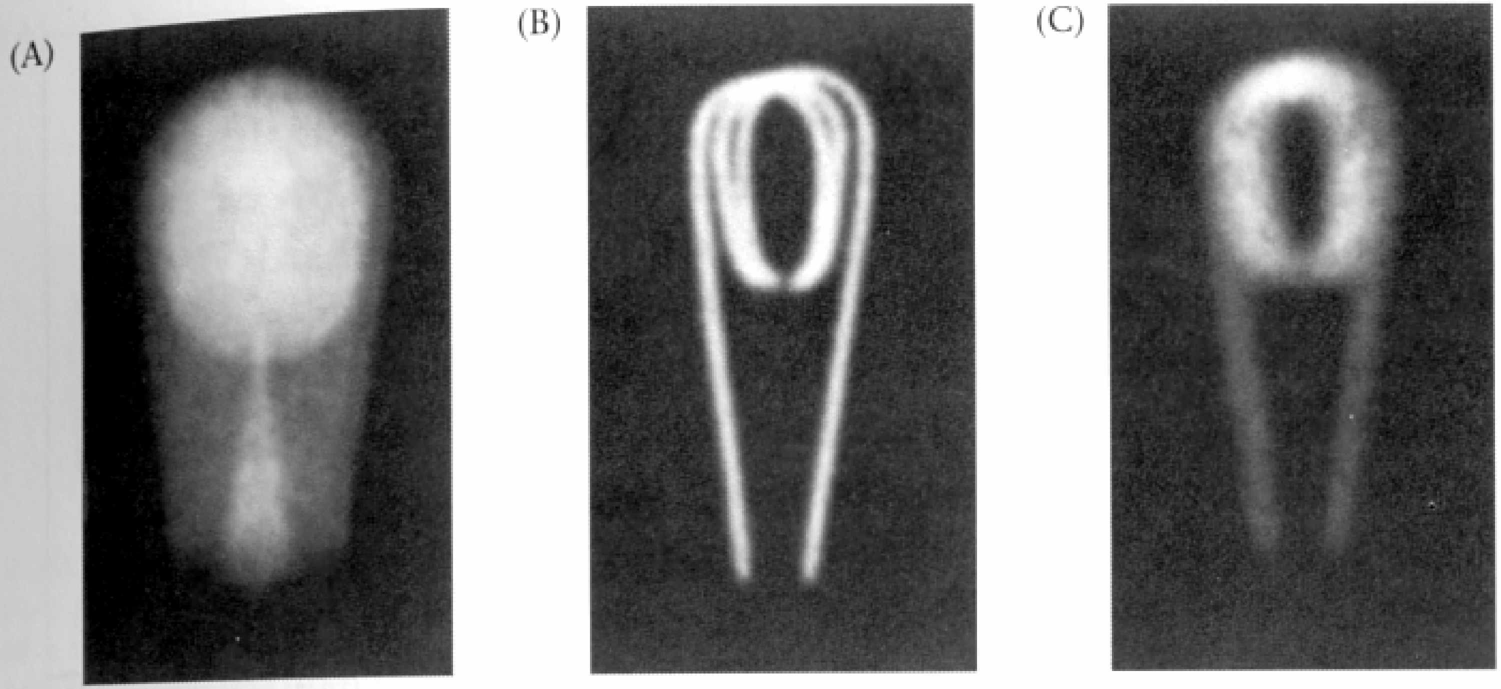




Wandell, Foundations of Vision, Sinauer, 1995

Pinhole camera demonstrations

- Film camera, box, demo. Apertures, lens.
- The image is the convolution of the aperture with the scene.

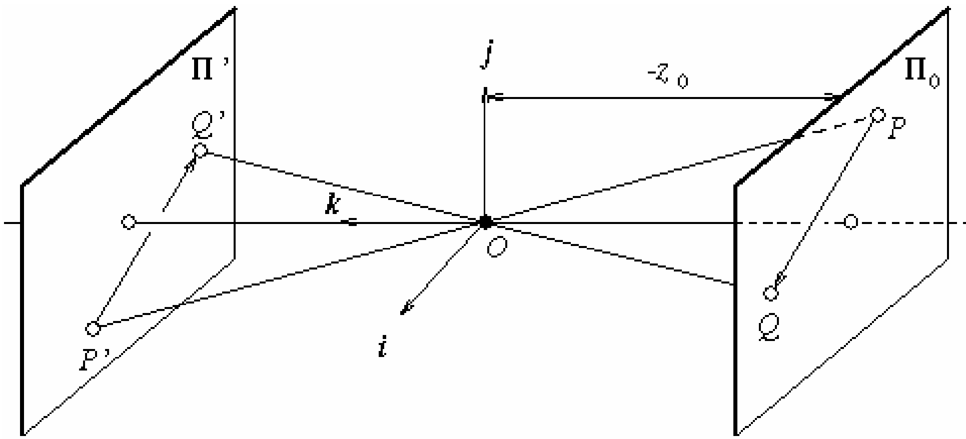


2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

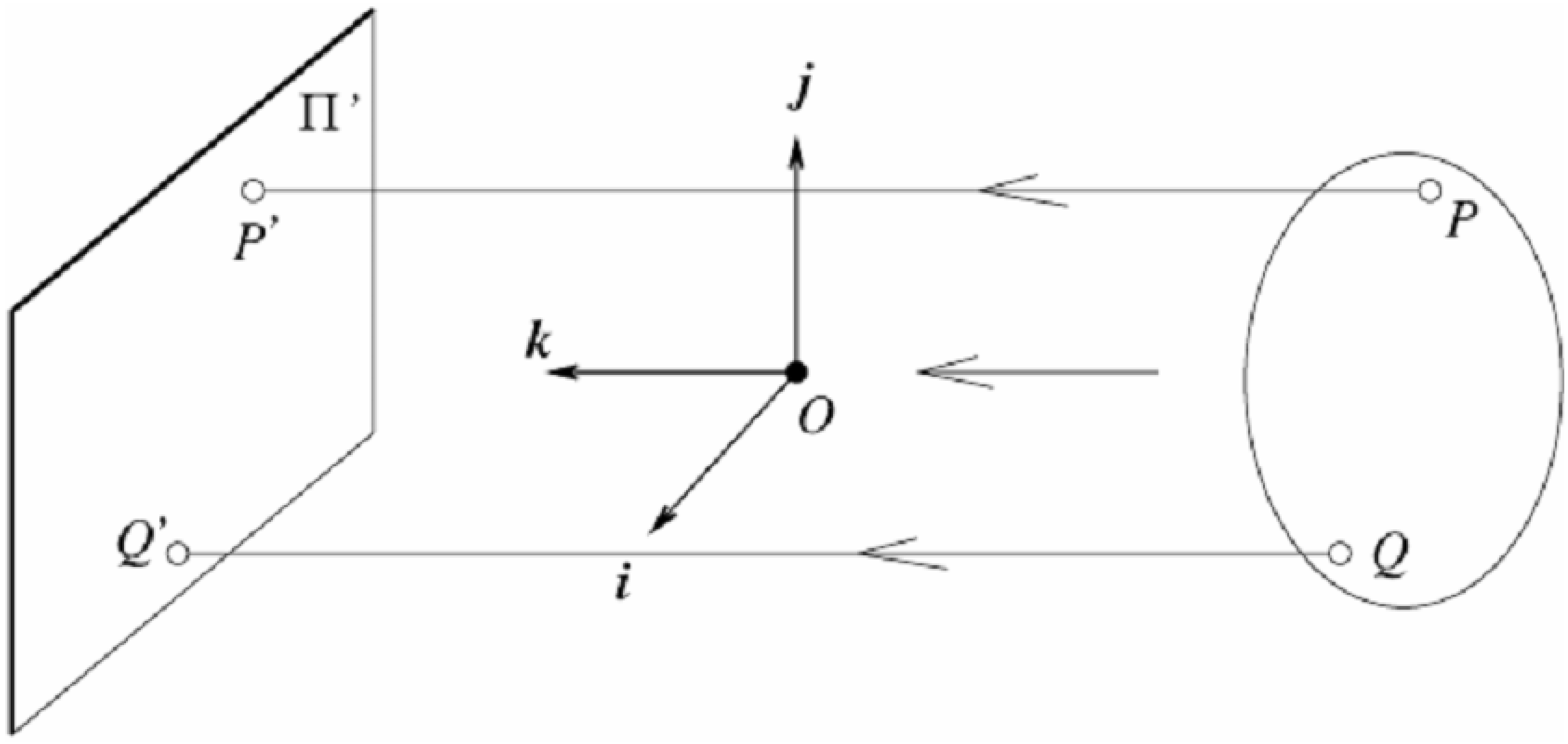
Wandell, Foundations of Vision, Sinauer, 1995

Weak perspective

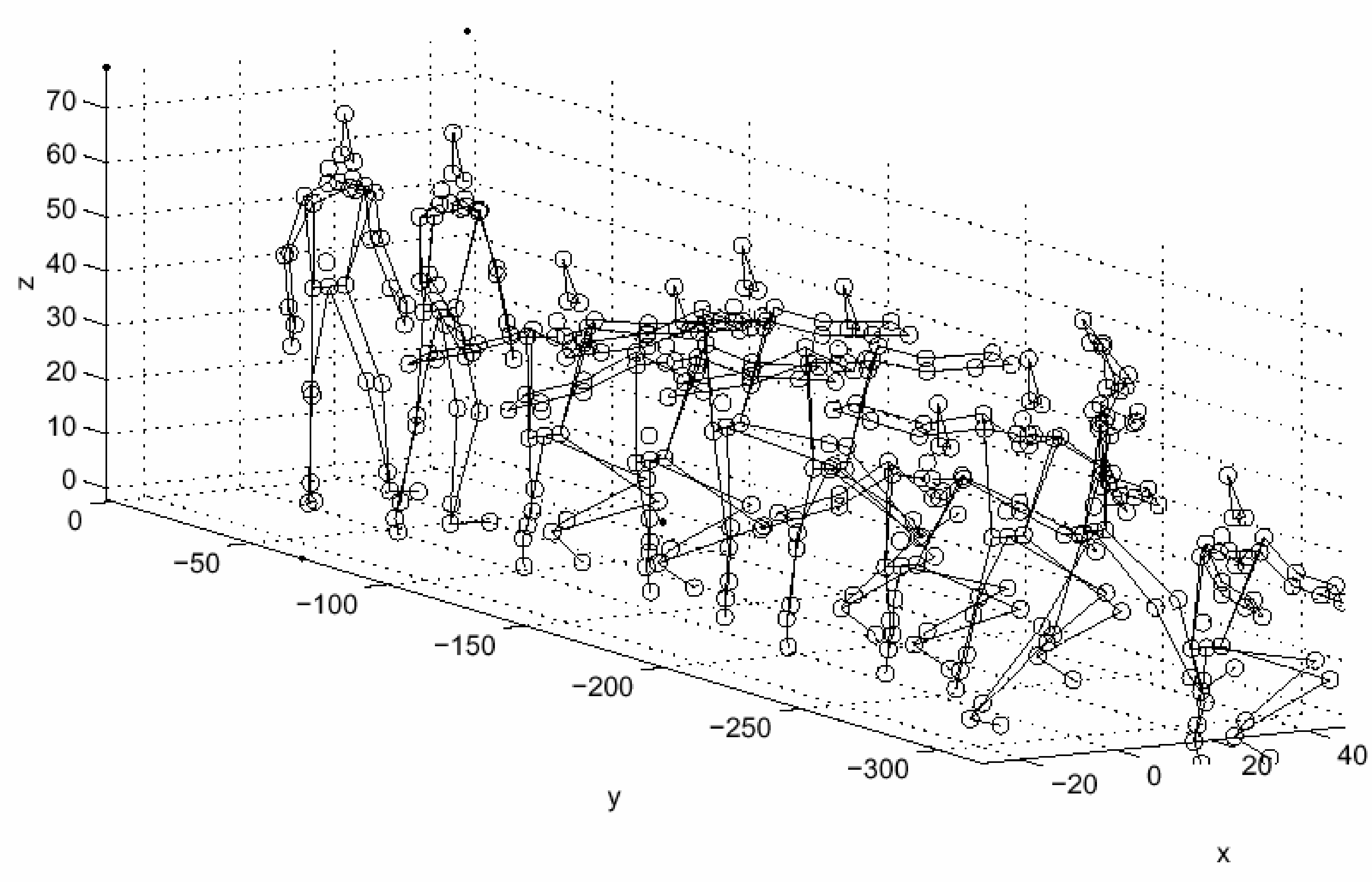
- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong



Orthographic projection



Example use of orthographic projection: inferring human body motion in 3-d



Advantage of orthographic projection

Our simplified rendering conditions are as follows: the body is transparent, and each marker is rendered to the image plane orthographically. For figural motion described by human motion basis coefficients $\vec{\alpha}$, the rendered image sequence, $\vec{y}$, is:

$$\vec{y} = PU\vec{\alpha}, \tag{1}$$

where P is the projection operator which collapses the y dimension of the image sequence $U\vec{\alpha}$.

Leventon and Freeman, Bayesian Estimation of Human Motion, MERL TR98-06

Orthography can lead to analytic solutions

have our multi-dimensional gaussian,

Prior probability

$$P(\vec{\alpha}) = k_2 e^{-\vec{\alpha}' \Lambda^{-1} - \vec{\alpha}}, \tag{3}$$

where k_2 is another normalization constant. If we model the observation noise as i.i.d. gaussian with variance σ, we have, for the likelihood term of Bayes theorem,

Likelihood function

$$P(\vec{y}|\vec{\alpha}) = k_3 e^{-|\vec{y} - PU\vec{\alpha}|^2/(2\sigma^2)}, \tag{4}$$

with normalization constant k_3.

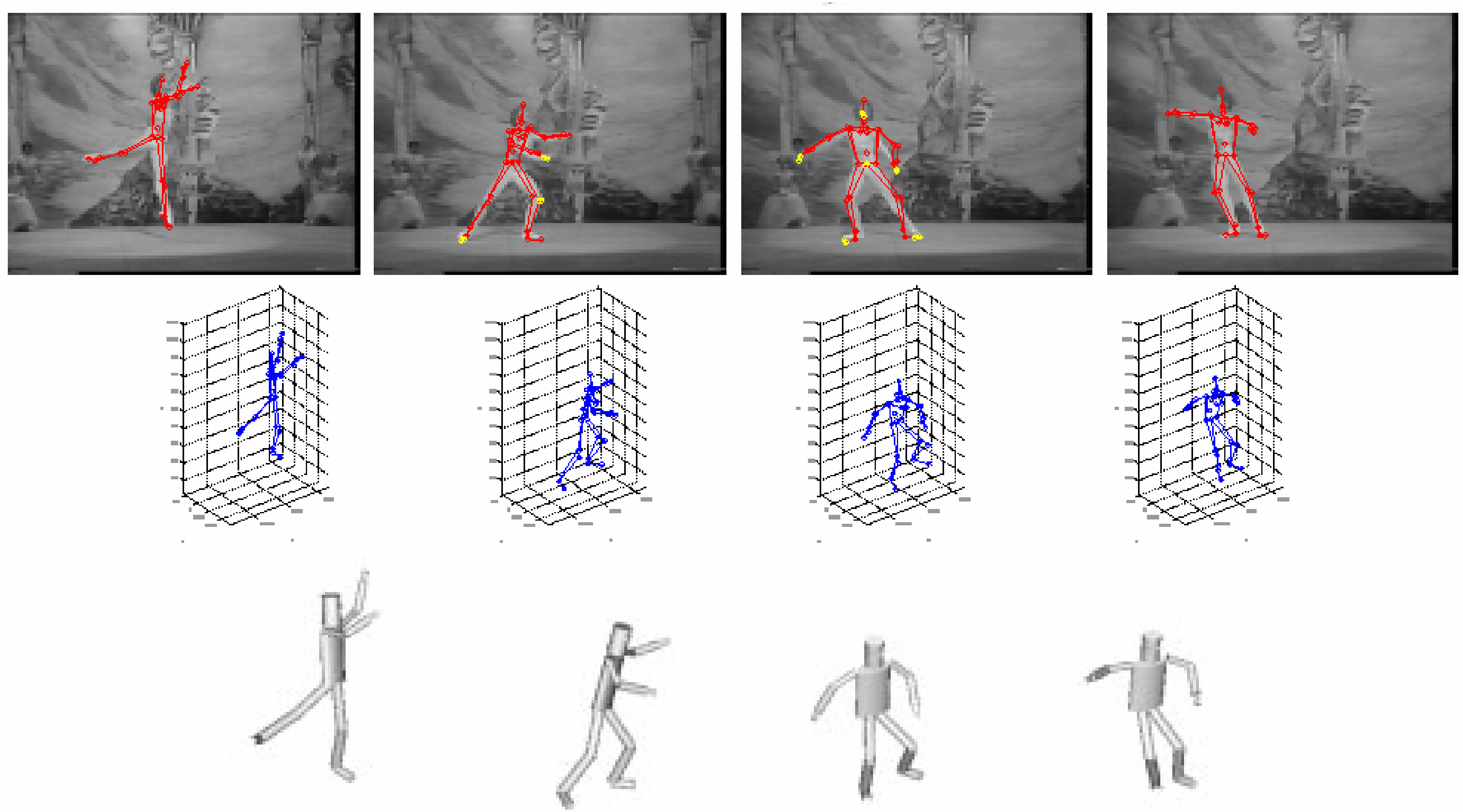
The posterior distribution is the product of these two gaussians. That yields another gaussian, with mean and covariance found by a matrix generalization of "completing the square" [7]. The squared error optimal estimate for α is then

$$\alpha = SU'P'(PUSU'P' + \sigma I)^{-1}(\vec{y} - (P\vec{m})) \tag{5}$$

Analytic solution for inferred 3-d motion

Leventon and Freeman, Bayesian Estimation of Human Motion, MERL TR98-06

Results



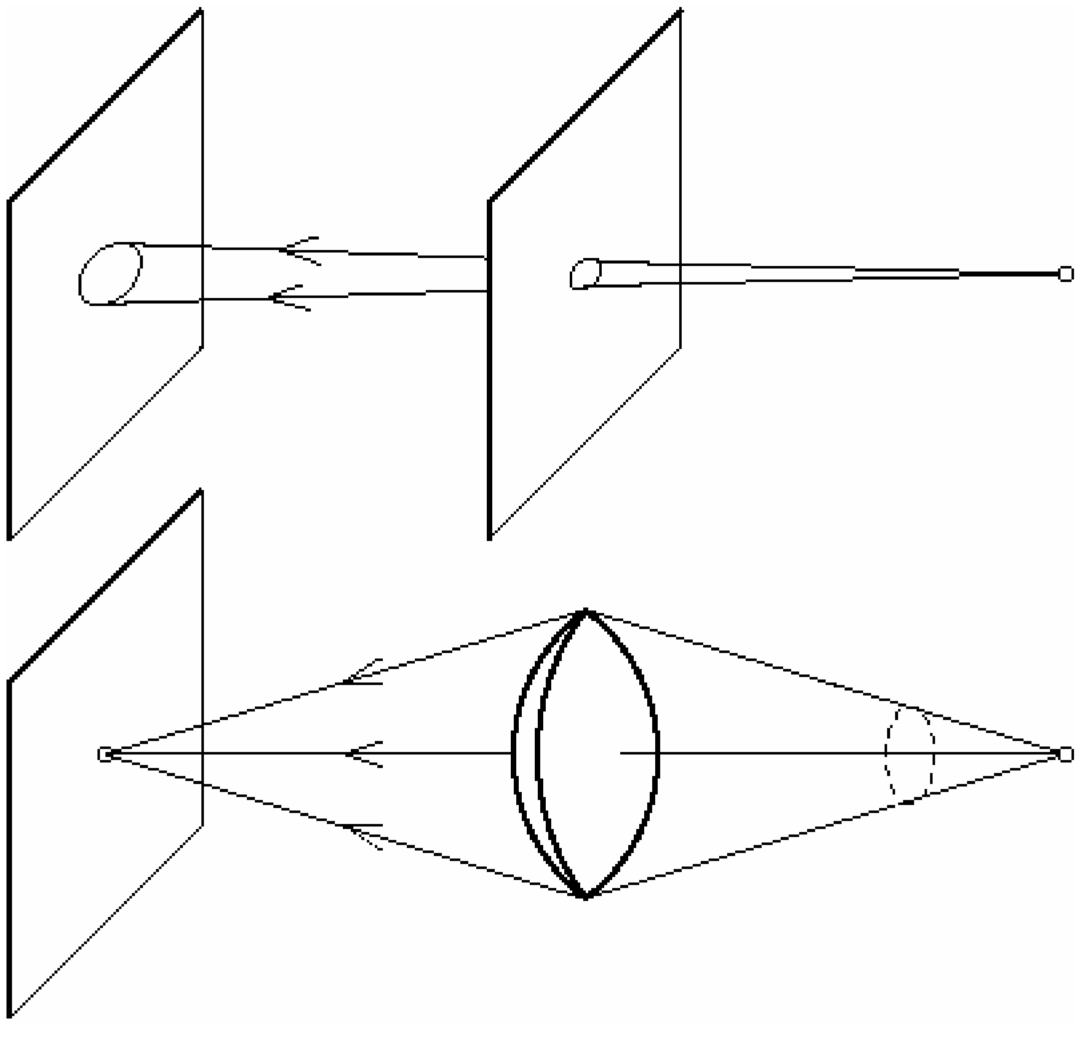
Leventon and Freeman, Bayesian Estimation of Human Motion, MERL TR98-06

But, alas

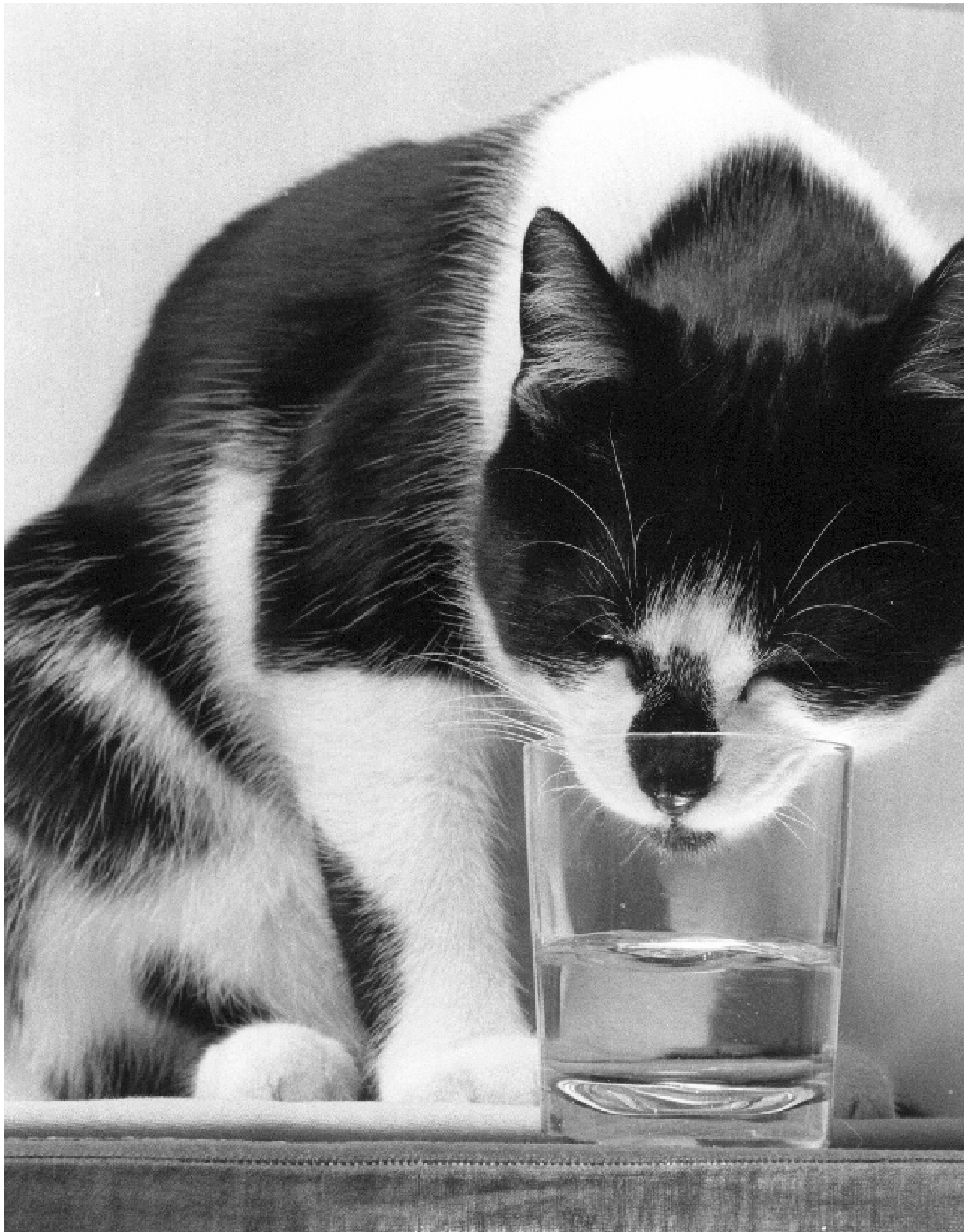
"The results for the simplified problem appear promising. However serious questions arise because of the simplifying assumptions, which trivialize a number of the hard issues of the problem in the real world. Eg. scaling effects that arise from perpective projection are ignored, by assuming orthographic projection. …"

Reviewer's comments

The reason for lenses

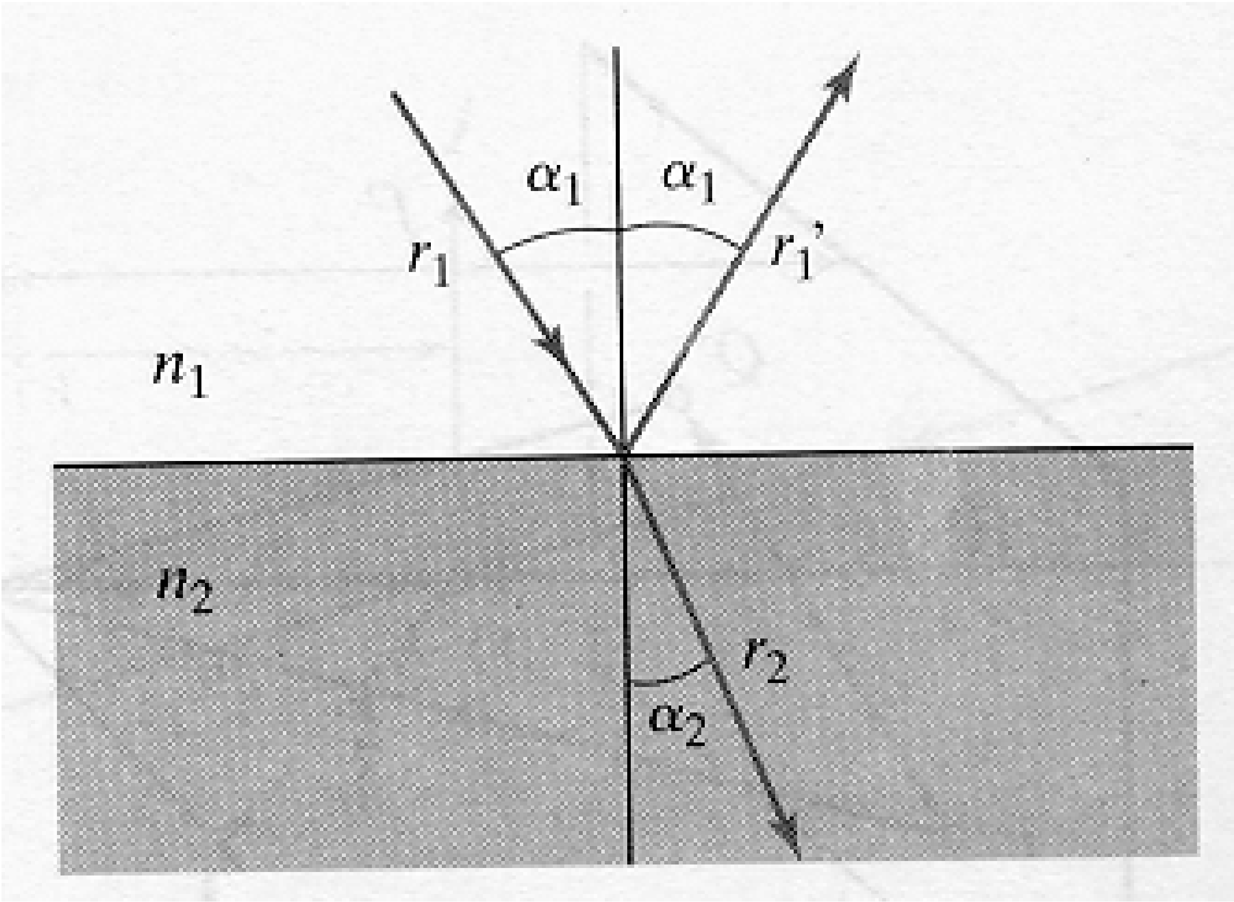


Water glass refraction



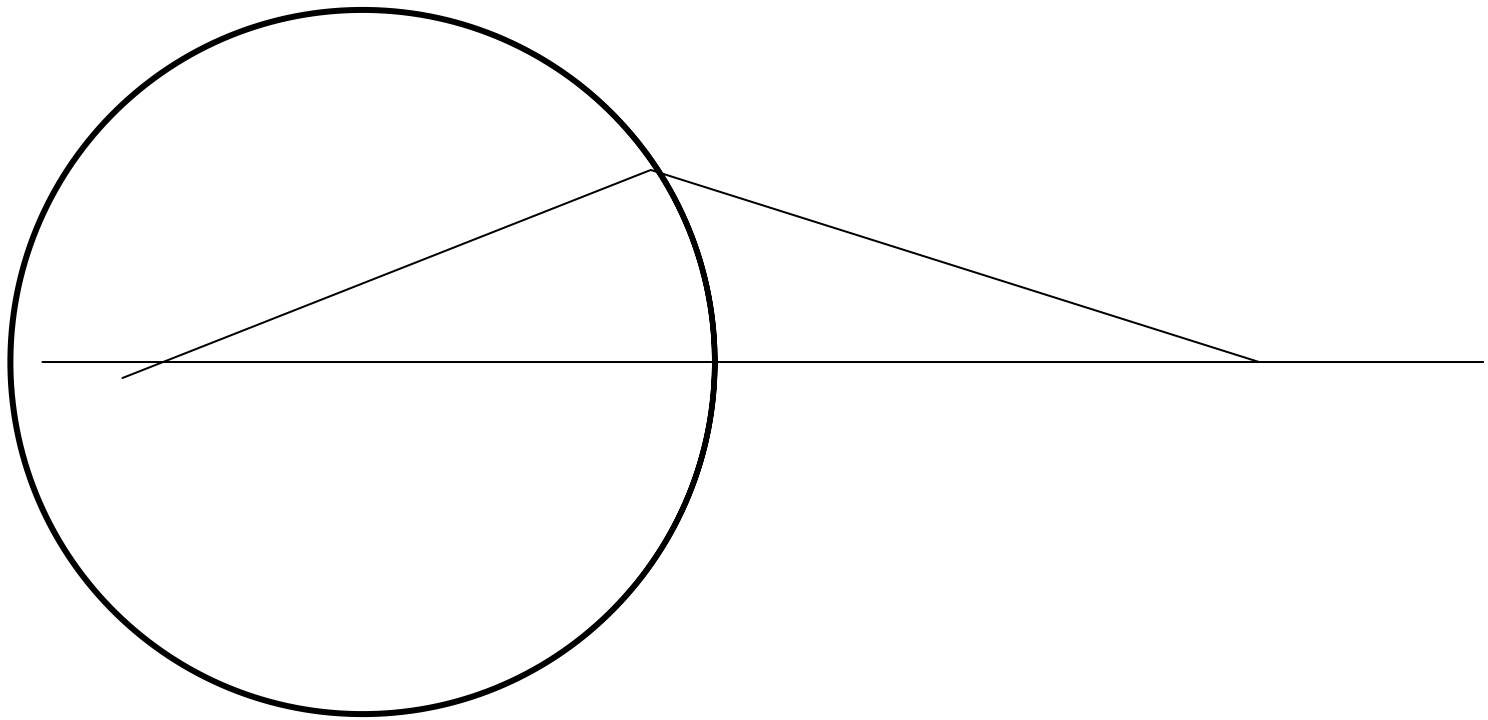
http://data.pg2k.hd.org/_exhibits/natural-science/cat-black-and-white-domestic-short-hair-DSH-with-nose-in-glass-of-water-on-bedside-table-tweaked-mono-1-AJHD.jpg

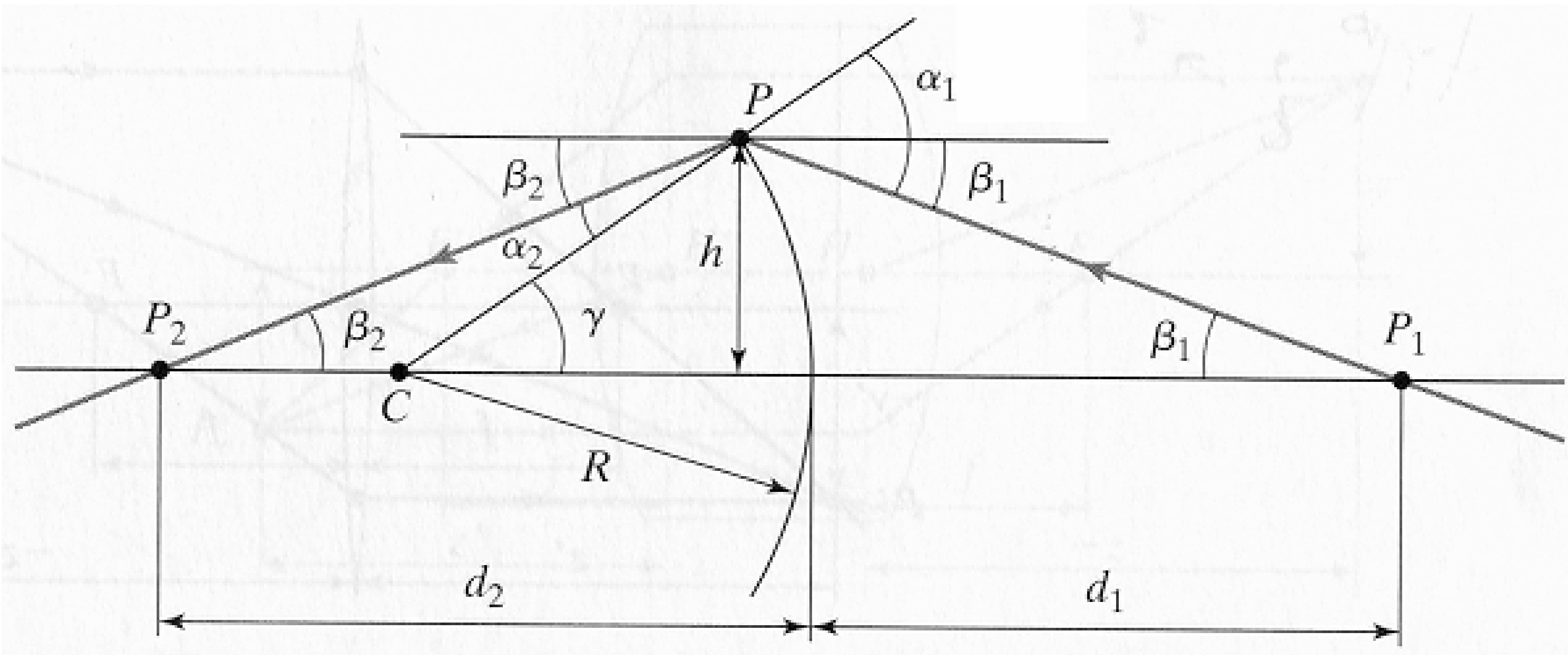
Snell's law



$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$$

Spherical lens

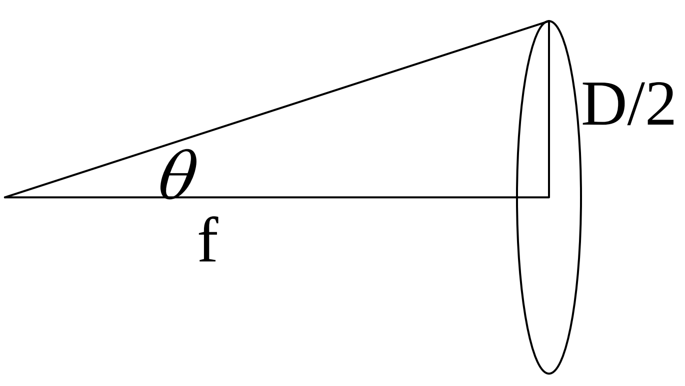




Forsyth and Ponce

First order optics

$$\sin(\theta) \approx \theta$$



$$\theta \approx \frac{D/2}{f}$$

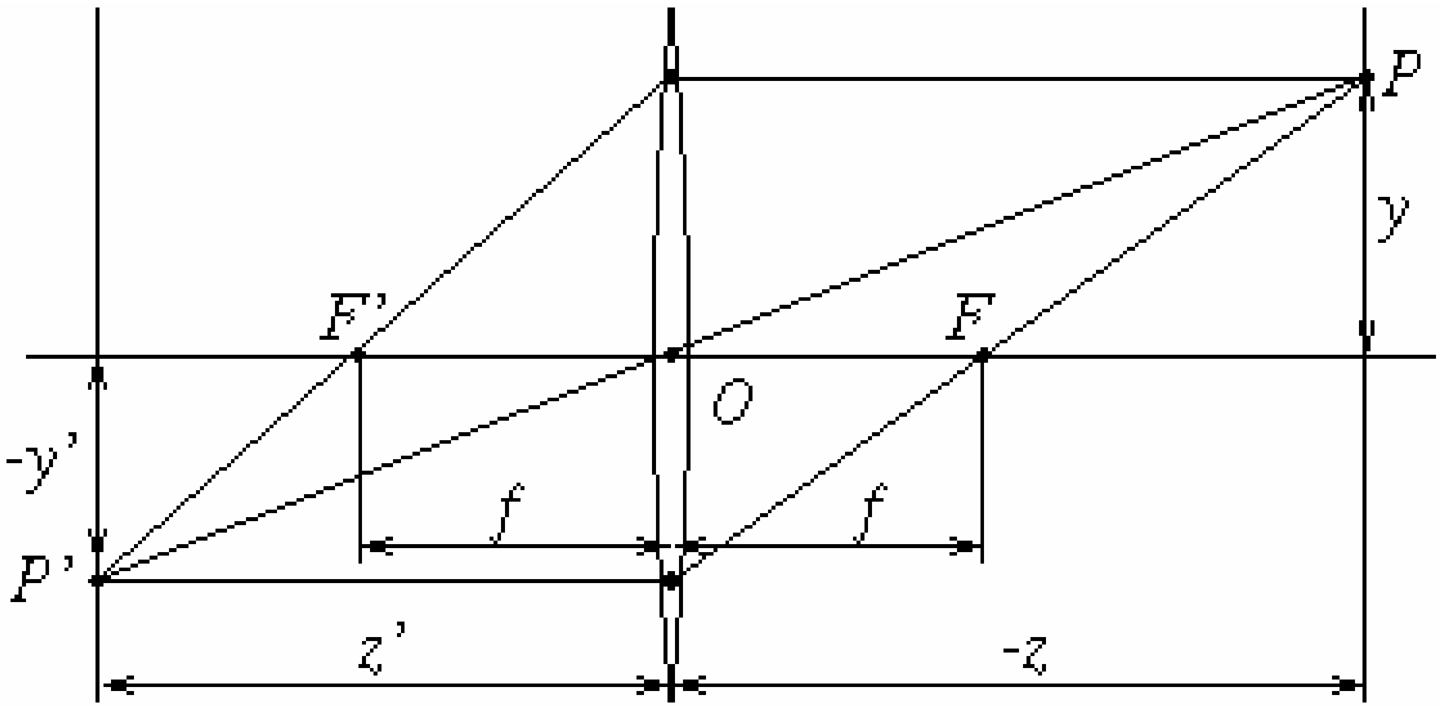
Paraxial refraction equation

$$\alpha_1 = \gamma + \beta_1 \approx h\left(\frac{1}{R} + \frac{1}{d_1}\right)$$

$$\alpha_2 = \gamma - \beta_2 \approx h\left(\frac{1}{R} - \frac{1}{d_2}\right)$$

$$n_1\alpha_1 \approx n_2\alpha_2 \Leftrightarrow \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$$

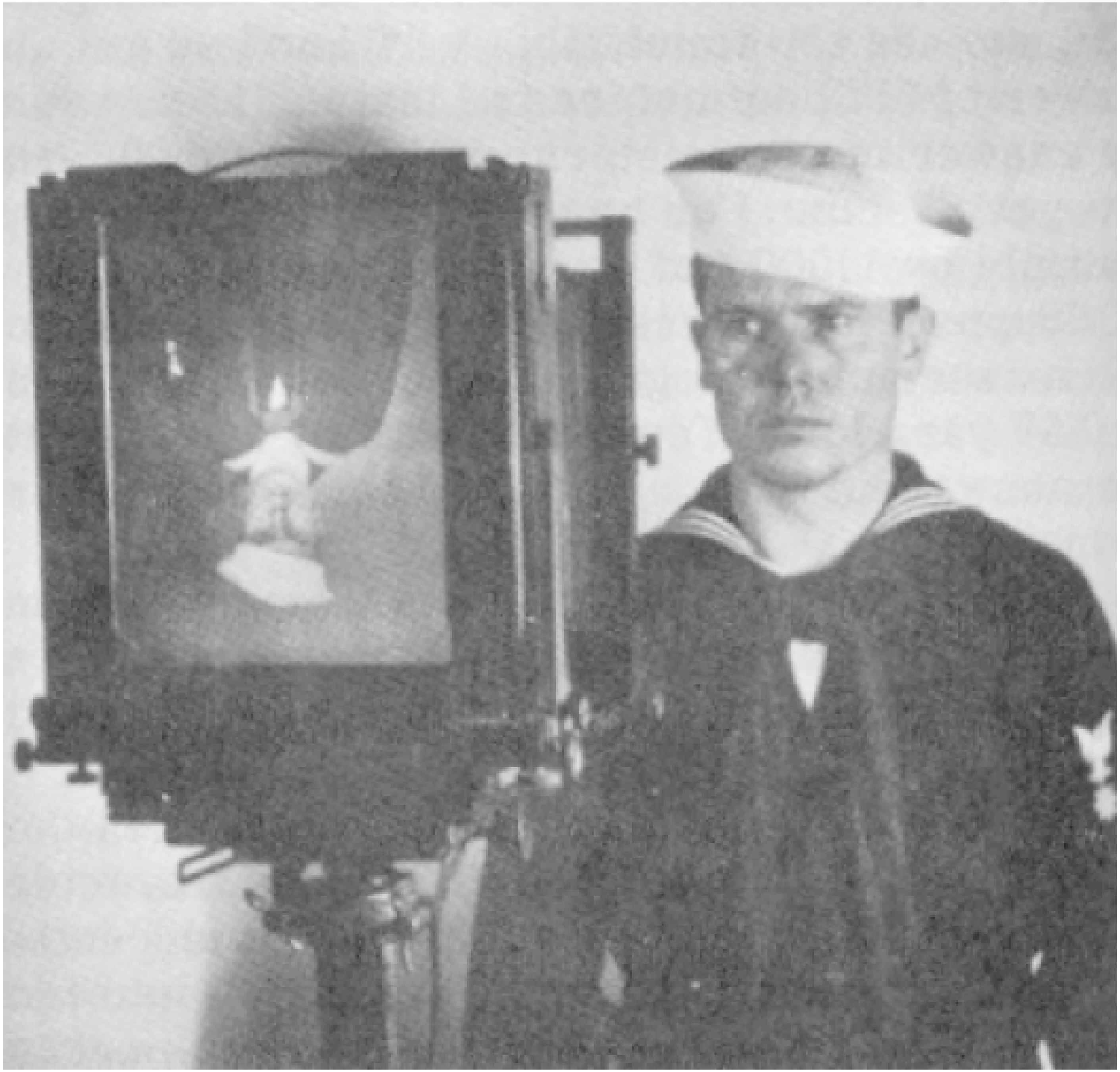
The thin lens, first order optics



$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$f = \frac{R}{2(n-1)}$$

Forsyth&Ponce



US Navy Manual

What camera projection model applies for a thin lens?

Candle and laser pointer demo

More accurate models of real lenses

- Finite lens thickness
- Higher order approximation to $\sin(\theta)$
- Chromatic aberration
- Vignetting

Thick lens

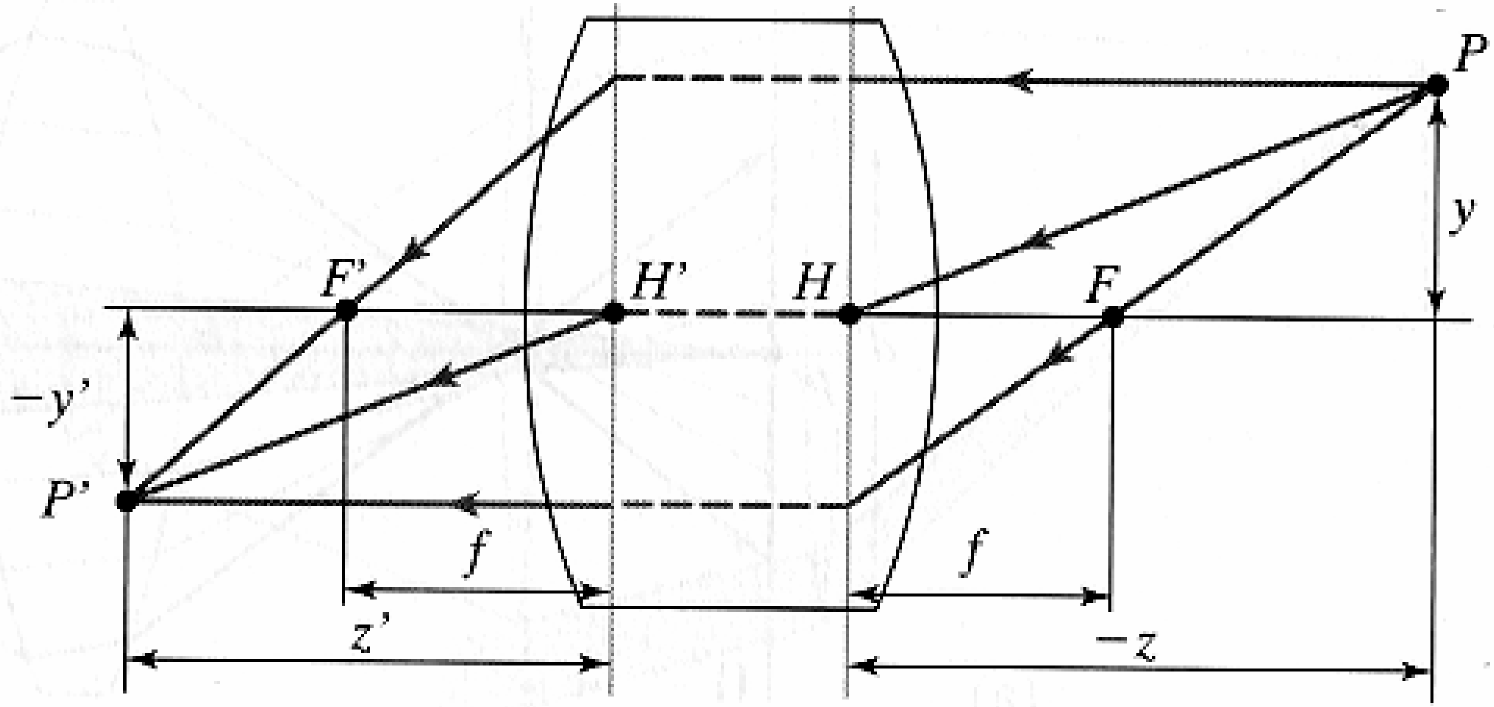
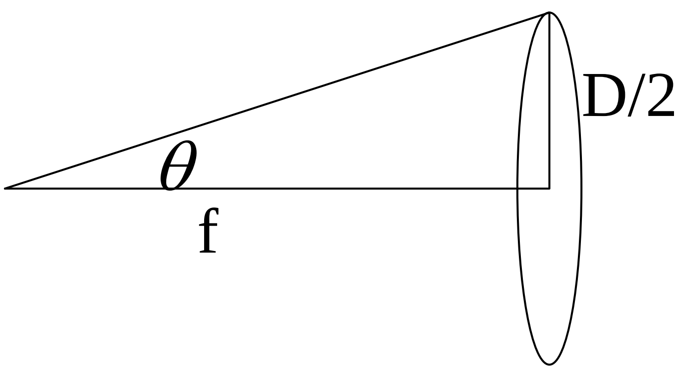


Figure 1.11 A simple thick lens with two spherical surfaces.

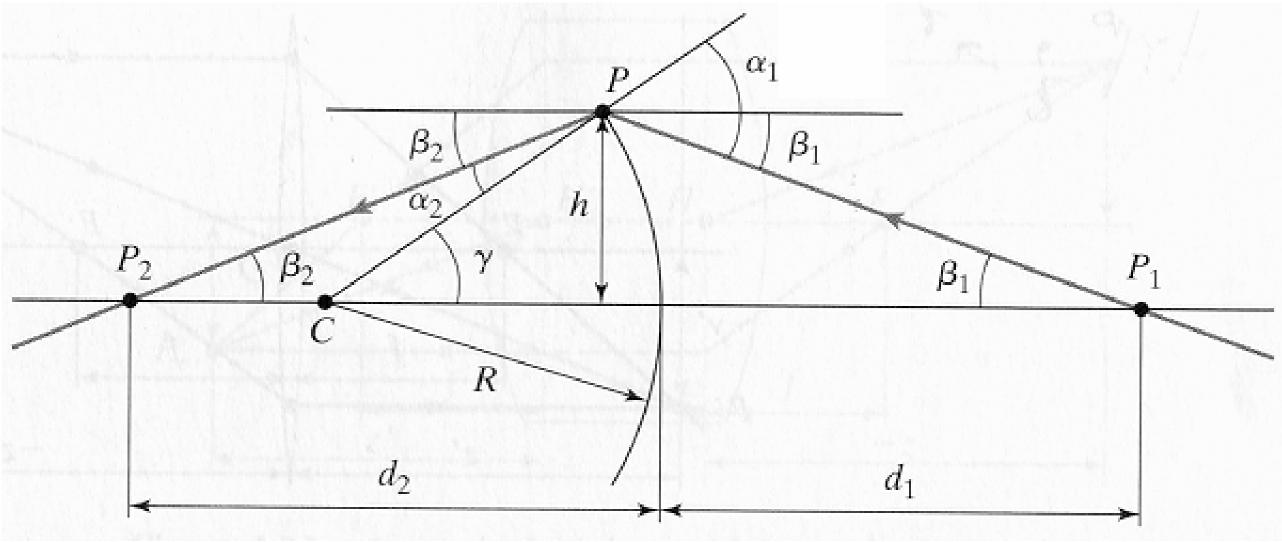
Third order optics

$$\sin(\theta) \approx \theta - \frac{\theta^3}{6}$$



$$\theta \approx \frac{D/2}{f} - \frac{\left(\frac{D/2}{f}\right)^3}{6}$$

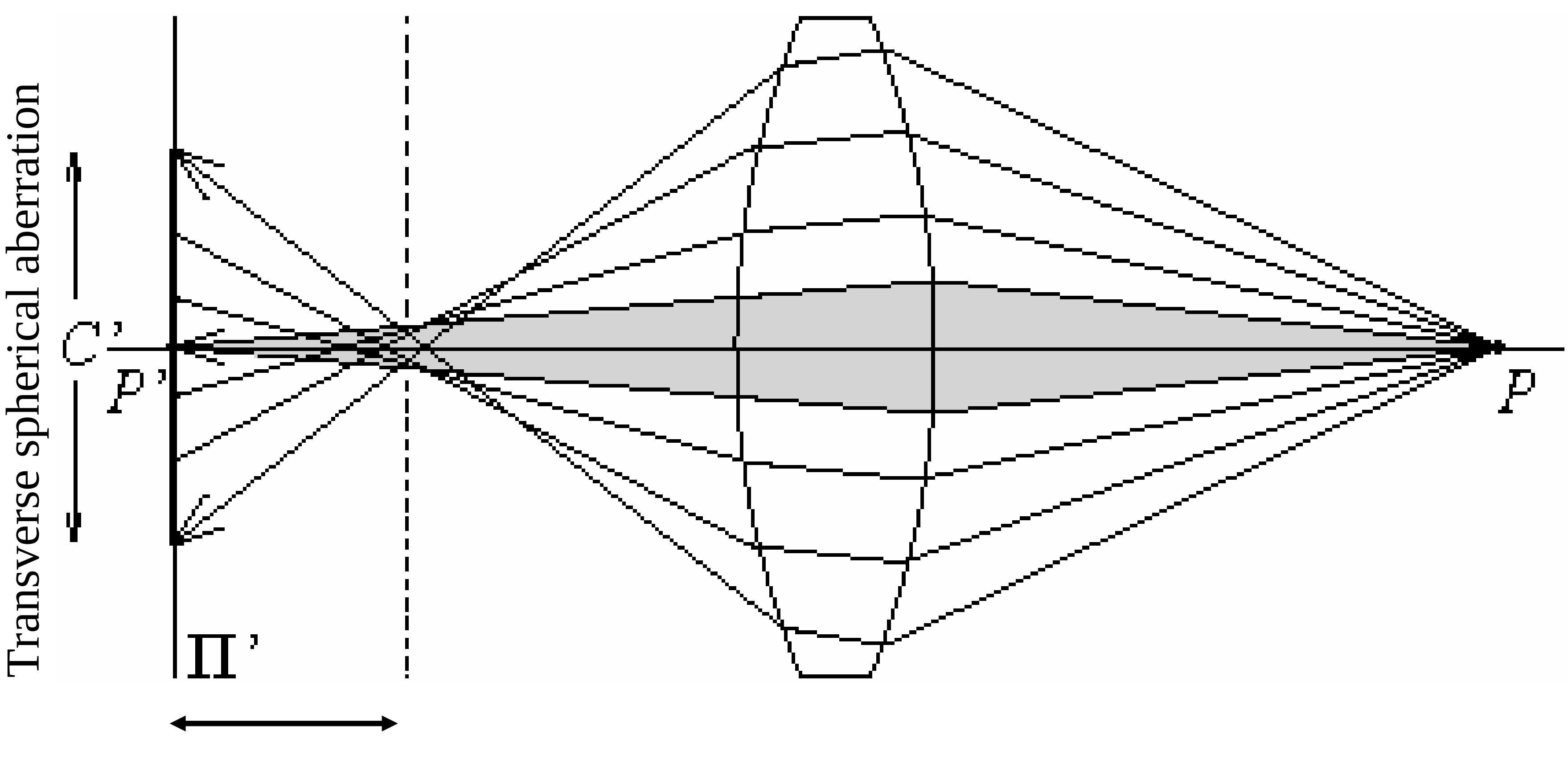
Paraxial refraction equation, 3rd order optics



$$\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R} + h^2 \left[\frac{n_1}{2d_1} \left(\frac{1}{R} + \frac{1}{d_1} \right)^2 + \frac{n_2}{2d_2} \left(\frac{1}{R} - \frac{1}{d_2} \right)^2 \right]$$

Forsyth&Ponce

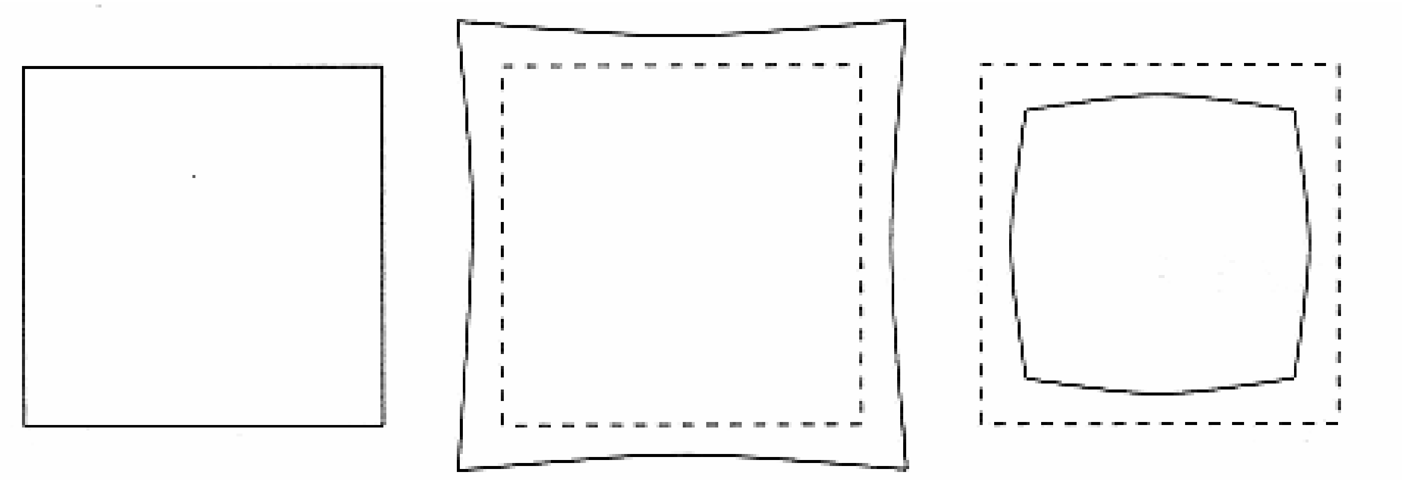
Spherical aberration (from 3rd order optics



Forsyth&Ponce

Other 3rd order effects

- Coma, astigmatism, field curvature, distortion.



no distortion

pincushion distortion

barrel distortion

Forsyth&Ponce

Astigmatic distortion

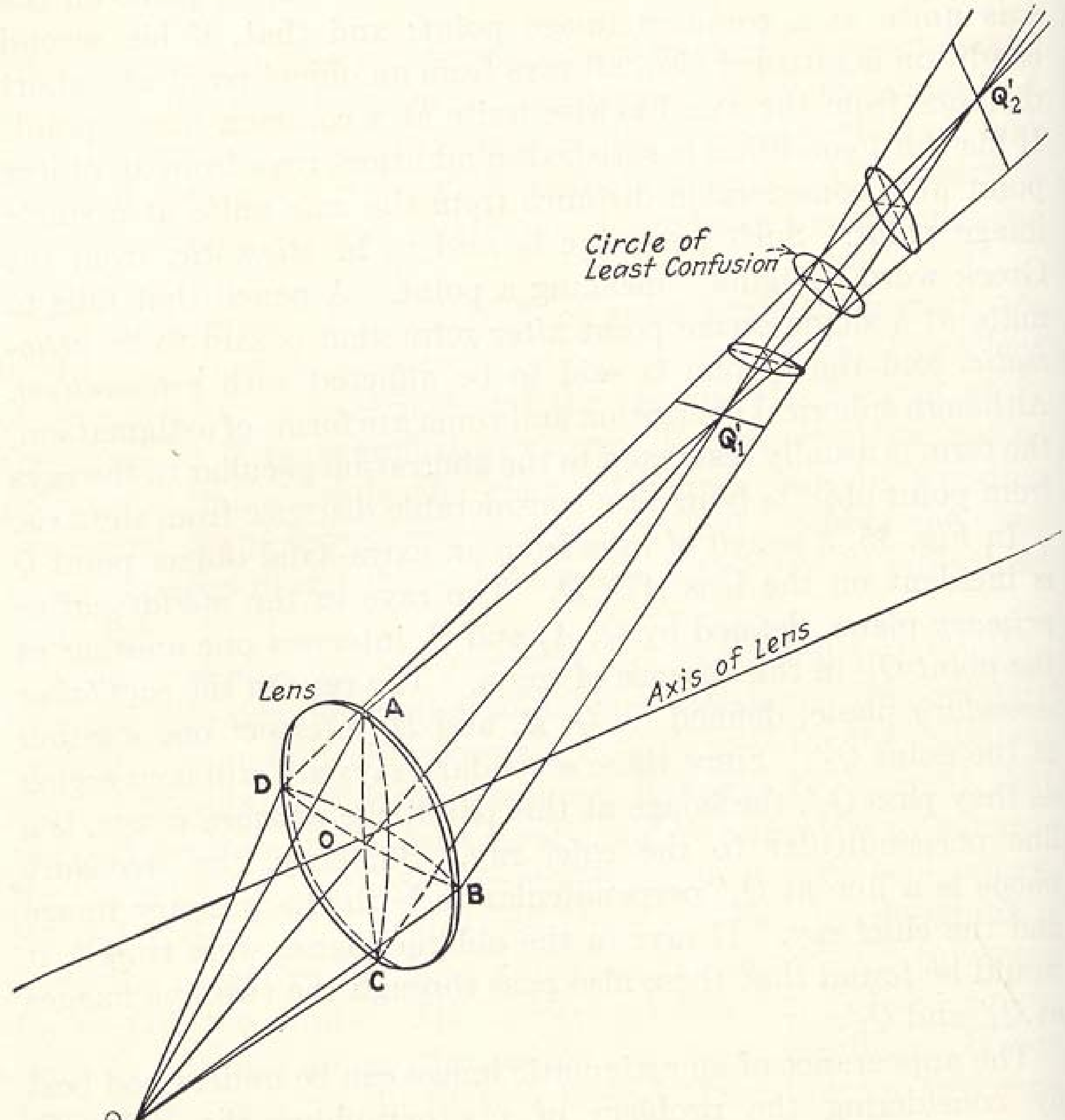
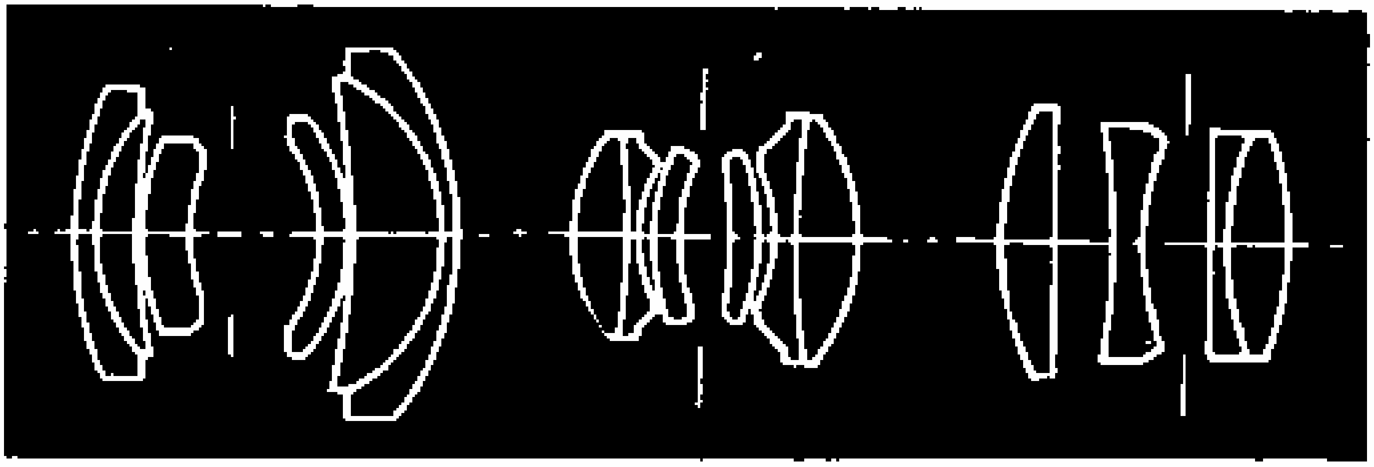


FIG. 45.—An illustration of the character of astigmatic images.

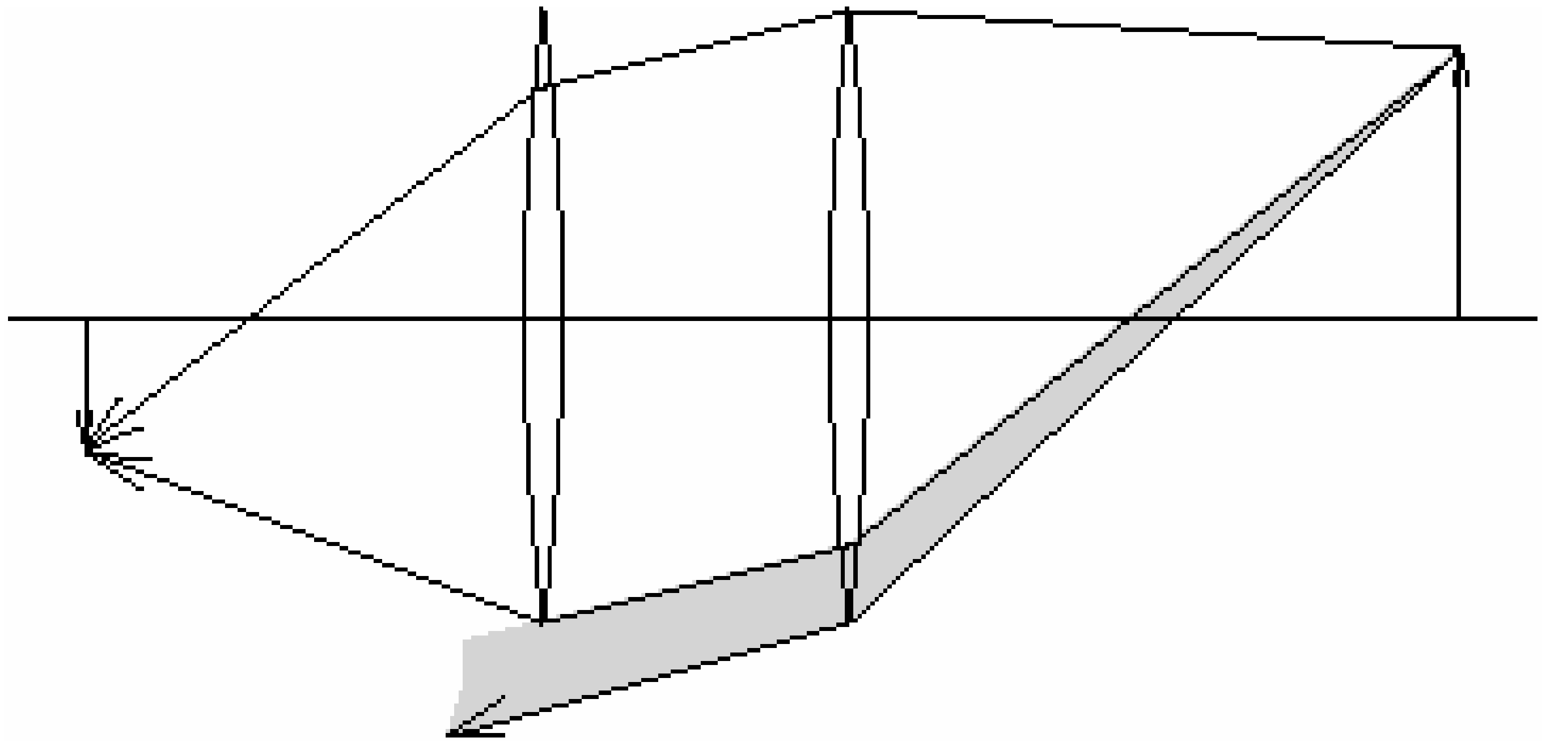
Hardy & Perrin,
The Principles of Optics, 1932

Lens systems



Lens systems can be designed to correct for aberrations described by 3rd order optics

Vignetting



Chromatic aberration

(great for prisms, bad for lenses)

Other (possibly annoying) phenomena

- Chromatic aberration
  - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  - Machines: coat the lens
  - Humans: live with it
- Scattering at the lens surface
  - Some light entering the lens system is reflected off each surface it encounters (Fresnel's law gives details)
  - Machines: coat the lens, interior
  - Humans: live with it (various scattering phenomena are visible in the human eye)

Summary

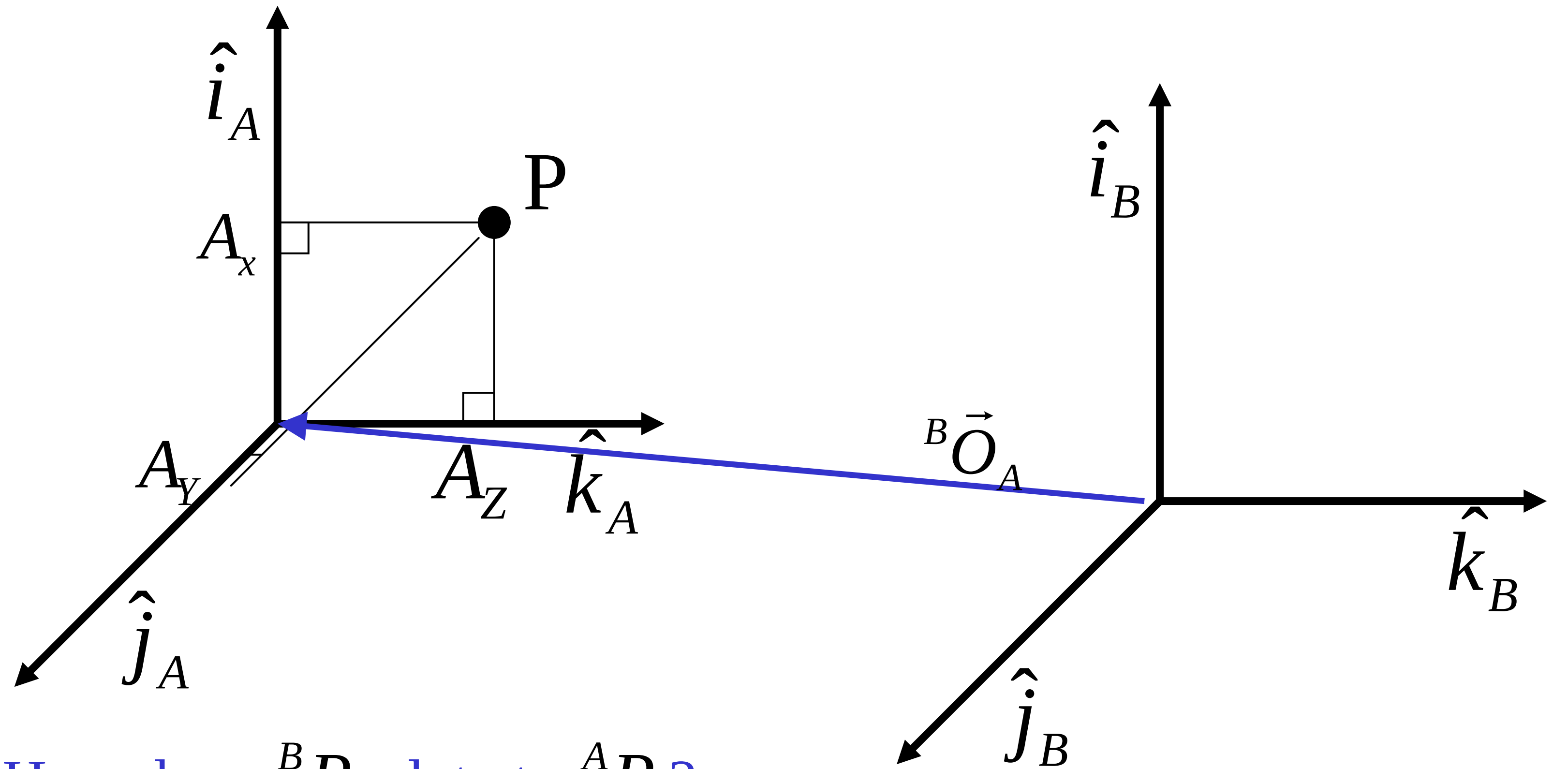
- Want to make images
- Pinhole camera models the geometry of perspective projection
- Lenses make it work in practice
- Models for lenses
  - Thin lens, spherical surfaces, first order optics
  - Thick lens, higher-order optics, vignetting.

Next

- how *positions* in the image relate to 3-d positions in the world.

Translation

$$^{A}P = \begin{pmatrix} A_x \\ A_Y \\ A_Z \end{pmatrix} \qquad ^{B}P = \begin{pmatrix} B_x \\ B_Y \\ B_Z \end{pmatrix}$$

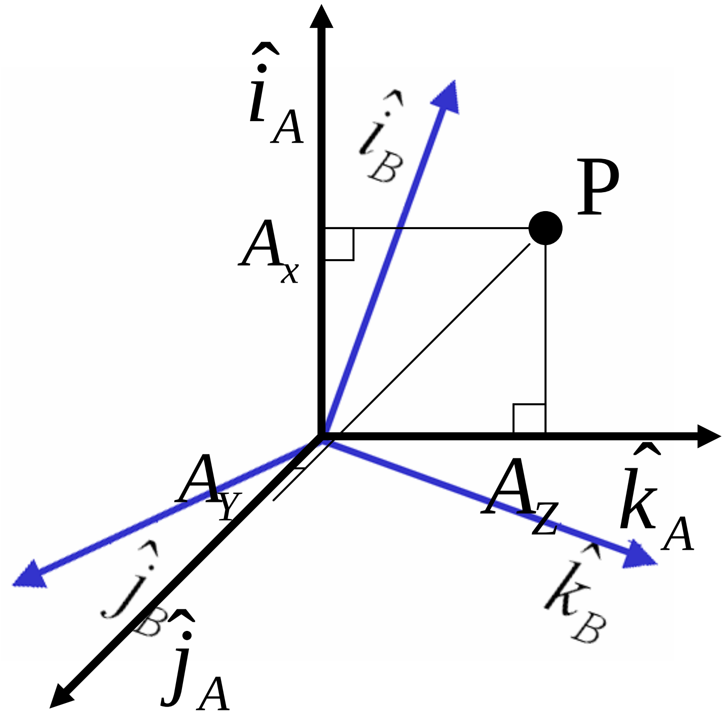


How does ^{B}P relate to ^{A}P ?

$$^{B}P = {}^{A}P + {}^{B}O_A$$

$$ {}^{A}P = \begin{pmatrix} A_x \\ A_Y \\ A_Z \end{pmatrix} \qquad {}^{B}P = \begin{pmatrix} B_x \\ B_Y \\ B_Z \end{pmatrix} $$

Rotation

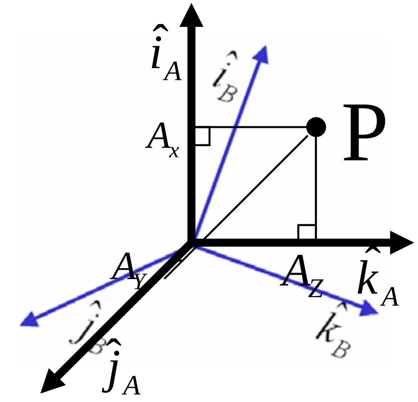


How does ${}^{B}P$ relate to ${}^{A}P$?

$$ {}^{B}P = {}^{B}_{A}R \; {}^{A}P $$

Find the rotation matrix

Project $\overrightarrow{OP} = \begin{pmatrix} \hat{i}_A & \hat{j}_A & \hat{k}_A \end{pmatrix} \begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix}$



onto the B frame's coordinate axes.

$$\begin{pmatrix} B_X \\ B_Y \\ B_Z \end{pmatrix} = \begin{pmatrix} \hat{i}_B \bullet \hat{i}_A A_X & \hat{i}_B \bullet \hat{j}_A A_Y & \hat{i}_B \bullet \hat{k}_A A_Z \\ \hat{j}_B \bullet \hat{i}_A A_X & \hat{j}_B \bullet \hat{j}_A A_Y & \hat{j}_B \bullet \hat{k}_A A_Z \\ \hat{k}_B \bullet \hat{i}_A A_X & \hat{k}_B \bullet \hat{j}_A A_Y & \hat{k}_B \bullet \hat{k}_A A_Z \end{pmatrix}$$

Rotation matrix

this

$$\begin{pmatrix} B_X \\ B_Y \\ B_Z \end{pmatrix} = \begin{pmatrix} \hat{i}_B \bullet \hat{i}_A A_X & \hat{i}_B \bullet \hat{j}_A A_Y & \hat{i}_B \bullet \hat{k}_A A_Z \\ \hat{j}_B \bullet \hat{i}_A A_X & \hat{j}_B \bullet \hat{j}_A A_Y & \hat{j}_B \bullet \hat{k}_A A_Z \\ \hat{k}_B \bullet \hat{i}_A A_X & \hat{k}_B \bullet \hat{j}_A A_Y & \hat{k}_B \bullet \hat{k}_A A_Z \end{pmatrix}$$

implies

$$^{B}P = {}^{B}_{A}R \; {}^{A}P$$

where

$$^{B}_{A}R = \begin{pmatrix} \hat{i}_B \bullet \hat{i}_A & \hat{i}_B \bullet \hat{j}_A & \hat{i}_B \bullet \hat{k}_A \\ \hat{j}_B \bullet \hat{i}_A & \hat{j}_B \bullet \hat{j}_A & \hat{j}_B \bullet \hat{k}_A \\ \hat{k}_B \bullet \hat{i}_A & \hat{k}_B \bullet \hat{j}_A & \hat{k}_B \bullet \hat{k}_A \end{pmatrix}$$

Translation and rotation

Let's write $^{B}P = {}^{B}_{A}R \; {}^{A}P + {}^{B}O_{A}$

as a single matrix equation:

$$\begin{pmatrix} B_X \\ B_Y \\ B_Z \\ 1 \end{pmatrix} = \begin{pmatrix} - & - & - & | \\ - & {}^{B}_{A}R & - & {}^{B}O_{A} \\ - & - & - & | \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_X \\ A_Y \\ A_Z \\ 1 \end{pmatrix}$$