6.045J/18.400J: Automata, Computability and Complexity	Prof. Nancy Lynch

Homework 2

Due: 18 February 2004 — *Susan Hohenberger*

Problem 1: **Designing DFAs and NFAs.** For each of the following, draw a state diagram for a DFA or NFA (as required) that recognizes the specified language. In all cases the alphabet is $\{0, 1\}$.

(a) Sipser exercise 1.4, part e. $L_a = \{w|w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$. Provide a DFA recognizing L_a.

(b) Sipser exercise 1.4, part i. $L_b = \{w|$ every odd position of w is a 1$\}$. Provide a DFA recognizing L_b.

(c) Sipser exercise 1.5, part b. $L_c = \{w|w$ contains the substring 0101, i.e., $w = x0101y$ for some x and $y\}$. Provide an NFA recognizing L_c.

(d) Sipser exercise 1.5, part c. $L_d = \{w|w$ contains an even number of 0s, or exactly two 1s$\}$. Provide an NFA with **exactly six states** that recognizes L_d.

(e) The language described in Sipser problem 1.26. Let

$$\Sigma_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}.$$

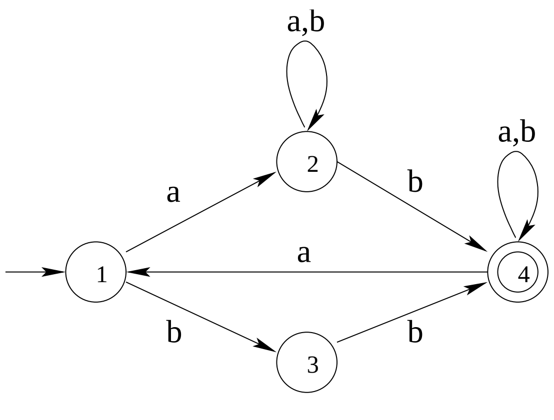
Here, Σ_2 contains all columns of 0s and 1s of height two. A string of symbols in Σ_2 gives two rows of 0s and 1s. Consider each row to be a binary number and let

$$L_e = \{w \in \Sigma_2^*|\text{ the bottom row of } w \text{ is three times the top row}\}.$$

For example, $\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \in L_e$, but $\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} \notin L_e$. Provide an NFA that recognizes L_e.

Problem 2: **Proving an FA recognizes a language.** For one of the automata you designed in problem 1, prove that the machine recognizes *exactly* the specified language. To do this, you will need to prove that your automaton (1) accepts all strings in L_i and (2) does not accept any string not in L_i.

Problem 3: **NFA to DFA.** Consider the following state diagram.



(a) The state diagram above represents an NFA $N = (Q, \Sigma, \delta, q_0, F)$. Say what each of the components of the 5-tuple is, for this NFA.

(b) Apply the subset construction described in class to obtain a DFA $M = (Q', \Sigma, \delta', q_0', F')$ that is equivalent to N. State the corresponding element(s) in Q', δ', q_0', F', then describe δ' via either a transition table or a state diagram.

Problem 4: **Regular languages are closed under the star operation.** Sipser exercise 1.11. Give a counterexample to show that the following construction fails to prove Theorem 1.24, the closure of the class of regular languages under the star operation.[1] Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

(a) The states of N are the states of N_1.

(b) The start state of N is the same as the start state of N_1.

(c) $F = \{q_1\} \cup F_1$. The accept states F are the old accept states plus its start state.

(d) Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q, a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \end{cases}$$

(Suggestion: convert this formal construction to a picture, as in Figure 1.26 of Sipser.)

Problem 5: **Showing closure under other operations.** Similar to Sipser problem 1.32. Say that a string x is a **prefix** of string y if a string z exists where $xz = y$ and that x is a **proper prefix** of y if in addition $x \neq y$. In each of the following parts we define an operation on a language A. Show that the class of regular languages is closed under that operation.

(a) $\text{NOPREFIX}(A) = \{w \in A |$ no proper prefix of w is a member of $A\}$.

(b) $\text{HASPREFIX}(A) = \{w \in A |$ at least one proper prefix of w is a member of $A\}$.

(c) $\text{NOSUFFIX}(A) = \{w \in A |$ no proper suffix of w is a member of $A\}$.

[1] In other words, you must present a finite automaton, N_1, for which the constructed automaton N does not recognize the star of N_1's language.