6.045J/18.400J: Automata, Computability and Complexity	Nancy Lynch

Homework 8.5 (FAKE)

Due: Never	*Vinod Vaikuntanathan*

If every NP-completeness proof had to be as complicated as that for SAT*, it is doubtful that the class of known NP-complete problems would have grown as fast as it has.*
– Garey and Johnson

This fake homework is intended as a study guide covering the material in lectures 20 and 21 (NP-complete problems).

Readings: Sipser, Section 7.5. Also (optionally) see Garey and Johnson's book, "Computers and Intractability: a Guide to NP-Completeness".

Problem 1: In class, we covered constructions reducing 3SAT directly to four other problems:

- CLIQUE $= \{\langle G, k\rangle |\ G$ is an undirected graph with a k-clique$\}$,
- HAMPATH $= \{\langle G, s, t\rangle |\ G$ is a directed graph with a Hamiltonian path from s to $t\}$,
- SUBSET-SUM $= \{\langle S, t\rangle |\ S = \{x_1, \ldots, x_k\}$ and for some $\{y_1, \ldots, y_\ell\} \subseteq \{x_1, \ldots, x_k\}$, we have $\Sigma y_i = t\}$, and
- 3-DIMENSIONAL-MATCHING$=\{\langle A, B, C, M\rangle |\ A, B, C$ are disjoint sets of size n, $M \subseteq A \times B \times C$, a set of acceptable triples, such that $\exists M' \subseteq M, |M'| = n$, and each element of A, B, C appears exactly once in $M'\}$.

In this problem, we propose variations on the constructions that were presented and ask you whether they work or not, and why.

1. We modify the construction reducing 3SAT to CLIQUE by adding an edge between each pair of nodes in the same triple, unless the pair is contradictory (e.g., x and $\overline{x}$).
2. In the construction reducing 3SAT to HAMPATH, we constructed a diamond for each variable. The horizontal row contains $3k + 1$ nodes in addition to the two nodes on the ends belonging to the diamond (here k is the number of clauses in ϕ). Now, we try to make the reduction more efficient by cutting out the "separator" nodes in the diamond, reducing the size of the horizontal row by $\frac{1}{3}$.
3. In the construction reducing 3SAT to SUBSET-SUM, we used multisets, by including, for each clause C_j, two copies of a vector with a 1 in the position corresponding to C_j. Now we try to avoid the use of multisets by replacing one of these copies with a vector having a 2 in the position corresponding to C_j.
4. In the construction reducing 3SAT to RELAXED-3-DIMENSIONAL-MATCHING, we include all the same Truth Assignment triples as before. But we eliminate some of the Clause Satisfaction triples: now, for each clause C_j, we include only one of the three triples $(*, b'_j, c'_j)$ that were included before.

Problem 2: Two graphs are *isomorphic* if, by renaming the nodes of one, we get a graph that is identical to the other.

Formulated as a language, the graph isomorphism problem is

$$\text{ISO} = \{\langle G, H\rangle |\ \text{G and H are undirected graphs and G and H are isomorphic }\}.$$

Now, consider a restricted version of graph isomorphism on bipartite graphs – BIPARTITE-ISO.

$$\text{BIPARTITE-ISO} = \{\langle G, H\rangle |\ \text{G and H are undirected } \textit{bipartite} \text{ graphs and G and H are isomorphic }\}.$$

Show that ISO $\leq_p$ BIPARTITE-ISO.

Problem 3: The "Set Packing" problem is defined by the language SET-PACKING, which is

$$\{\langle C, k\rangle |\ C \text{ is a collection of finite sets, } k \text{ is a positive integer, and } C \text{ contains at least } k \text{ disjoint sets}\}.$$

Prove that SET-PACKING is NP-complete, by a reduction from 3-DIMENSIONAL-MATCHING or EXACT-3-COVER.

Problem 4: (Sipser 7.27) A **coloring** of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let

$$\text{3COLOR} = \{\langle G\rangle |\ \text{the nodes of } G \text{ can be colored with three colors such that no two nodes joined by an edge have the same color}\}.$$

Show that 3COLOR is NP-complete. (Hint: Use the following three subgraphs.)