6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch, Nati Srebro

Quiz 3

April 28, 2004 *Susan Hohenberger*

Please write your name in the upper corner of each page.

Problem	Points	Grade
1	18	
2	12	
3	10	
4	20	
5	20	
6	20	
Total	100	

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Problem 1: **True, False, or Unknown (18 points).** In each case, say whether the given statement is known to be TRUE, known to be FALSE, or currently not known either way. Full credit will be given for correct answers. If you include justification for your answers, you may obtain partial credit for incorrect answers.

1. True, False, or Unknown: P $\neq$ NP.

2. True, False, or Unknown: The Hamiltonian path problem for undirected graphs is in P (i.e., UHAMPATH=$\{\langle G, s, t\rangle \mid G$ is an *undirected* graph with a Hamiltonian path from s to $t\}$).

3. True, False, or Unknown: NP $\cap$ coNP = P.

4. True, False, or Unknown: If SAT $\in$ P, then coNP $\neq$ P.

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5. True, False, or Unknown: There exists a language that is not decidable in time 3^n but is decidable in time 10^n (where n is the length of the input).

6. True, False, or Unknown: *Both* of the following exist:

   (a) A language A such that $P^A = coNP^A$.

   (b) A language B such that $P^B \neq coNP^B$.

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Problem 2: **(12 points)** The proof that SAT is NP-complete appears in Sipser's book, p. 254-259. Part of the main construction involves constructing a formula ϕ_{move}, which is expressed as the conjunction of formulas saying that 2×3 windows of the tableau are "legal". For each of the following, state whether they represent legal windows. You do not need to justify your answers.

Assume that the underlying polynomial-time NTM is of the form $(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$, where $\Sigma = \{0, 1, 2, 3\}$ and $\Gamma = \{0, 1, 2, 3, \sqcup\}$.

1. Legal or not: $\begin{array}{|c|c|c|}\hline 0 & q_1 & 1 \\ \hline 0 & 2 & q_2 \\ \hline\end{array}$, where $(q_2, 2, R) \in \delta(q_1, 1)$.

2. Legal or not: $\begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline q_2 & 1 & 2 \\ \hline\end{array}$, where $(q_2, 3, R) \in \delta(q_1, 0)$ for some q_1.

3. Legal or not: $\begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 1 & 1 & 2 \\ \hline\end{array}$, where $(q_2, 1, L) \in \delta(q_1, 0)$ for some q_1 and q_2.

4. Legal or not: $\begin{array}{|c|c|c|}\hline 0 & 1 & 2 \\ \hline 0 & 1 & 3 \\ \hline\end{array}$, where $(q_2, 3, R) \in \delta(q_1, 2)$ for some q_1 and q_2.

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Problem 3: **(10 points)** Suppose that L_1, L_2, and L_3 are nontrivial languages over $\Sigma = \{0, 1\}$. Prove that if:

1. $L_1 \leq_P L_2 \cap L_3$,
2. $L_2 \in$ NP, and
3. $L_3 \in$ P,

then $L_1 \in$ NP. You may invoke theorems proved in class and in the book, but if you do this, cite them explicitly.

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Problem 4: **(20 points)** For any k, the language k-COLOR is defined to be the set of (undirected) graphs whose vertices can be colored with at most k distinct colors, in such a way that no two adjacent vertices are colored the same color. In class, we learned that 2-COLOR $\in$ P and 3-COLOR is NP-complete.

1. Prove that 4-COLOR is NP-complete.

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2. Assuming $P \neq NP$, for which values of $k \in \mathbb{N}$ is k-COLOR NP-complete? Briefly explain.

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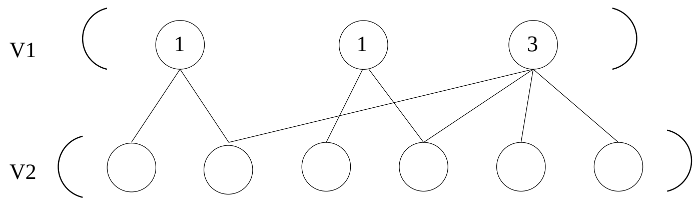
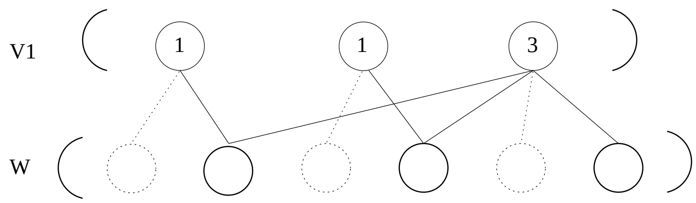
Problem 5: **(20 points)** Let $G = (V_1, V_2, E)$ be a "bipartite" undirected graph, that is, a graph whose nodes are divided into two sets, V_1 and V_2, such that every edge in E connects a node in V_1 to a node in V_2. The nodes in V_1 are all labeled with non-negative integers. For example, G might look like the following:



Some of the nodes in V_2 can be removed to leave a subset $W \subseteq V_2$. A subset W is called *consistent* with G if every node in V_1 which has been assigned a number m is connected to exactly m nodes in W. The problem is to determine whether a consistent W exists. For example, one consistent W for the graph above would be:



We formulate this problem as the language:

$MATCH = \{\langle G, N\rangle |$ G is a bipartite graph (V_1, V_2, E), N is an assignment of nonnegative integers to V_1, and there exists $W \subseteq V_2$ that is consistent with G and $N\}$.

(a) Show that $MATCH$ is in NP, using the certificate and verifier method.

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(b) Show that $MATCH$ is NP-hard, using a reduction from 3SAT.

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Problem 6: **(20 points)** Suppose that we are given a polynomial time algorithm M (formally, a basic deterministic TM) that decides membership in the language VERTEX-COVER = $\{\langle G, k\rangle \mid G$ is an undirected graph and G has a vertex cover of size $\leq k\}$.

1. Describe a polynomial time algorithm that, given the representation of an undirected graph G, finds the size of the smallest vertex cover of G. Your algorithm may use M as a "subroutine". Explain why your algorithm takes polynomial time.

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2. Describe a polynomial time algorithm that, given the representation of an undirected graph $G = (V, E)$, finds a smallest-size vertex cover of G (that is, a subset $V' \subseteq V$ such that for each edge $\{u, v\} \in E$ at least one of u and v belongs to V').