6.045J/18.400J: Automata, Computability and Complexity	Prof. Nancy Lynch

Homework 1

Due: February 11, 2004 — *Susan Hohenberger*

Note: Recitation meets on Thursday, Feb 5 at 10am in 34-302, and 1pm and 4pm in 34-304.

Problem 1: Construct truth tables for each of the following formulae. Additionally, for each pair of formulae, state which of the following holds:

- They are equivalent,
- They are not equivalent, but one implies the other (make sure to state which is which), or
- Neither of the above:

(a) $(p \oplus q) \Rightarrow p$

(b) $p \vee (\neg q \Rightarrow p)$

(c) $(p \vee q) \Rightarrow p$

(d) $p \wedge q \wedge (p \Rightarrow \neg q)$

Problem 2: Fix any finite set S and let the power set of S be denoted as $P(S)$. Let R be the relation between elements of $P(S)$ such that ARB if and only if there is a bijection between A and B.

(a) Show that R is an equivalence relation.

(b) Define a relation $R_1 \subseteq R$ that is reflexive and symmetric but not transitive.

(c) Define a relation $R_2 \subseteq R$ that is reflexive and transitive but not symmetric.

(d) Define a relation $R_3 \subseteq R$ that is symmetric and transitive but not reflexive.

Problem 3: Practicing Proofs.

Part (a). Prove that for every natural number $n \geq 108$, there exist integers a, b such that $n = 7a + 9b$.

Part (b). Let function $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+$ be defined recursively as follows:

$$\forall n, k \geq 0,\ f(n,k) = \begin{cases} 1, & k = 0 \\ 1, & k = n \\ f(n-1,k) + f(n-1,k-1), & \text{otherwise.} \end{cases}$$

Prove that for every $n, k \geq 2$, $f(n, k)$ is equal to the binomial coefficient ("n choose k"), which is defined as

$$\frac{n!}{k!(n-k)!}$$

(HINT: use strong induction on *both* variables.)