6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 6: Decidability and Undecidability

March 10, 2005 *Vinod Vaikuntanathan*

Problem 1: These are the key concepts from lecture this week:

1. Undecidability - p. 172-176 have great example proofs.
2. Reductions - p. 171-172 will help with the terminology (e.g., "reduce A from B", etc.)
3. Computation history - p. 176, 179, 185 give the definition and some examples.
4. Diagonalization method - p. 160-168. This concept is both elegant and difficult; make sure you understand it.

Problem 2: Show that the following language is undecidable:

$$HALT_{TM} = \{< M > \mid M \text{ halts and accepts or rejects on all inputs}\}$$

Problem 3: Show that the following language is undecidable:

$$E_{TM} = \{< M > \mid M \text{ is a TM which accepts no strings}\}$$

Recall that E_{DFA} was decidable.

Problem 4: Show that the following language is undecidable:

$$EQ_{TM} = \{< M, N > \mid M \text{ and } N \text{ are TMs such that } L(M) = L(N)\}$$

Reduce from both E_{TM} and A_{TM}. Recall that EQ_{DFA} was decidable.

Problem 5: It is good practice to get a feeling for which languages are undecidable, and which variations turn them into decidable problems.

Consider the Post Correspondence Problem over small alphabets.

1. Show that the problem is decidable over the unary alphabet $\{0\}$.
2. Show that the problem is undecidable over the binary alphabet $\{0, 1\}$.

Problem 6:**(PCP Unleashed)** Some facts: PCP with one or two tiles is known to be decidable. With more than 6 tiles, it is undecidable. For $3 \leq m \leq 6$ tiles, the decidability of PCP is open !!

Let us try to prove the simplest case : PCP is decidable if we have just one tile.