6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch, Nati Srebro

Homework 6

Due: April 7, 2004 *Susan Hohenberger*

Reading: Sipser, Section 6.3

For the next two problems, you will have to consider some new definitions that were not covered in lecture, but are presented in Section 6.3 of Sipser's book. Namely, an *Oracle Turing Machine* is a variant of a Turing machine that has the ability to query an external source – an "oracle" – about membership of a string in some particular *oracle language.* A language is *decidable relative to* a language L if it can be decided by an oracle Turing machine that uses L as its oracle set.

Problem 1: (From Sipser Problem 6.20.)
Recall the Post Correspondence Problem discussed in class and in Section 5.2 of Sipser. Show that PCP is decidable relative to A_{TM}, the acceptance problem for basic Turing machines.

Problem 2: (From Sipser Problem 6.22.)
Let $Z = \{\langle M, w\rangle \mid M$ is an oracle Turing machine and $M^{A_{TM}}$ accepts $w\}$. Thus, Z can be thought of as the "acceptance problem for oracle TMs relative to the acceptance problem for ordinary TMs".

Use a proof by diagonalization to show that Z is not decidable relative to A_{TM}. That is, even with an oracle for the ordinary acceptance problem, the relative version of the acceptance problem still cannot be decided!

(Hint: Use techniques similar to the proof by diagonalization for the undecidability of the ordinary acceptance problem that we saw in lecture and Sipser discusses on page 165.)