6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Recitation 3: Regular Expressions and Non-regular Languages

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Problem 1: **Key terms.** Regular expression, generalized NFA, pigeon-hole principle, pumping lemma, pumping length, pumping up, pumping down.

Problem 2: **True or False?**

1. If L_1 and L_2 are regular, then $L_1 \cup L_2$ is regular.
2. If L_1 and L_2 are non-regular, then $L_1 \cap L_2$ is non-regular.
3. If L_1 is regular and L_2 is non-regular, then $L_1 \cup L_2$ is non-regular.
4. If L_1 is regular, L_2 is non-regular, and $L_1 \cap L_2$ is regular, than $L_1 \cup L_2$ is non-regular.
5. The following language is regular: The set of strings in $\{0, 1\}^*$ having the property that the number of 0's and the number of 1's differ by no more than 2.
6. The following language is regular: The set of strings in $\{0, 1\}^*$ having the property that in every prefix, the number of 0's and the number of 1's differ by no more than 2.

Problem 3: **Regular Expressions.** Write regular expressions for the following languages. The alphabet is $\{0, 1\}^*$.

1. $A_1 = \{w | w$ contains at least two 0's$\}$.
2. $A_2 = \{w | w$ contains an even number of 0's$\}$.
3. (from Fake HW 2.5) $A_3 = \{w | w$ does not contain 100 as a substring$\}$.

Problem 4: **Proving non-regularity: the Pumping Lemma.** Prove that the following languages are not regular.

1. $L_1 = \{0^i 1^j 0^k | \; k > i + j\}$.
2. $L_2 = \{0^i 1^j | \; j$ is a multiple of $i\}$.
3. $L_3 = \{0^i 1^j \mid i > j\}$.
4. $L_4 = \{0^i 1^j 2^k \mid i, j, k \geq 0$ and if $i = 1$ then $j = k\}$.

Problem 5: **The size of the minimal DFA for a regular language L.** Consider the regular language $L = \{w \mid w$ contains at least three $1's\}$. Prove that any DFA for this language has at least 4 states.