Massachusetts Institute of Technology
6.046J/18.410J: Introduction to Algorithms
Professors Piotr Indyk and Madhu Sudan

Handout 19
May 3, 2001

Problem Set 6

This problem set is due in class on *Thursday, May 10th* at the end of class.

There are three problems. **Each problem is to be done on a separate sheet (or sheets) of paper. Separate problems should *not* be stapled together.** Mark the top of **each and every sheet** with your name, 6.046J/18.410J, the **problem number**, your **recitation section, your TA's name**, and the date. It is very important to hand in problems separately and to indicate your recitation section **and** TA name on every sheet. Failure to do so will result in delays in returning your work and possible confusions with entering your grade. All problem sets throughout the semester should be handed in this way.

Reading: Chapter 35

Problem 6-1. Same side or not?

Give a data structure that represents n points in the plane using only $O(n)$ space and can perform the following operation in $O(\log n)$ time:

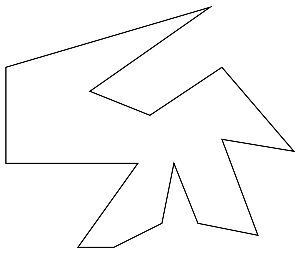
query(l) – returns "same side" if all the points are on the same side of line l, otherwise "not same side"

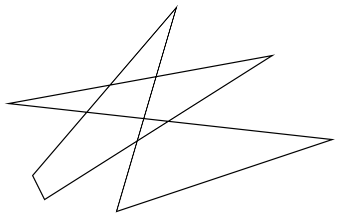
You may use a lot of time (say $O(n^2)$) to build the data structure initially, but you must end up with a data structure that uses only $O(n)$ space and can answer each query in only $O(\log n)$ time.

(**Hint:** It might help to look at a convex hull.)

Problem 6-2. Make this one simple

Give an $O(n \log n)$ time algorithm that takes n points in the plane and constructs a simple polygon that has each point as a vertex and no other points as vertices. (A simple polygon is a polygon that does not cross itself. For example, the figure on the left is simple, but the figure on the right is not.)





Problem 6-3. Approximate Furthest Pair

Give an $O(n)$ time algorithm that takes n points in the plane and finds a pair of points who are at least half as far apart as the furthest pair. That is, suppose that a and b are the two points in the inputs that are furthest apart. Find a pair c, d such that $dist(c, d) \geq dist(a, b)/2$.