Problem Set 5

This problem set is due **AT THE BEGINNING OF** class on *April 11, 2002*.

Both exercises and problems should be solved, but *only the problems* should be turned in. Exercises are intended to help you master the course material and will be useful in solving the assessed problems. You are responsible for material covered by the exercises. Each problem is to be done on a separate sheet (or sheets) of paper. Mark the top of each sheet with your name, 6.046J/18.410J, the problem number, your TA and recitation section, the date, and the names of any students with whom you collaborated.

Exercise 5-1. Like you guys actually do any of these!! Well.. take some time off and prepare for the quiz.

Problem 5-1. Bottleneck Spanning Tree

Solve parts (a) and (b) in Problem 23-3 in CLRS page 577.

Problem 5-2. Boy-Girl Matching

For the end of term, we will be teaching tango dancing. Before we can do that, we need to pair up every male and female in the class (pretend that due to some miracle, the number of males and females in 6.046 is equal). We know the height of every student i to be h_i.

(a) Design an efficient algorithm that makes boy-girl pairs with minimal maximum height difference. Specifically, given a pair of students (i, j), let $\Delta(i, j) = |h_i - h_j|$. Your algorithm should make sure that the largest $\Delta(i, j)$ over all your pairings is as small as possible.

To be more clear, let us say that you are given three boys of height $(6, 4, 9)$ and three girls of height $(8, 7, 3)$. If you paired them like $(6, 8), (4, 7), (9, 3)$ then the maximum Δ is 6, whereas if you paired them as $(6, 3), (4, 7), (9, 8)$, the maximum Δ is 3. We want your algorithm to spit out the pairing with the smallest maximum delta.

Prove that your algorithm is correct, i.e., that it finds the pairing with the smallest value of maximum Δ.

(b) Does your algorithm minimize the sum of Δ? (for the examples above, the total Δ were 11 and 7). Explain why or give a counterexample where your algorithm does not provide the optimal solution for the sum of Δ.

(c) Does your algorithm from part (a) maximize the total number of of equal-height pairs (where the $\Delta = 0$) ? Explain why or give a counterexample.

Problem 5-3. Disjoint Sets Remembered

Recall the 'linked-list' representation of disjoint sets from class (also covered in page 501 of the book).

Argue that the total time spent on performing n MAKE-SET and m UNION operations is $O(n + m \log n)$.