6.045J/18.400J: Automata, Computability and Complexity	Nancy Lynch

Homework 10: Fake

Due: Never — *Vinod Vaikuntanathan*

Readings: Sipser, Sections 8.1-8.4

Problem 1: (Sipser exercise 8.1) Show that for any function $f : N \to N$, where $f(n) \geq n$, the space complexity class SPACE($f(n)$) is the same whether you define the class by using the single-tape TM model or the two tape read-only TM model.

Problem 2: The japanese game *go-moku* is played by two players, "X" and "O", on a 19×19 grid. Players take turns placing markers, and the first player to achieve 5 of his markers consecutively in a row, column or diagonal, is the winner. Consider this game generalized to an $n \times n$ board. Let

$$GM = \{\langle P \rangle \mid P \text{ is a position in generalized go-moku, where player "X" has a winning strategy}\}.$$

By a *position*, we mean a board with markers placed on it, such as may occur in the middle of a play of the game. Show that $GM \in \mathsf{PSPACE}$.

Problem 3: The proof of Savitch's theorem, in Section 8.2, describes in general how one can simulate any $f(n)$-space-bounded nondeterministic Turing machine N with an $f^2(n)$-space-bounded deterministic Turing machine M. The key is a recursive computation of the CANYIELD relation, which reuses space.

Give a good upper bound on the *running time* of M on input w.

Problem 4: (Sipser 8.20) An undirected graph is *bipartite* if its nodes may be divided into two sets so that all edges go from a node in one set to a node in the other set. Show that a graph is bipartite if and only if it does not contain a cycle that has an odd number of nodes. Let

$$\mathsf{BIPARTITE} = \{\langle G \rangle \mid G \text{ is a bipartite graph }\}.$$

Show that $\mathsf{BIPARTITE} \in NL$.