6.045J/18.400J: Automata, Computability and Complexity

Prof. Nancy Lynch

Recitation 8: Oracle Turing Machines

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Vinod Vaikuntanathan

Reading: Section 6.3

Problem 1: Quiz Questions? Quiz Statistics ...

Problem 2: Define the following new terms and concepts:

- 1. An oracle for a language.
- 2. An oracle Turing machine.
- 3. Turing reducibility.
- 4. A language A is **decidable relative to** a language B.
- 5. $T^{A_{TM}}$.

Summary: If T^B decides A, then we say that A is decidable relative to B. A language A is *Turing* reducible to language B, written $A \leq_T B$, if A is decidable relative to B. If $A \leq_T B$ and B is decidable, then A is decidable.

How is this different from mapping reducibility? Turing reducibility is a generalization of mapping reducibility. If $A \leq_m B$, then $A \leq_T B$, but there is no reverse implication.

Problem 3: Let's explore a case where $A \leq_T B$, but $A \not\leq_m B$. First, recall that $E_{TM} \not\leq_m A_{TM}$. Why? We know that A_{TM} is recognizable. If $E_{TM} \leq_m A_{TM}$, then by Thm 5.22 E_{TM} would also be recognizable. However, we know that E_{TM} is *not* recognizable.

Now, let's see why $E_{TM} \leq_T A_{TM}$. Given an oracle for A_{TM} we show how to build an oracle TM to decide E_{TM} . (Details on the board; see Sipser page 212.)

Problem 4: Show $HALT_{TM}$ is decidable relative to A_{TM} .