Master Theorem Worksheet Solutions

This is a worksheet to help you master solving recurrence relations using the Master Theorem. For each recurrence, either give the asympotic solution using the Master Theorem (state which case), or else state that the Master Theorem doesn't apply. You should be able to go through these **25** recurrences in **10** minutes.

Problem 1-1. $T(n) = 3T(n/2) + n^2$
$T(n) = \Theta(n^2)$ (case 3).

Problem 1-2. $T(n) = 7T(n/2) + n^2$
$T(n) = \Theta(n^{\lg 7})$ (case 1).

Problem 1-3. $T(n) = 4T(n/2) + n^2$
$T(n) = \Theta(n^2 \lg n)$ (case 2).

Problem 1-4. $T(n) = 3T(n/4) + n \lg n$
$T(n) = \Theta(n \lg n)$ (case 3).

Problem 1-5. $T(n) = 4T(n/2) + \lg n$
$T(n) = \Theta(n^2)$ (case 1).

Problem 1-6. $T(n) = T(n-1) + n$
M.T. doesn't apply. Iteration gives $T(n) = \Theta(n^2)$.

Problem 1-7. $T(n) = 4T(n/2) + n^2 \lg n$
$T(n) = \Theta(n^2 \lg^2 n)$ (extended case 2).

Problem 1-8. $T(n) = 5T(n/2) + n^2 \lg n$
$T(n) = \Theta(n^{\lg 5})$ (case 1).

Problem 1-9. $T(n) = 3T(n/3) + n/\lg n$
M.T. case 1 doesn't apply since $f(n) = n/\lg n$ is not polynomially smaller than $n^{\log_3 3-\varepsilon}$ for any $\varepsilon > 0$.

Problem 1-10. $T(n) = 2T(n/4) + c$
$T(n) = \Theta(n^{1/2})$ (case 1).

Problem 1-11. $T(n) = T(n/4) + \lg n$
$T(n) = \Theta(\lg^2 n)$ (extended case 2).

Problem 1-12. $T(n) = T(n/2) + T(n/4) + n^2$
M.T. doesn't apply. Recursion tree gives guess $T(n) = \Theta(n^2)$.

Problem 1-13. $T(n) = 2T(n/4) + \lg n$
$T(n) = \Theta(n^{1/2})$ (case 1).

Problem 1-14. $T(n) = 3T(n/3) + n \lg n$
$T(n) = \Theta(n \lg^2 n)$ (extended case 2).

Problem 1-15. $T(n) = 8T((n - \sqrt{n})/4) + n^2$
M.T. doesn't apply. Using Akra-Bazzi can ignore $\sqrt{n}/4$, which gives $\Theta(n^2)$. Could also use M.T. to get an upper bound of $O(n^2)$ by removing the $\sqrt{n}/4$ term and a lower bound of $\Omega(n^2)$ by replacing the $(n - \sqrt{n})/4$ term by $0.24n$.

Problem 1-16. $T(n) = 2T(n/4) + \sqrt{n}$
$T(n) = \Theta(n^{1/2} \lg n)$ (case 2).

Problem 1-17. $T(n) = 2T(n/4) + n^{0.51}$
$T(n) = \Theta(n^{0.51})$ (case 3).

Problem 1-18. $T(n) = 16T(n/4) + n!$
$T(n) = \Theta(n!)$ (case 3).

Problem 1-19. $T(n) = 3T(n/2) + n$
$T(n) = \Theta(n^{\lg 3})$ (case 1).

Problem 1-20. $T(n) = 4T(n/2) + cn$
$T(n) = \Theta(n^2)$ (case 1).

Problem 1-21. $T(n) = 3T(n/3) + n/2$
$T(n) = \Theta(n \lg n)$ (case 2).

Problem 1-22. $T(n) = 4T(n/2) + n/\lg n$
$T(n) = \Theta(n^2)$ (case 1).

Problem 1-23. $T(n) = 7T(n/3) + n^2$
$T(n) = \Theta(n^2)$ (case 3).

Problem 1-24. $T(n) = 8T(n/3) + 2^n$
$T(n) = \Theta(2^n)$ (case 3).

Problem 1-25. $T(n) = 16T(n/4) + n$
$T(n) = \Theta(n^2)$ (case 1).