Massachusetts Institute of Technology — Handout 24
6.046J/18.410J: Introduction to Algorithms — April 11, 2002
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Problem Set 6

This problem set is due **AT THE BEGINNING OF** class on *April 25, 2002*.

Both exercises and problems should be solved, but *only the problems* should be turned in. Exercises are intended to help you master the course material and will be useful in solving the assessed problems. You are responsible for material covered by the exercises. Each problem is to be done on a separate sheet (or sheets) of paper. Mark the top of each sheet with your name, 6.046J/18.410J, the problem number, your TA and recitation section, the date, and the names of any students with whom you collaborated.

Problem 6-1. Six Degrees of Separation from Kevin Bacon

A *diameter* of a (possibly directed) graph $G = (V, E)$ is the minimum value Δ such that for all $p, q \in V$, the distance from p to q is at most Δ (the distance is simply the number of edges along the shortest path). If the graph is not connected (or is directed and not strongly connected), then $\Delta = \infty$. If the diameter is finite, then we know that any two nodes are within distance Δ from each other, which is a useful thing to know.

From the lecture we know that a diameter of a graph can be computed in time $O(|V||E|)$. However, if the graph is large, this is too costly. The goal of this problem is to design a faster (but approximate) algorithm for this problem,

(a) Assume that G is undirected. Consider an algorithm which computes the shortest path distances $D(u, v)$ from an arbitrary vertex u to all other $v \in V$. Then, the algorithm reports

$$\tilde{\Delta} = \max_{v \in V} D(u, v)$$

as an approximation to the diameter Δ. Show that $\tilde{\Delta} \le \Delta \le 2\tilde{\Delta}$, and that $\tilde{\Delta}$ can be computed in time $O(|E|)$.

(b) Modify the algorithm and result from part (a) to apply to *directed* graphs.

(c) Go to the web page `http://www.cs.virginia.edu/oracle/` and click on `Bacon numbers`. Based on the just acquired wisdom and the information stored at the web page, give an estimation for the diameter of the *Bacon graph*, defined over the set of all actors reachable from Kevin Bacon (see the web page for details).

Problem 6-2. Shortest Path With Small Weights

You are given a weighted graph, where the weights are integers in the range from $\{1 \ldots 10\}$. Give a linear time algorithm for finding the shortest path from a given node u to a given node v in such a graph.

Problem 6-3. Paintball^{TM}

You and your friends thought that playing Paintball^{TM} over a network of platforms situated in the jungle would be a good way to forget about 6.046 for a few hours. No such luck! The platforms are connected by rickety bridges, each of which has an independent failure probability. As every Paintballer^{TM} knows, the goal of each player to get from platform s to platform t in one piece in order to gain a better defensive or offensive position. You've created a graph with each platform as a node and each bridge as an edge in order to better analyze your situation.

Give a polynomial time algorithm to find the path from node s to node t with minimum failure probability.

Problem 6-4. Boy-Girl Matching Revisited

Last week we attempted to match students up for tango dancing based on their heights h_i. Having done so, we realized that matching students of similar ages a_i would also be useful. Design an efficient algorithm that maximizes the number of student pairs, such that in each pair, the partners' heights differ by no more than c and their ages differ by no more than d. (Hint: A good place to start on this problem would be to read section 26.3 in CLRS on maximum bipartite matching).