

Recitation 9: Time Complexity, P, and NP

April 7, 2005

Vinod Vaikuntanathan

Readings: Sections 7.1, 7.2, 7.3

Problem 1: Let's review the following new terms and concepts.

1. Time complexity. The complexity classes $TIME(f(n))$.
2. Asymptotic, worst-case analysis
3. Polynomial vs exponential bounds
4. The class P : the class of languages where membership can be *decided* in polynomial time.
5. The class NP : the class of languages where membership can be *verified* in polynomial time.

Problem 2: Let's get some practice with asymptotic bounds. Roughly, you can think of these notations as follows (see Section 7.1 for precise definitions):

1. **Big-O:** $a(n) = O(f(n))$ means that $a(n)$ is less than or equal to a constant multiple of $f(n)$ for every n , once n is sufficiently large (i.e., an "upper bound").
2. **Big-Ω:** $c(n) = \Omega(f(n))$ means that $c(n)$ is greater than or equal to a constant multiple of $f(n)$ for every n , once n is sufficiently large (i.e., a "lower bound").
3. **Θ:** $d(n) = \Theta(f(n))$ means that $d(n) = O(f(n))$ and $d(n) = \Omega(f(n))$.
4. **Small-o:** $b(n) = o(f(n))$ means that $b(n) = O(f(n))$ and $b(n) \neq \Omega(f(n))$.

Now, answer TRUE or FALSE for each of the following.

1. $n^2 = O(n^2 + n)$.
2. $2^n = 5^{O(n)}$.
3. $n^{1000000} = o(1.0000001^n)$.
4. For $c_1 < c_2$, $n^{c_1} = o(n^{c_2})$.

Problem 3: Prove that NP is closed under the star operation.

Problem 4:(NP) Let $MAXCUT = \{\{G, k\} \mid G = (V, E) \text{ is an undirected graph and } V \text{ can be partitioned into disjoint sets } V_L \text{ and } V_R \text{ such that the number of edges in } E \text{ with one endpoint in } V_L \text{ and the other in } V_R \text{ is at least } k\}$. Prove that $MAXCUT$ is in NP .

Problem 5: Describe the error in the following fallacious proof that $P \neq NP$. Consider an algorithm for the problem $3COLOR = \{\{G\} \mid G \text{ is a graph that can be colored "properly" with at most 3 colors}\}$: "On input a graph G , try all possible colorings of the nodes with 3 colors. If any of these colorings is proper, accept. Else, reject." Clearly, this algorithm requires exponential time. Thus $3COLOR$ has exponential time complexity. Therefore $3COLOR$ is not in P . Because $3COLOR$ is in NP , it must be true that $P \neq NP$. (Aha, thats it !! where is my million-dollar prize ?)¹

¹http://www.claymath.org/millennium/P_vs_NP/