Secret Messages in Juggling and Card Shuffling

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Abstract

We present several puzzles around the idea of hiding messages within two types of mathematical performance: juggling and card shuffling.

1 Introduction

Over the past dozen years, we have developed several different typefaces/fonts that express text through mathematical theorems or open problems in broadly accessible forms, often through the use of puzzles. The fonts are all free to play with on the web.[1]

Two of our most recent such fonts are based on ball-juggling patterns and perfect (Faro) shuffling of cards. These fonts were originally prepared for the 80th birthday party of Ron Graham [DD], who Martin Gardner wrote about on several occasions and to whom Martin dedicated his book *Wheels, Life and Other Mathematical Amusements*. Here we explore how to develop standalone puzzles based on variations of these fonts.

2 Juggling Font Puzzles

Our juggling font[2] consists of one three-ball juggling pattern for each letter of the alphabet. The idea is that the trajectories of the balls trace out the desired letter. But seeing just the pattern animated, it is a puzzle to figure out the letter:

Puzzle 1 *The juggling patterns in Figure 1 spell what word?*

Puzzle 2 *The juggling patterns in Figure 2 spell what word?*

3 Card Shuffling Font Puzzles

In our card-shuffling font,[3] the magician starts with a sorted deck of 26 cards labeled A through Z, and given a word (e.g., from the audience), the magician can bring the word's letters to the top of the deck, one at a time in order, by repeatedly shuffling the deck. The trick is to use *perfect shuffles* (often called *Faro shuffles*), which are riffle shuffles where the cards exactly alternate between the two exact halves of the deck [Gar89]. The magician carefully chooses between making *outside* and

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[1] http://erikdemaine.org/fonts/

[2] http://erikdemaine.org/fonts/juggling/

[3] http://erikdemaine.org/fonts/shuffle/

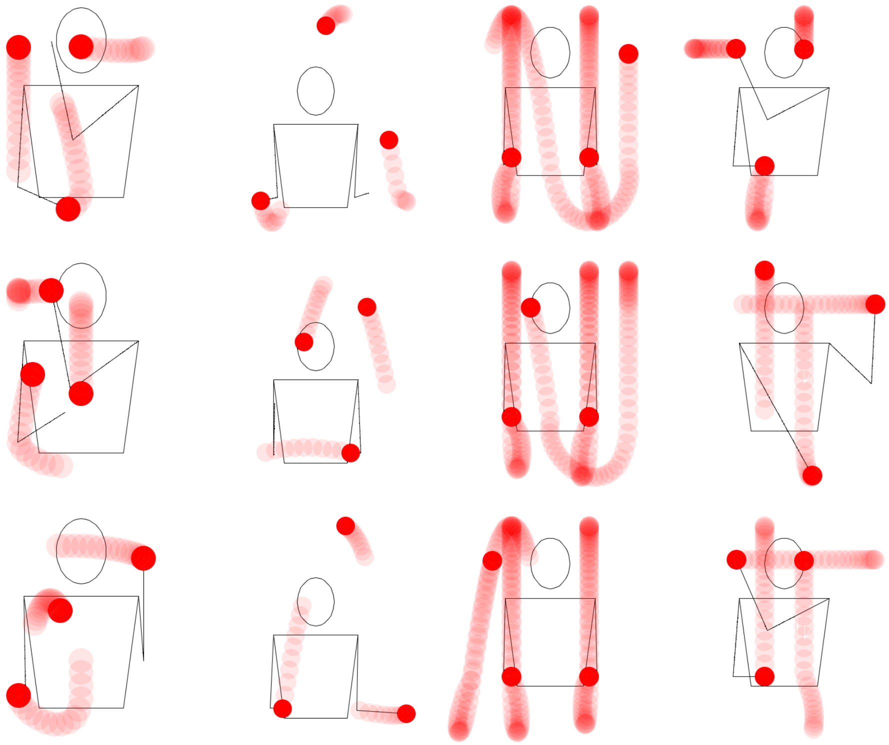

Figure 1: Four juggling patterns representing four letters, with time looping vertically. Underlying animations produced with Juggling Lab [B+14].

inside perfect shuffles, that is, choosing which half of the deck to start the shuffle with in order to keep the outside (top and bottom) cards on the outside or move them one card inside, respectively. Using an algorithm of Diaconis and Graham [DG07], any card can be brought to the top within $\lceil \log_2 26 \rceil = 5$ perfect shuffles. Figure 3 shows how this works to produce the initials of Martin Gardner.

Puzzle 3 *Starting from a sorted deck of 26 letters (A on top, Z on bottom), what card is on top after one inside perfect shuffle?*

Puzzle 4 *Starting from a sorted deck of 26 letters (A on top, Z on bottom), what card is on top after two inside perfect shuffles?*

A new and different type of trick involves stacking (re-arranging) the deck so that few operations produce large effects.

Puzzle 5 *Find a (nearly sorted) arrangement of a deck of 26 letters so that, after one inside perfect shuffle, the top cards are M, A, T, H, with M on top.*

Puzzle 6 *Find a (nearly sorted) arrangement of a deck of 26 letters so that, after one inside perfect shuffle, the top cards are M, A, R, T, I, N, with M on top.*

Puzzle 7 *Find a (nearly sorted) arrangement of a deck of 26 letters so that, after one inside perfect shuffle, the top cards are G, A, T, H, E, R, with G on top.*

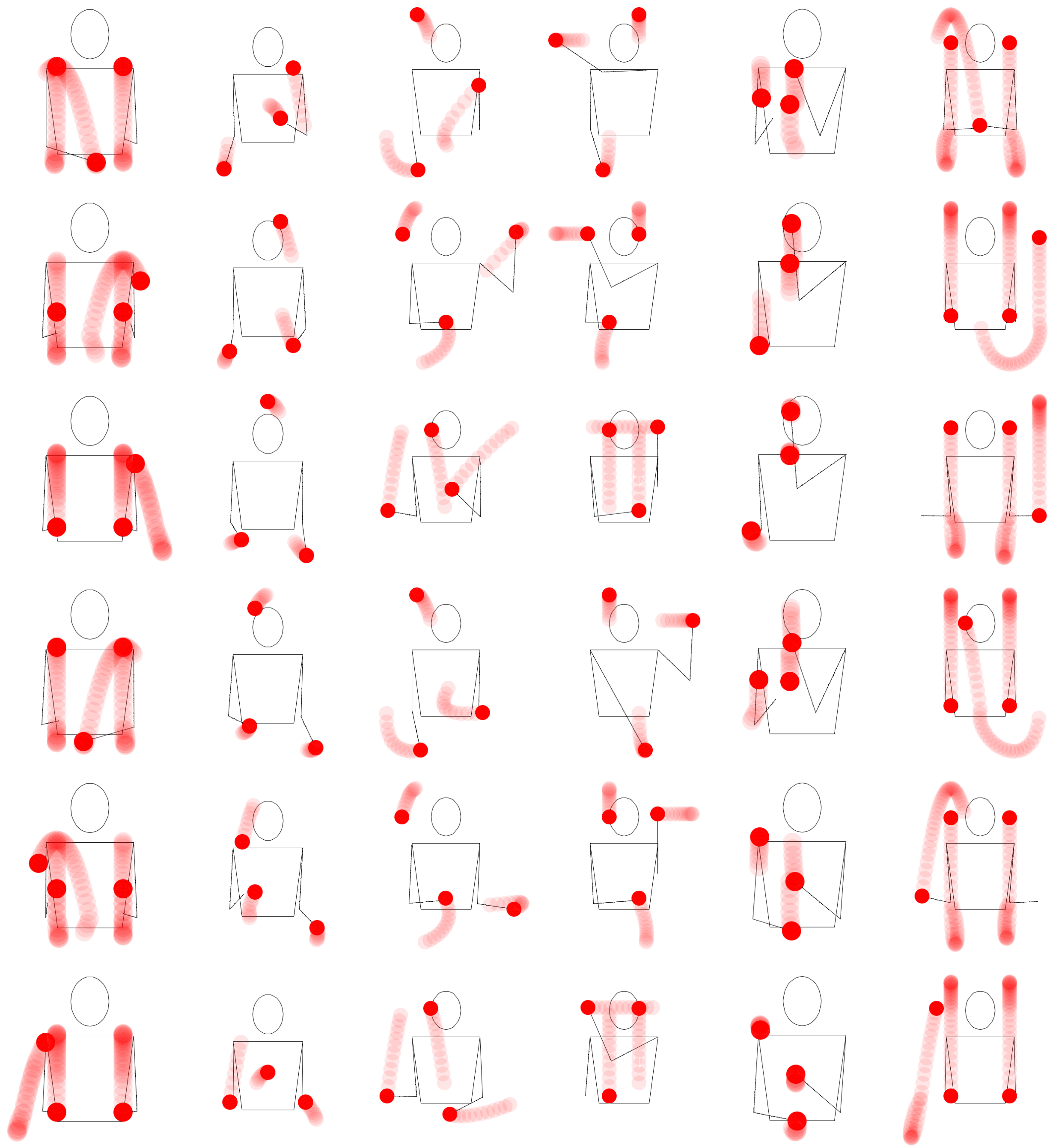

Figure 2: Six juggling patterns representing six letters, with time looping vertically. Underlying animations produced with Juggling Lab [B+14].

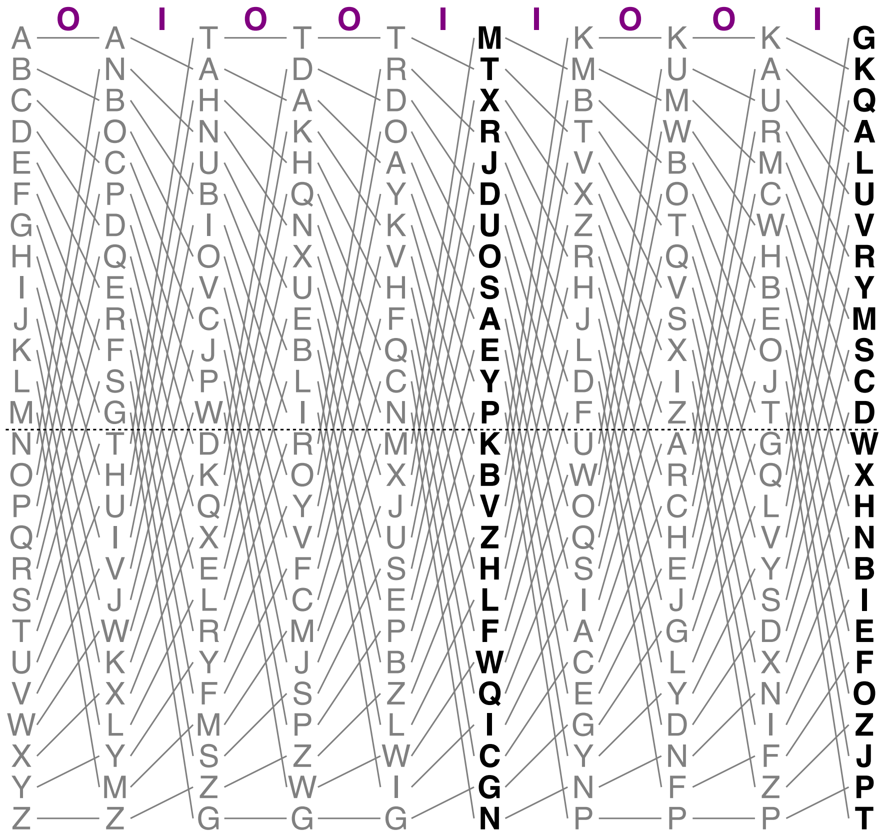

Figure 3: Starting from the sorted alphabet deck on the left, five perfect shuffles bring the letter M to the top, and four subsequent shuffles bring the letter G to the top. I and O denote inside and outside perfect shuffles, respectively.

References

[B+14] Jack Boyce et al. Juggling lab. http://jugglinglab.sourceforge.net/, 2014.

[DD] Erik D. Demaine and Martin L. Demaine. Juggling and card shuffling meet mathematical fonts. In *Connection in Discrete Mathematics: A celebration of the work of Ron Graham*. Cambridge University Press. To appear.

[DG07] Persi Diaconis and Ron Graham. The solutions to Elmsley's problem. *Math Horizons*, 14:22–27, February 2007.

[Gar89] Martin Gardner. Card shuffles. In *Mathematical Carnival*, chapter 10. Mathematical Association of America, 1989.

Puzzle Solutions

1. FONT • 2. MARTIN • 3. N • 4. G • 5. AHCDEFGBIJKLNMTPQRSOUVWXYZ
6. ATNDEFGHPIJKLCMRIQOSBUVWXYZ • 7. AHRDPFNBIJKLMGTEQCSOUVWXYZ