6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 5: Turing Machines

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Problem 1: Define the following words, phrases and symbols.

1. Turing Machine
2. $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$
3. (Turing) Decidable Language
4. (Turing) Recognizable Language
5. (Recursively) Enumerable Language
6. Looping/ Not Halting

Problem 2: Mark each of the following statements either true or false.

1. A Turing machine has a single start state, but may have many accept states.
2. It is possible to make a Turing machine with only one state.
3. A Turing machine halts when its head reaches the end of its input.
4. All decidable languages are regular languages.
5. A nondeterministic TM with k-heads can recognize more languages than a deterministic TM with k-tapes.
6. A Turing machine might not halt on a finite input string.
7. A language L can be both co-decidable and undecidable.

Problem 3: Describe the operation of a basic Turing machine that *recognizes* the language $L = \{ww^R : w \in \{0,1\}^*\}$. Explain how the Turing machine head(s) move and mark the tape(s), without listing the specific details of each transition. (You are allowed to use multiple heads/tapes).

How would you recognize the language $\{0^n 1 0^{2n} 1 0^{3n} : n \geq 1\}$ using a 3-headed, single tape TM?

Problem 4: Assume there exists a language L that is Turing-recognizable, but $\bar{L}$ is not.

1. Construct a language L_1 that is not Turing-recognizable, but $\bar{L_1}$ is.
2. Construct a language L_2 such that neither L_2 nor $\bar{L_2}$ is Turing-recognizable.

Problem 5:[**A Different Turing Machine Model**] A Turing Machine (TM) with doubly infinite tape is similar to an ordinary TM except that its tape is infinite to the right as well as left. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of TM recognizes the class of Turing-recognizable languages.