6.045J/18.400J: Automata, Computability and Complexity	Prof. Nancy Lynch
Homework 2.5 (FAKE)	
Due: never	*Susan Hohenberger*

This "fake" homework is intended as a study guide covering the material on lectures 4 and 5.

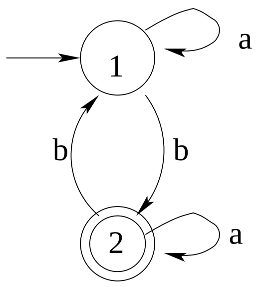
Problem 1: Taken from Sipser 1.13. Give regular expressions generating the following languages. In all cases the alphabet is $\{0, 1\}$.

1. $L_1 = \{w|w$ contains at least three 1s$\}$.
2. $L_2 = \{w|w$ has length at least 3 and its third symbol is a $0\}$.
3. $L_3 = \{w|w$ doesn't contain the substring $110\}$.
4. $L_4 = \{w|$ every odd position of w is a $1\}$.
5. $L_5 = \{w|w$ contains at least two 0s and at most one $1\}$.

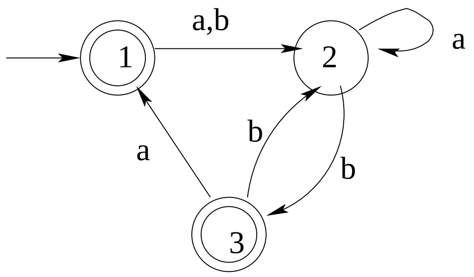
Problem 2: Sipser 1.14. Use the procedure described in Lemma 1.29 to convert the following regular expressions to nondeterministic finite automata.

1. $(0 \cup 1)^*000(0 \cup 1)^*$
2. $(((00)^*(11)) \cup 01)^*$
3. $\emptyset^*$

Problem 3: Sipser 1.16. Use the procedure described in Lemma 1.32 to convert the following finite automata to regular expressions.



(a)



(b)

Problem 4: Part of Sipser 1.17. Use the pumping lemma to show that the following languages are not regular.

1. $A_1 = \{0^n 1^n 2^n | \; n \geq 0\}$.
2. $A_2 = \{a^{2^n} | \; n \geq 0\}$. (Here, a^{2^n} means a string of 2^n a's.)

Problem 5: Sipser 1.18. Describe the error in the following "proof" that 0^*1^* is not a regular language. (An error must exist because 0^*1^* *is* regular.) The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Choose s to be the string 0^p1^p. You know that s is a member of 0^*1^*, but Example 1.38 (in text) shows that s cannot be pumped. Thus you have a contradiction. So 0^*1^* is not regular.

Problem 6: Similar to Sipser 1.36. Let $\Sigma = \{1, =, +\}$ and

$$ADD_{UNARY} = \{x = y + z |\ x, y, z \text{ are written in unary, and } x \text{ is the sum of } y \text{ and } z\}.$$

For example, the unary representation of 3 is '111', and the string '111111=111+111' is in ADD_{UNARY}. Show that ADD_{UNARY} is not regular.