6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch

Homework 1

Due: February 9, 2005 *Vinod Vaikuntanathan*

Note: Recitation meets on Thursday, Feb 3 at 10am in 34-302 and 1pm and 4pm in 34-304.

Problem 1: Let **Z** be the set of Integers. Let R be the binary relation on **Z** such that $\{a, b\} \in R$ if and only if $ab \geq 0$.

(a) Is R reflexive? Explain.

(b) Is R symmetric? Explain.

(c) Is R transitive? Explain.

(d) Define a relation $R_1 \subseteq R$ that is reflexive and symmetric but not transitive.

(e) Define a relation $R_2 \subseteq R$ that is reflexive and transitive but not symmetric.

(f) Define a relation $R_3 \subseteq R$ that is symmetric and transitive but not reflexive.

(g) Define a relation $R_4 \subseteq R$ that is an equivalence relation.

Problem 2: Construct truth tables for each of the following formulas. Also, for each pair of formulas, state which of the following holds:

- They are equivalent,
- They are not equivalent, but one implies the other (say which is which), or
- Neither of the above.

(i) $p \oplus (q \Rightarrow \neg p)$

(ii) $(q \Rightarrow \neg p) \Rightarrow \neg p$

(iii) $(p \Rightarrow q) \Rightarrow \neg p$

(iv) $p \wedge \neg q \wedge (p \Rightarrow q)$

Problem 3:

(a) Prove that for every natural number $n \geq 12$, there exist integers a, b such that $n = 3a + 7b$.

(b) A *Hamiltonian cycle* in an undirected graph is a cycle that goes through every vertex in the graph exactly once.
Suppose that G is an undirected graph that has a Hamiltonian cycle. Suppose that H is another undirected graph that is obtained from G by adding one node at a time,

along with some edges between the new node and some of the old nodes.
More precisely, we have a sequence of graphs $G = G_0, G_1, G_2, \ldots, G_k = H$, where each graph G_{i+1} is obtained from the previous graph G_i by adding one node n_{i+1}, together with edges connecting the new node n_{i+1} to strictly more than half of the nodes in the previous graph G_i. Prove that if G has a Hamiltonian cycle, H also must have a Hamiltonian cycle.