6.045J/18.400J: Automata, Computability and Complexity Nancy Lynch, Nati Srebro

Homework 5

Due: March 17, 2004 *Susan Hohenberger*

Problem 1: Prove that the following languages are undecidable. Use reductions from A_{TM} or other problems already known to be undecidable. Here, $\Sigma = \{0, 1\}$.

1. $L_1 = \{< M >: M$ is a Turing machine and M accepts the empty string $\epsilon\}$.
2. $L_2 = \{< M >: M$ is a Turing machine and $L(M) = \Sigma^*\}$.
3. $L_3 = \{< M >: M$ is a Turing machine and M accepts exactly the even-length strings in $\Sigma^*\}$.

Problem 2: (From Sipser, problem 4.21)
Let $A = \{< D_1 >, < D_2 >, < D_3 >, \dots\}$ be an infinite language consisting of representations of Turing machines that are deciders, that is, each machine D_i halts (accepts or rejects) on every input. Suppose that A is Turing-recognizable, and therefore, enumerable by an enumerator machine E.

Show that there must be some decidable language that is not decided by any of the machines represented in A (i.e., some language $L(D')$ that is decided by a machine D' such that $< D' > \notin A$).

Note that this time, countability arguments are not going to help: there are only countably many machine descriptions in A, but then again, there are also only countably many decideable languages. Still, A cannot contain descriptions of deciders for *all* decideable languages.

(Hint: Recall the diagonalization method; try constructing D' using the enumerator E.)

Problem 3: (From Sipser, exercise 5.3)
Find a match in the following instance of the PCP. Please provide a match with less than 15 characters on the top row.

$$\left\{ \left[\frac{ab}{abab}\right], \left[\frac{b}{a}\right], \left[\frac{aba}{b}\right], \left[\frac{aa}{a}\right] \right\}$$

Problem 4: Consider the machine M_2 on page 132 of Sipser's book, which recognizes the language consisting of all strings of 0s whose length is a power of 2.

1. Write out the accepting computation history for the machine M_2 on input 00.
2. What are the dominoes for the instance of the Post Correspondence Problem defined for M_2 and input 00?
3. Write out your computation history from part (a) twice, one copy above the other. Draw lines indicating how your dominoes from part (b) can be used to establish a correspondence between these two copies.