Problem Set 8

This problem set is due **in recitation** on **Friday, December 6.**

Reading: §24.1–2; §24.4–5; Chapter 25; §26.1–2

Both exercises and problems should be solved, but *only the problems* should be turned in. Exercises are intended to help you master the course material. Even though you should not turn in the exercise solutions, you are responsible for material covered by the exercises.

Mark the top of each sheet with your name, the course number, the problem number, your recitation section, the date, and the names of any students with whom you collaborated.

Each problem should be done on a separate sheet (or sheets) of three-hole punched paper.

You will often be called upon to "give an algorithm" to solve a certain problem. Your write-up should take the form of a short essay. A topic paragraph should summarize the problem you are solving and what your results are. The body of your essay should provide the following:

1. A description of the algorithm in English and, if helpful, pseudocode.
2. At least one worked example or diagram to show more precisely how your algorithm works.
3. A proof (or indication) of the correctness of the algorithm.
4. An analysis of the running time of the algorithm.

Remember, your goal is to communicate. Graders will be instructed to take off points for convoluted and obtuse descriptions.

Exercise 8-1. No exercises this time! Take a break, or study for the final.

Problem 8-1. You are planning a world-tour, and you want to make the most of your money. Your bank gives you a table of ***exchange rates*** between different world currencies, which are the "prices" of currencies in terms of each other (e.g., "US$0.96 buys 1 euro"). However, these rates aren't perfect — to get the most euros for your dollar, you might need to convert from dollars to yen first, then from yen to euros.

(a) Sometimes a process called ***arbitrage*** is possible, in which one starts with one type of currency, puts it through a series of exchanges, and ends up with even more of the original currency! Give an efficient algorithm which, given the bank's exchange rates, tells whether arbitrage is possible.

(b) Now suppose the bank is smart enough to avoid offering exchange rates that lend themselves to arbitrage. Give an efficient algorithm that will compute the best exchange rates you can get between all types of currencies. Additionally, explain how to determine the exact exchanges you must make to get the best rate between any two given currencies.

Problem 8-2. A ***path cover*** of a directed graph $G = (V, E)$ is a set P of vertex-disjoint paths such that every vertex in V is included in exactly one path in P. Paths may start and end anywhere, and they may be of any length, including 0. A ***minimum path cover*** of G is a path cover containing the fewest possible paths.

(a) Give an efficient algorithm to find a minimum path cover of a directed acyclic graph $G = (V, E)$. (*Hint:* Assuming that $V = \{1, \ldots, n\}$, construct the graph $G' = (V', E')$, where

$$\begin{aligned} V' &= \{x_0, \ldots, x_n\} \cup \{y_0, \ldots, y_n\}, \\ E' &= \{(x_0, x_i) : i \in V\} \cup \{(y_i, y_0) : i \in V\} \cup \{(x_i, y_j) : (i, j) \in E\} \end{aligned}$$

and run a maximum-flow algorithm.)

(b) Does your algorithm work for directed graphs that contain cycles? Explain.