6.045J/18.400J: Automata, Computability and Complexity Prof. Nancy Lynch, Nati Srebro

Recitation 9: Time Complexity, P, and NP

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Readings: Sections 7.1, 7.2, 7.3

Problem 1: Let's review the following new terms and concepts.

1. Time complexity
2. Asymptotic, worst-case analysis
3. Polynomial vs exponential bounds
4. The class P: the class of languages where membership can be *decided* in polynomial time.
5. The class NP: the class of languages where membership can be *verified* in polynomial time.

Problem 2: Let's get some practice with asymptotic bounds. Roughly, you can think of these notations as follows (see Section 7.1 for precise definitions):

1. **Big-O:** $a(n) = O(f(n))$ means that $a(n)$ is less than or equal to a constant multiple of $f(n)$ for every n, once n is sufficiently large (i.e., an "upper bound").
2. **Big-Ω:** $c(n) = \Omega(f(n))$ means that $c(n)$ is greater than or equal to a constant multiple of $f(n)$ for every n, once n is sufficently large (i.e., a "lower bound").
3. **Θ:** $d(n) = \Theta(f(n))$ means that $d(n) = O(f(n))$ and $d(n) = \Omega(f(n))$.
4. **Small-o:** $b(n) = o(f(n))$ means that $b(n) = O(f(n))$ and $b(n) \neq \Omega(f(n))$.

Problem 3: Prove that NP and P are closed under the star operation.

Problem 4: Let $MAXCUT = \{\langle G, k\rangle |\ G = (V, E)$ is an undirected graph and V can be partitioned into disjoint sets V_L and V_R such that the number of edges in E with one endpoint in V_L and the other in V_R is at least $k\}$. Prove that $MAXCUT$ is in NP.